



# Integrated Optimisation of Train Timetables and Maintenance Windows under Mixed Passenger and Freight Train Operation Mode

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## ABSTRACT

The train timetable and maintenance windows are closely interrelated and mutually constrained, creating a coupled relationship. Optimising the train timetable can greatly enhance the operational efficiency of trains. This study focuses on the mixed passenger-freight train operation mode and investigates the integrated optimisation of train timetables and maintenance windows in this context. The problem is modelled as a multi-objective mixed-integer programming problem. The model's objectives are twofold: to minimise the total travel time of all trains and to maximise the density of train paths. Several constraints are incorporated, including those related to train stopping, train safety intervals, passenger train departure time windows and maintenance windows. These constraints ensure that the model reflects practical operational requirements while achieving optimal efficiency. The model places particular emphasis on station capacity constraints. To facilitate solving, these constraints are linearised. A case study is conducted to compare scenarios with and without considering station capacity constraints. The results demonstrate the effectiveness of the proposed model. This study provides theoretical support for the integrated optimisation of train timetables and maintenance windows under mixed passenger-freight train operation mode and offers valuable insights for improving the efficiency of railway transportation.

## KEYWORDS

integrated optimisation; train timetable; maintenance windows; mixed passenger and freight train; multi-objective mixed-integer programming model.

## 1. INTRODUCTION

China's demand for railway transportation continues to grow, the network coverage is expanding, and railway transport capacity has significantly improved. Against this backdrop, the mixed passenger-freight train operation mode has become one of the key organisational forms in China's railway transportation. As the core scheduling tool of railway operations, the train timetable is directly related to the transport capacity and operational efficiency of railway lines, thereby imposing higher requirements on the coordinated planning of train timetables and maintenance windows. However, train timetables and maintenance windows are often planned separately, which can lead to uneven utilisation of railway resources, increased train waiting times and insufficient maintenance window durations. How to achieve an integrated optimisation of train timetables

and maintenance windows, fully considering the operational needs of passenger and freight trains as well as the requirements for track maintenance, has become a key research focus in the academic community. Such optimisation not only meets transportation demands but also improves the utilisation efficiency of railway lines.

The optimisation of train timetables has been widely explored by researchers both in China and abroad. These studies can generally be divided into two main categories: the development of optimisation models for train timetables and the creation of efficient algorithms for solving these models. In terms of optimisation models, Jiang et al. [1] and Yang et al. [2] constructed a space-time network to transform train timetable planning into a path optimisation problem, effectively addressing the complexities of freight train scheduling. Zhang et al. [3] proposed a collaborative optimisation linear model aimed at maximising satisfaction with train arrival and departure times while minimising the disruptions caused by maintenance windows. To achieve this, they used big-M constants and binary variables to linearise the constraints. Several studies have focused on minimising total train travel time. For instance, Zhang et al. [4], Lan et al. [5], Yang et al. [6], Mu et al. [7] and Li et al. [8] developed models that incorporate various constraints, including train running times in sections, station dwell times, headway intervals, integrated maintenance windows and reasonable time limits. For research involving maintenance windows in train timetable optimisation, Liu Min et al. [9], Peng et al. [10], Boland et al. [11] and Zhang et al. [12] mainly focused on minimising maintenance costs. However, other researchers addressed dual objectives, such as reducing total train travel time and minimising maintenance operation delay costs. For example, LIDÉN et al. [13] and Yang et al. [14] developed bi-objective optimisation models for train timetable planning, using linear weighting methods to convert the bi-objective problem into a single-objective problem for easier solution.

The solution methods for train timetable optimisation models can generally be categorised into two main approaches. Due to the high complexity of these models, many researchers have developed efficient algorithms tailored to the specific characteristics of the problem. Xu et al. [15] and Yang et al. [16-17] proposed a mixed-integer programming model for the integrated optimisation of train timetables and maintenance windows. To tackle the complexity, they designed a two-stage solution approach. In the first stage, a heuristic algorithm based on expert knowledge was applied to generate an initial timetable framework. In the second stage, a tabu search algorithm was used to refine this framework and identify the global optimal solution. Zhang et al. [18] introduced an iterative algorithm that decomposes the main problem into smaller subproblems, focusing separately on train scheduling with or without maintenance tasks. This approach enabled them to achieve near-optimal solutions within shorter computation times. Zhao et al. [19], Ni et al. [20], Guo et al. [21] and Meng et al. [22] proposed a solution method based on an enhanced particle swarm optimisation algorithm, which accounted for the specific features of the model. Xu et al. [23] developed a space-time state network-based integrated scheduling model for train timetables and high-speed train fleet deployment. They solved the model using a Lagrangian heuristic algorithm. Additionally, Shi et al. [24] focused on operations on single-track railways. They established a sequencing model for single-line train timetables and solved it using a time-cyclic optimisation method.

Some researchers have utilised commercial solvers to solve optimisation models after preprocessing the problem. For instance, Yang et al. [25] and Wang et al. [26] proposed a mixed-integer programming model aimed at minimising the total delay between actual and scheduled train departure times. They used the Cplex solver to obtain solutions, successfully validating the model's correctness. Similarly, Mi et al. [27] developed an integer linear programming model for the integrated optimisation of high-speed railway timetables and freight assembly plans under a mixed passenger-freight operation scenario. The model was applied to a case study on the Beijing-Shanghai High-Speed Railway and solved using Cplex. Zhang et al. [4] and Li et al. [28] addressed the integrated scheduling problem of maintenance windows and double-track railway timetables. They adopted a column generation method in combination with the commercial solver Gurobi to solve their model, effectively tackling the complexity of the problem.

In summary, most existing literature treats the optimisation of train timetables and maintenance window scheduling as separate objectives. In these studies, the timetable planning problem is often considered the primary challenge, with maintenance windows treated as known conditions or model inputs, thus downplaying the importance of maintenance window planning. In contrast, this paper proposes an integrated optimisation model for train timetables and maintenance windows under a mixed passenger-freight operation mode. Specifically, by considering train categories, the objective is to minimise the total travel time of all trains while maximising the density of the operational timetable. An integrated optimisation model for the passenger and freight train timetable and maintenance window scheduling is therefore developed.

## 2. PROBLEM DESCRIPTION AND NOTATION

### 2.1 Problem description

To enhance the quality of train timetable planning, this paper adopts an integrated optimisation approach for train timetables and maintenance windows. The arrival and departure times of trains at stations, along with the start and end times of maintenance windows, are treated as decision variables. By formulating an objective function and corresponding constraints, this method enables the determination of an overall optimal train timetable solution.

The traditional step-by-step approach for compiling train timetables and maintenance windows often leads to numerous inflexibilities in practical railway operations. In sections with high train operation density, when a maintenance window is required, constraints on train operations often result in significant waiting times for trains at stations. On the other hand, in sections with lower train density, or even no trains passing through, if maintenance windows are not optimally scheduled, it leads to a waste of available track time resources. As shown in Figure 1(a), where  $t$  represents a train (with Train 1 and Train 3 being freight trains and Train 2 and Train 4 being passenger trains) and  $S$  represents a station, in a pre-established timetable, to avoid conflicts with trains, the maintenance window can only be scheduled between Train 2 and Train 3, which results in a short maintenance window with insufficient time for maintenance and potential safety risks. However, by adopting an integrated optimisation approach for both train timetables and maintenance windows, where both the train departure times at stations and the maintenance window start and end times are treated as adjustable variables, it allows for more flexible coordination between the timetable and maintenance schedules. This integrated approach enables simultaneous optimisation of train departure times and maintenance window schedules. Additionally, it better accommodates the needs of passenger and freight trains, which often have different track requirements. Passenger trains typically have higher priority and occupy tracks for shorter periods, while freight trains require longer durations. Through integrated optimisation, the spatial-temporal network can be more effectively coordinated to meet the transportation demands of both passenger and freight trains, ensuring efficient, stable and safe operations. As shown in Figure 1(b), by adjusting the station dwell time of Train 1 at Station 3, Train 2 can overtake Train 1, thereby reasonably adjusting the train timetable. This not only maintains train operational efficiency but also provides more time for the maintenance window.

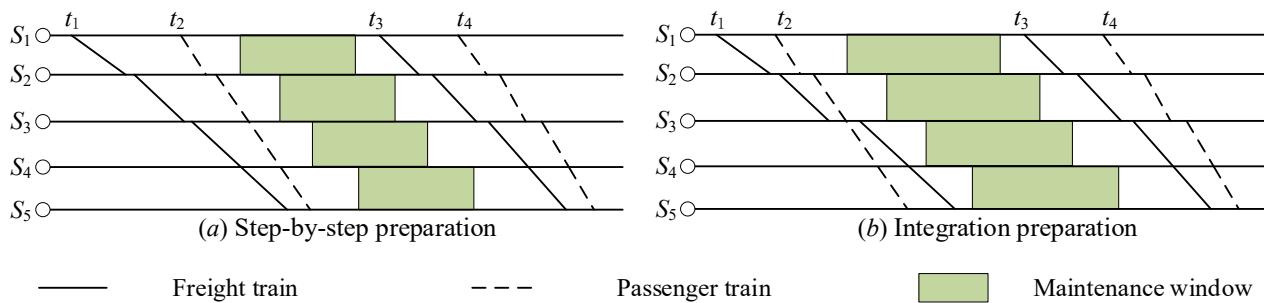


Figure 1 – Step-by-step compilation and integrated compilation of train timetables and maintenance windows

### 2.2 Model parameters and variables

- 1) Symbolic representation of sets, indices and decision variables in the model.

Table 1 – Symbolic representation of sets

Sets	Definition
$T$	Set of all trains
$K$	Set of stations
$P$	Set of maintenance windows
$L_k$	Set of departure and arrival tracks at station $k$

Table 2 – Symbolic representation of indices

Indices	Definition
$i, j$	Index of trains, $i, j \in T$
$k$	Index of stations, $k \in K$
$p$	Index of maintenance windows, $p \in P$
$l_k$	Index of departure and arrival tracks at stations, $l_k \in L_k$

Table 3 – Symbolic representation of decision variables

Decision variables	Definition
$d_{ik}$	Departure time of train $i$ from station $k$
$a_{ik}$	Arrival time of train $i$ from station $k$
$mot_p^s$	Start time of the maintenance window $p$
$mot_p^e$	End time of the maintenance window $p$

## 2) Symbolic representation of parameters in the model.

Table 4 – Symbolic representation of parameters

Parameters	Definition
$S_i^k$	Binary variable indicating whether train $i$ stops at station $k$ (1 if it stops, 0 otherwise).
$r_{ik}$	Travel time of train $i$ on section $(k, k+1)$ .
$\beta_i$	Additional time for train $i$ start-up.
$\gamma_i$	Additional time for train $i$ stops.
$y_{ik}$	Minimum required dwell time of train $i$ at station $k$ .
$\omega_{ij}^k$	Binary variable indicating whether train $i$ departs station $k$ later than train $j$ (1 if $i$ departs later than $j$ , 0 otherwise).
$O_{ij}^k$	Binary variable indicating whether train $i$ arrives at station $k$ later than train $j$ (1 if train $i$ arrives later than train $j$ , 0 otherwise).
$a_k$	Minimum time interval required between the arrivals of two consecutive trains at a station.
$d_k$	Minimum time interval required between the departures of two consecutive trains from a station.
$td_{ik}^-$	The earliest possible start time for a passenger train at its origin station.
$td_{ik}^+$	The latest possible start time for a passenger train at its origin station.
$\lambda_i^{mot_p}$	Binary variable indicating whether train $i$ departs station $k$ before the start of maintenance window $p$ (1 if train $i$ departs before the maintenance window starts, 0 otherwise).
$mot_p$	Duration of maintenance window $p$ .
$mot^s$	Earliest allowable start time of maintenance window $p$ .
$mot^e$	Latest allowable end time of maintenance window $p$ .

$M$	A sufficiently large positive integer.
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### 3. MATHEMATICAL MODEL

#### 3.1 Model assumptions

In constructing the integrated optimisation model for train scheduling and maintenance windows under the mixed passenger-freight train operation mode, the following assumptions are made:

- 1) The research focuses on the mixed passenger-freight train operation mode.
- 2) The train operation and stopping plans are known.
- 3) The various parameters required for creating the train timetable are known, including the names of stations along the route, the section distances and the number of tracks at each station.
- 4) The model developed is applicable only to the train timetable formulation during the planning phase, and not to the adjustment or re-planning of the train timetable.

#### 3.2 Systematic constraints

The model developed in this paper involves a significant number of constraints, and this section will provide a detailed explanation of them.

- 1) Interval minimum running time constraint.

The train's operation on a track section must satisfy the minimum running time requirement for that section. Specifically, the arrival time of train  $i$  at station  $k+1$  minus its departure time from station  $k$  must be no less than the minimum running time for the section. When train  $i$  departs from station  $k$ , the running time for the section should be increased by the additional startup time  $\beta_{ik}$ . When the train  $i$  stops at station  $k+1$ , the running time for the section should be increased by the additional dwell time  $\gamma_{ik+1}$ .

$$a_{ik+1} - d_{ik} \geq r_{ik} + \beta_{ik} \cdot S_{ik} + \gamma_{ik+1} \cdot S_{ik+1} \quad \forall i \in T, k \in K \quad (1)$$

- 2) According to regulations, trains must stop at certain necessary stations.

Based on the train's requirements, the train must stop at certain necessary stations to perform related operations.  $S_i^k$  is a 0-1 variable, indicating that train  $i$  stops at station  $k$ , and 0 indicates that it does not stop.

$$S_i^k = \{ 0, 1 \} \quad \forall i \in T, k \in K \quad (2)$$

- 3) Train dwell time constraint.

The dwell time of a train is limited to a reasonable time range. After the train arrives at the station and completes the required operations, it can depart. For trains without any stopping operation requirements, the minimum dwell time is 0. The corresponding dwell time constraint can be described as:

$$d_{ik} - a_{ik} \geq S_i^k \cdot y_{ik} \quad \forall i \in T, k \in K \quad (3)$$

- 4) Train safety interval constraint.

To ensure the safety of train operations, the departure times of trains must meet certain minimum departure time interval requirements, as shown in *Figure 2*. When trains  $i$  and  $j$  depart from station  $k$ , *Equation (4)* represents the time interval between the departure of train  $i$  before train  $j$ , where  $\omega_{ij}^k = 1$  implies a time interval of  $d_{ik} - d_{jk} \geq d_k$ . *Equation (5)* represents the time interval when train  $j$  departs before train  $i$ , where  $\omega_{ji}^k = 1$  implies a time interval of  $d_{jk} - d_{ik} \geq d_k$ . *Equation (6)* ensures that either  $\omega_{ij}^k$  or  $\omega_{ji}^k$  is equal to 1, but not both simultaneously.

$$d_{ik} - d_{jk} + (1 - \omega_{ij}^k) \cdot M \geq d_k \quad \forall i, j \in T \quad k \in K \quad (4)$$

$$d_{jk} - d_{ik} + (1 - \omega_{ji}^k) \cdot M \geq d_k \quad \forall i, j \in T \quad k \in K \quad (5)$$

$$\omega_{ij}^k + \omega_{ji}^k = 1 \quad \forall i, j \in T \quad k \in K \quad (6)$$

Similarly, the arrival times of trains must meet certain minimum arrival time interval requirements. When trains  $i$  and  $j$  are scheduled to arrive at station  $k$ , Equation (7) represents the time interval between the arrival of train  $i$  before train  $j$ , where  $O_{ji}^k = 1$  implies a time interval of  $a_{jk} - a_{ik} \geq a_k$ . Equation (8) represents the time interval when train  $j$  arrives before train  $i$ , where  $O_{ij}^k = 1$  implies a time interval of  $a_{ik} - a_{jk} \geq a_k$ . Equation (9) ensures that either  $O_{ij}^k$  or  $O_{ji}^k$  is equal to 1, but not both simultaneously.

$$a_{jk} - a_{ik} + (1 - O_{ji}^k) \cdot M \geq a_k \quad \forall i, j \in T \quad k \in K \quad (7)$$

$$a_{ik} - a_{jk} + (1 - O_{ij}^k) \cdot M \geq a_k \quad \forall i, j \in T \quad k \in K \quad (8)$$

$$O_{ij}^k + O_{ji}^k = 1 \quad \forall i, j \in T \quad k \in K \quad (9)$$

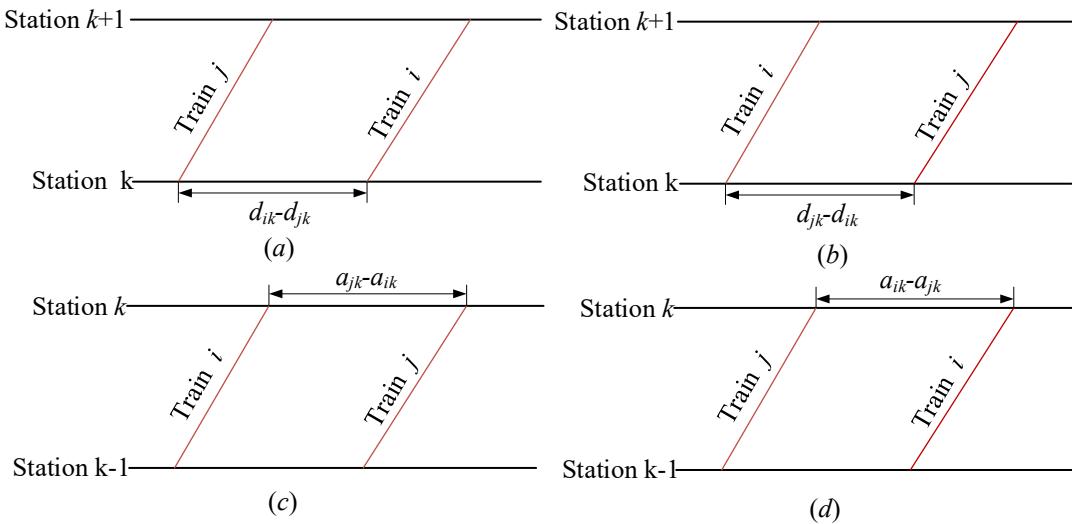


Figure 2 – Interval time constraints

5) Overtaking constraint within track sections.

Overtaking is prohibited between trains travelling in the same direction within a track section. Specifically, if train  $i$  departs from station  $k$  before train  $j$ , then train  $i$  must arrive at the subsequent station  $k'$  before train  $j$ , where station  $k'$  is the next adjacent station to station  $k$ , and station  $k'$  cannot be the last station on the line.

$$\omega_{ij}^k = O_{ij}^{k'} \quad \forall i, j \in T \quad k', k \in K \quad (10)$$

$$\omega_{ji}^k = O_{ji}^{k'} \quad \forall i, j \in T \quad k', k \in K \quad (11)$$

6) Passenger train departure time window constraint.

To improve service quality and meet the daily commuting needs of passengers, a departure time window for passenger trains is defined.

$$td_{ik}^- \leq d_{ik} \leq td_{ik}^+ \quad \forall i \in T \quad k \in K \quad (12)$$

7) Station arrival and departure track capacity constraint.

For any station, at any given time, the number of tracks available for train arrivals and departures must be no less than the number of trains scheduled to arrive or depart from the station. If the station's capacity is insufficient, the station will be unable to accommodate train arrivals or departures. For example, consider a station where, for each time, a 3-minute time window before and after the given time (with a time step of 1 minute) is used to determine track occupation. If the arrival time  $a_{ik}$  of train  $i$  at station  $k$  falls within the 3-minute window before or after a specific time, then train  $i$  is considered to occupy the tracks at station  $k$ , as shown in Figure 3. Therefore, the following function is defined:

$$w(a_{ik}, t) = \begin{cases} 1, & x \in [t-3, t+3] \\ 0, & x > t \end{cases} \quad (13)$$

In the equation:  $w(a_{ik}, t)$  is a 0-1 variable, where it equals 1 if the arrival time of train  $i$  at station  $k$  is within the time interval  $[t-3, t+3]$ , and 0 otherwise.

The total number of tracks occupied at all stations on the line at any given time must not exceed the number of tracks available for train arrivals and departures at each station. The constraint is as follows:

$$\sum_t \sum_{i=1}^{t+6} \sum_{k=1}^N w(a_{ik}, t) \leq l_k \quad i \in T \quad i < N \quad t \in [0, 1440] \quad l_k \in L_k \quad (14)$$

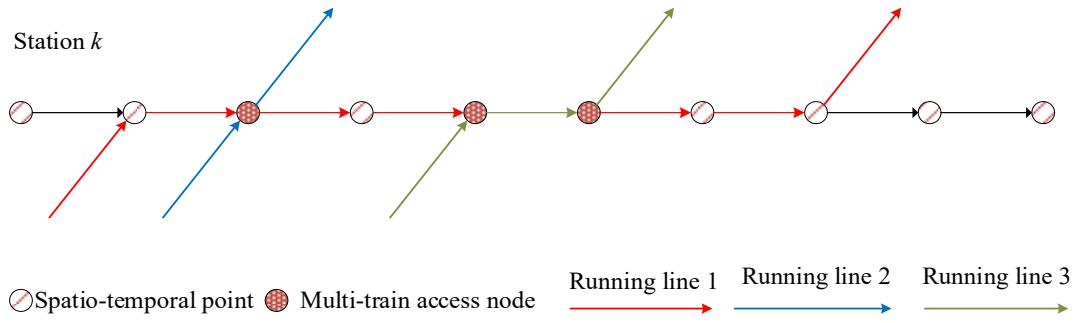


Figure 3 – Station capacity constraint

#### 8) Maintenance window constraint.

To meet the transportation demands of the railway line, ensure the safety and stability of the transportation process and balance the daily maintenance, inspection and equipment resource utilisation, the maintenance window must meet a certain duration. Furthermore, the start time of each section's maintenance window must be after the specified earliest start time, and the end time must be before the latest allowable end time.

$$mot_p^e - mot_p^s \geq mot_p \quad p \in P \quad (15)$$

$$mot_p^s \geq mot^s \quad p \in P \quad (16)$$

$$mot_p^e \leq mot^e \quad p \in P \quad (17)$$

#### 9) Conflict constraint between maintenance windows and trains.

During the maintenance window period, trains are not allowed to enter the section. If a train enters the section before the maintenance window starts, the maintenance window's start time must be after the train has passed through the section. Conversely, if a train enters the section after the maintenance window has started, the train must wait until the maintenance window ends before entering the section.

$$a_{ik} + 1 - M(1 - \lambda_i^{mot_p}) < mot_p^s \quad p \in P \quad (18)$$

$$d_{ik} + M\lambda_i^{mot_p} > mot_p^e \quad p \in P \quad (19)$$

#### 10) Variable constraints.

$$\lambda_i^{mot_k} \in \{0,1\} \quad (20)$$

$$\omega_{ij}^k, O_{ij}^k \in \{0,1\} \quad (21)$$

$$a_{ik}, d_{ik}, mot_p^s, mot_p^e \in [0, 1440] \text{ and are integers} \quad (22)$$

### 3.3 Objective function

To address the practical demands of railway operations, this paper proposes an optimisation model for train timetabling under a mixed passenger-freight operation mode. The model integrates train timetable compilation and maintenance window planning while accounting for station capacity constraints. To improve the quality of the timetable, enhance railway operational efficiency and reduce train travel times, the objective function is designed to minimise the total travel time of all trains within a given time horizon. This approach enables a direct assessment of the optimisation performance of the proposed model.

$$Z_1 = \sum_{i=1}^N (a_{iD} - d_{iO}) \quad (23)$$

In the equation:  $a_{iD}$  denotes the arrival time of train  $i$  at its terminal station,  $d_{iO}$  denotes the departure time of train  $i$  from its origin station, and  $(a_{iD} - d_{iO})$  represents the total travel time of train  $i$ , measured in minutes.

During the process of timetable formulation, the train paths should be arranged as densely as possible to improve the capacity of the timetable. Accordingly, the objective function can be defined as follows:

$$Z_2 = \sum_{i=1}^N |d_{i+1O} - d_{iO}| \quad (24)$$

In the equation:  $d_{i+1O}$  represents the departure time of train  $i + 1$  from the origin station,  $d_{iO}$  represents the departure time of train  $i$  from the origin station, and  $|d_{i+1O} - d_{iO}|$  denotes the time difference between the departure times of two consecutive trains at the origin station, measured in minutes.

### 3.4 Linearisation process

From the station arrival and departure track capacity constraints, it can be observed that this constraint involves a special piecewise function and a binary function, which significantly increase the model's size and complexity, making the problem more difficult to solve. To reduce the model's size and computational complexity, the arrival and departure track constraints are linearised. First, the arrival and departure track constraints for each station at each time step are represented by introducing two intermediate variables,  $C_{ikt}^1$  and  $C_{ikt}^2$ , to assist in the judgement. When  $a_{ik} \geq t - 3$ ,  $C_{ikt}^1 = 1$ ; when  $a_{ik} \leq t + 3$ ,  $C_{ikt}^1 = 0$ . Similarly, the condition applies to  $C_{ikt}^2$ . The arrival and departure track constraints for each station at each time step are then expressed as shown in Equations (25)-(32).

$$a_{ik} \leq (t - 3) + C_{ikt}^1 M \quad (25)$$

$$a_{ik} \geq (t - 3) - (1 - C_{ikt}^1) M \quad (26)$$

$$a_{ik} \geq (t + 3) - C_{ikt}^2 M \quad (27)$$

$$a_{ik} \leq (t + 3) + (1 - C_{ikt}^2) M \quad (28)$$

$$w(a_{ik}, t) \leq C_{ikt}^1 \quad (29)$$

$$w(a_{ik}, t) \leq C_{ikt}^2 \quad (30)$$

$$w(a_{ik}, t) \geq C_{ikt}^1 + C_{ikt}^2 - 1 \quad (31)$$

$$w(a_{ik}, t) \in \{0,1\} \quad C_{ikt}^1 \in \{0,1\} \quad C_{ikt}^2 \in \{0,1\} \quad (32)$$

In the equation:  $M$  is a sufficiently large number,  $C_{ikt}^1$  and  $C_{ikt}^2$  are two auxiliary variables introduced to determine the value of  $w(a_{ik}, t)$ .

The sum of the trains occupying the arrival and departure tracks at each station during each time period must not exceed the number of arrival and departure tracks available for train operations at that station. The station capacity constraint can thus be clearly expressed by *Equation (33)*.

$$\sum_t^{t+6} \sum_{i=1}^N w(a_{ik}, t) \leq l_k \quad i \in T \quad i < N \quad t \in [0, 1440] \quad l_k \in L_k \quad (33)$$

### 3.5 Analysis of model complexity

The integrated optimisation model for train timetabling and maintenance windows under a mixed passenger-freight operation mode, which incorporates station capacity constraints, includes integer variables for determining train departure times at stations and the start and end times of maintenance windows, as well as binary variables for train departure order and whether a train enters a maintenance window. After linearisation, both the constraints and objective functions of the model are expressed as linear equations, making it a typical mixed-integer linear programming (MILP) model. The complexity of solving this model primarily depends on the number of trains and stations. *Table 5* provides an analysis of the number of constraints and variables in the model.

Table 5 – Number of variables and constraints in the model

Variables or constraints	Types of variables		Total number at most
Variables	Integer variables	$a_{ik}$	$ T  \times ( K  - 1)$
	Integer variables	$d_{ik}$	$ T  \times ( K  - 1)$
	Integer variables	$mot_P^s$	$ P $
	Integer variables	$mot_P^e$	$ P $
	Binary variables	$S_i^k$	$ T  \times  K - 2 $
	Binary variables	$O_{ij}^k, \omega_{ij}^k$	$ T  \times ( T  - 1) \times ( K  - 1)$
Constraints	Minimum running time constraint for sections	(1)	$ T  \times  K - 1 $
	Train dwell time and stopping constraints	(2)-(3)	$ T  \times  K - 2 $
	Train safety interval time constraint	(4)-(9)	$6 \times  T  \times ( T  - 1) \times ( K  - 2)$
	No overtaking constraint within sections	(10)-(11)	$2 \times  T  \times ( T  - 1) \times ( K  - 1)$
	Passenger train departure time window constraints	(12)	$ T $
	Station capacity constraints	(14)	$1440 \times  T  \times  K $

Variables or constraints	Types of variables		Total number at most
	Maintenance window constraints	(15)-(17)	$3 \times  P $
	Conflict constraint between maintenance windows and trains	(18)-(19)	$2 \times  I  \times  K  \times  P $

#### 4. CASE ANALYSIS

The Lin-Ha Railway, serving as the northern corridor for coal transportation from Xinjiang, holds significant potential for outbound coal shipments. Stretching 1,328 km from Linhe Station on the Baolan Railway in Inner Mongolia to Hami Station on the Lanzhou-Xinjiang Railway in Xinjiang, the Lin-Ha Railway plays a critical role in transporting coal from major mining areas along its route, including the Turpan-Hami Coalfield, Naomaohu Mining Area, Dananhu Mining Area, Zhungeer Coalfield and Sandaoling Mining Area. The exported coal primarily supplies regions such as Ningxia, the Two Lakes and One River (Hunan, Hubei, Jiangxi) and the Beijing-Tianjin-Hebei area. Therefore, studying the integrated optimisation of train timetables and maintenance windows on the Lin-Ha Railway is of great importance for enhancing the transport capacity of this northern corridor. This research also holds significant value for the railway sector in improving efficiency and service quality.

Based on the functional positioning and transportation demand analysis of the Lin-Ha Railway, it can be concluded that after the capacity expansion of the Ejin to Linhe section, the railway operates under a freight-dominated, mixed passenger-freight transportation organisation mode. The railway features heavily loaded trains in the uphill direction and empty trains in the downhill direction. The entire line consists of 28 stations, with an average station spacing of 25 km and a maximum station spacing of 39 km. An analysis of the existing line's capacity and utilisation reveals that the bottleneck section is the Huzhuobuqi-Suhongtu section, with a capacity of only 12.5 train pairs per day. The long round-trip travel time for trains in this section makes it the capacity-limiting section for the entire line. As shown in *Table 5*, the existing line's capacity and utilisation rates indicate that the capacity utilisation rate of the bottleneck section has reached 85.7%, suggesting that the capacity is already saturated.

Table 5 – Existing line capacity and utilisation rate

Sections	Linhe–Tian’ehu West	Tian’ehu West–Ceke	Tian’ehu West–Ejin
Bottleneck sections	Huzhuobuqi–Suhongtu	Tian’ehu West–Juyanghai	Tian’ehu West–Ejin
Plain line capacity (train pairs/day)	12.5	14.0	14.5
Passenger and freight trains (train pairs/day)	Passenger train	1	2
	Through a freight train	8	3
	Pick-up and drop-off trains	1	1
	Total	10	3
Capacity utilisation rate (%)	85.7	21.3	62.3

Therefore, the Linhe-Ejin section of the Lin-Ha Railway, under the context of double-track reconstruction, is selected as the research object, focusing on the capacity-limiting section between Yagan and Huzhuobuqi. The schematic diagram of the line is shown in *Figure 4*. Utilising the model established in this study and based on the long-term forecast of passenger and freight demand, an optimised timetable is developed for this section.

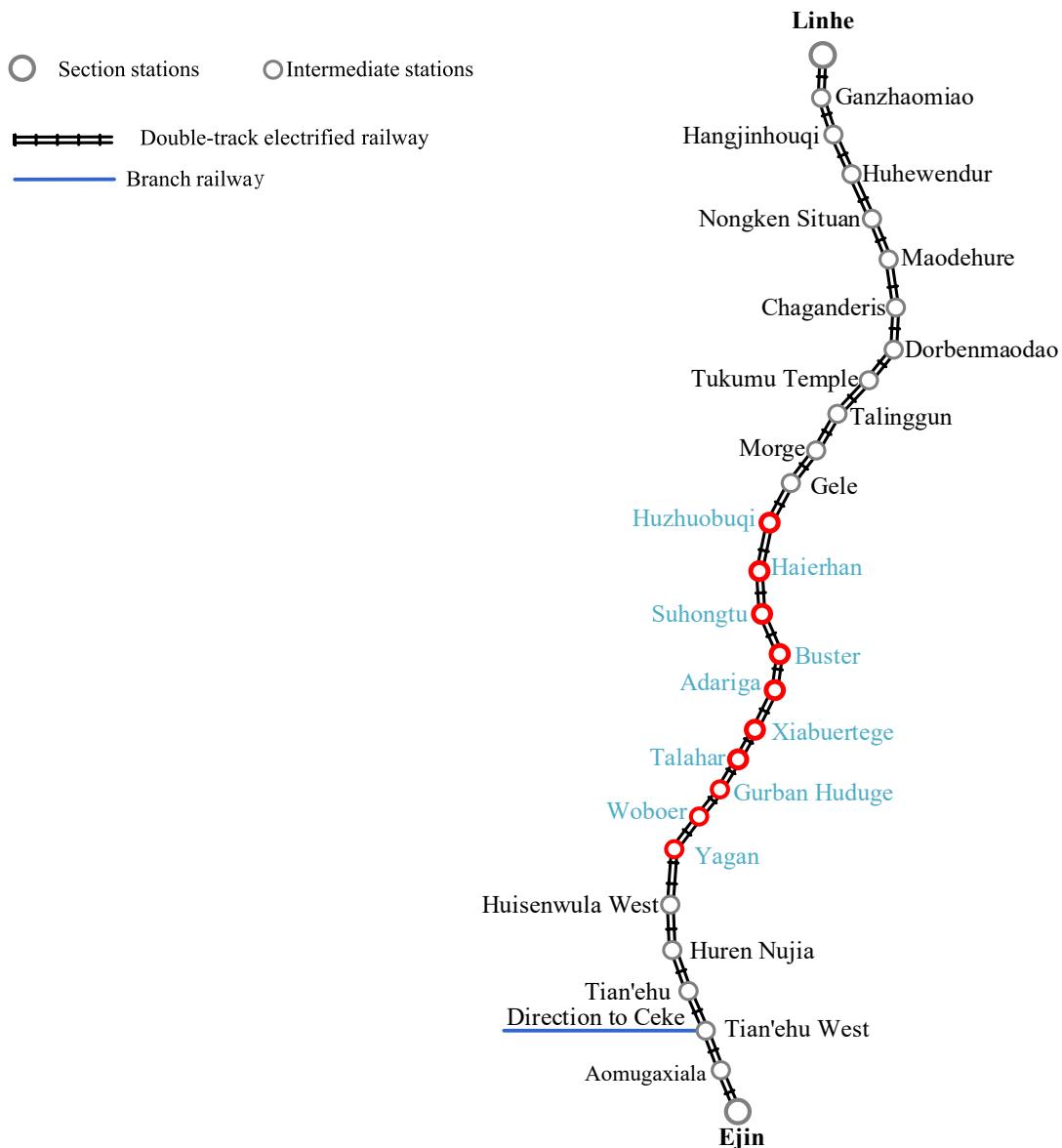


Figure 4 – Schematic diagram of the Linhe-Ejin section of the Lin-Ha Railway

#### 4.1 Basic data

This section of the railway includes 10 stations, such as Yagan, Woboer, Gurban Huduge and Tarahar, and 9 railway segments (numbered A–I). The train operating direction is defined from Yagan to Huzhuobuqi. The distances between stations and the train travel times for each section are shown in *Table 7*. The timetable planning period is 24 hours, during which a total of 60 trains (numbered 1–60) are scheduled to operate, including 4 passenger trains and 56 freight trains. A maintenance window is allocated for each section, with a minimum duration of 150 minutes. Both passenger and freight trains are scheduled to stop at designated stations for necessary operations, and the stopping plans for all trains are shown in *Figure 5*. The minimum headway between train arrivals and departures is 10 minutes, while the additional time required for train startup and stopping operations is 2 minutes each.

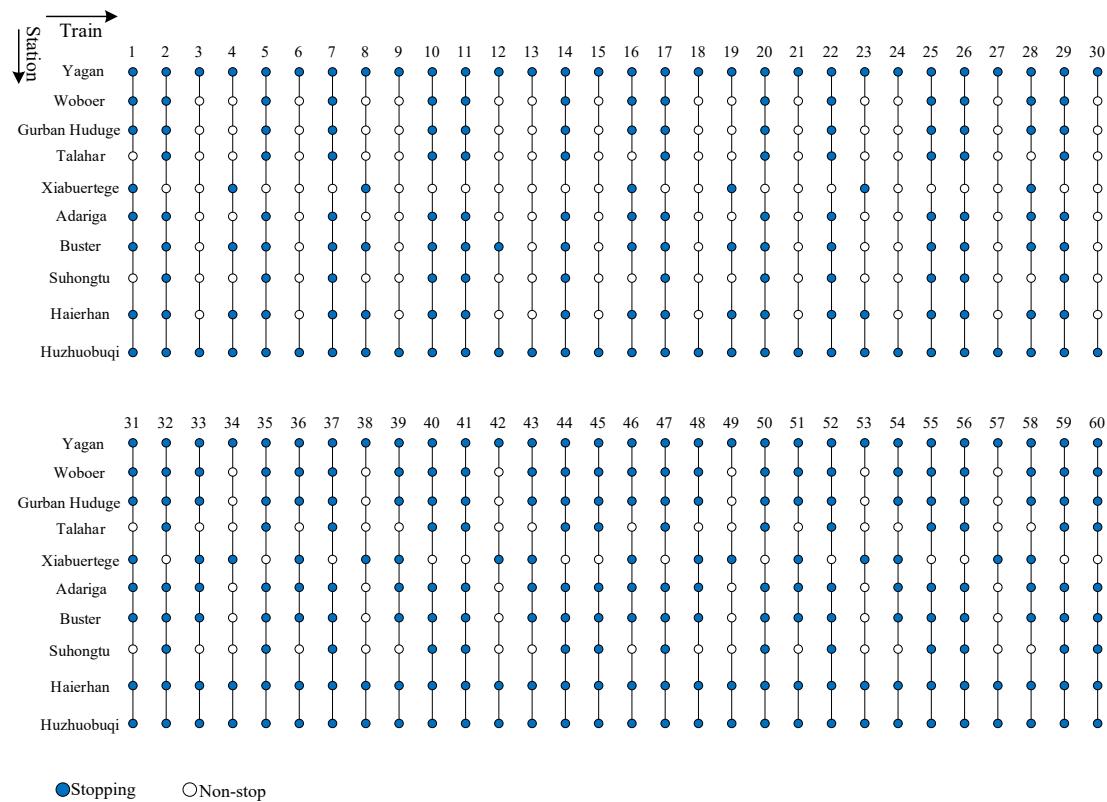


Figure 5 – Train stopping plan

Table 7 – Train running time and station spacing

Station	Section	Station spacing	Freight train sectional running time	Passenger train sectional running time
Yagan	A	25.6 km	20 min	15 min
Woboer	B	25.6 km	25 min	20 min
Gurban Huduge	C	23.6 km	23 min	20 min
Talahar	D	25.1 km	25 min	21 min
Xiabuertege	E	23.6 km	23 min	20 min
Adariga	F	23.9 km	24 min	20 min
Buster	G	25.7 km	25 min	21 min
Suhongtu	H	25.3 km	25 min	21 min
Haierhan	I	22.3 km	22 min	18 min
Huzhuobuqi				

## 4.2 Result analysis

The model was solved on a personal computer equipped with an Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, using Python programming on the PyCharm 2022.2.2 platform and the Gurobi 11.0.0 solver. Due to the inclusion of station arrival and departure track capacity constraints in the proposed model, the number of

constraints and variables increased, resulting in longer computation times. To evaluate the performance of the proposed integrated optimisation model for train timetabling and maintenance windows under the mixed passenger-freight operation mode, two scenarios were analysed: (A) without considering station capacity constraints and (B) with station capacity constraints. The solutions obtained were subjected to statistical analysis, and the results for the scenario with station capacity constraints were visualised. The train timetable is shown in *Figure 6*, and a portion of the train schedule is presented in *Table 8*.

The results validate the correctness and feasibility of the proposed model. Statistical analysis revealed that in both scenarios, train overtaking occurred at stations. This phenomenon is caused by differences in sectional running times, station dwell times and train priorities. Trains with shorter dwell times and higher priorities overtake trains with longer dwell times and lower priorities at stations, improving timetable efficiency. It is worth noting that both passenger and freight trains operate at relatively low speeds, which is due to the harsh surrounding environment of the studied railway line, which limits the travel speed of the trains. In both scenarios, the train dwell times and maintenance window durations met the minimum time standards. The difference between the two scenarios lies in the decision variables, such as the start and end times of maintenance windows and the departure times of trains at stations, which resulted in different layouts of train paths in the timetable.

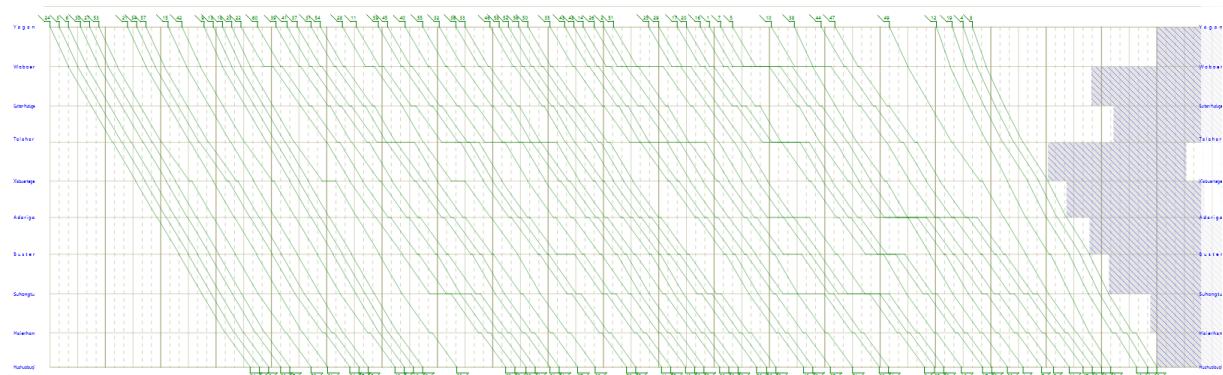


Figure 6 – Train timetable for the Yagan-Huzhuobuqi section

Table 8 – Train timetable

Departure	Yagan									
Arrival	Huzhuobuqi									
Station Train number	Yagan	Woboer	Gurban Huduge	Talahar	Xiabuertege	Adariga	Buster	Suhongtu	Haierhan	Huzhuobuqi
1 11:55	...	12:20 13:00	13:30 13:32	...	14:25 14:27	14:55 15:40	16:09 16:11	...	17:06 17:08	...
2 10:00	...	10:25 11:05	11:35 11:37	...	...	12:58 13:43	14:12 14:14	14:44 14:49	15:19 15:21	...
3 00:10	...	00:30	00:55	01:18	01:43	02:06	02:30	02:55	03:20	03:47
4 16:30	...	16:45	17:05	17:25	17:52	18:12	18:37	18:58	19:24 19:27	...
5 12:20	...	12:45 13:25	13:55 13:57	...	...	15:18 16:03	16:32 16:34	17:04 17:09	17:39 17:41	...
6 00:20	...	00:40	01:05	01:28	01:53	02:16	02:40	03:05	03:30	03:57
7 12:07	...	12:32 13:12	13:42 13:44	...	...	15:05 15:50	16:19 16:21	16:51 16:56	17:26 17:28	...
8 ...	...	...	...	...	...	...	...	...	...	...
9 02:47	...	03:07	03:32	03:55	04:20	04:43	05:07	05:32	05:57	06:24
10 13:02	...	13:27 14:07	14:37 14:39	...	...	16:00 16:45	17:14 17:16	17:46 17:51	18:21 18:23	...
11 05:32	...	05:57 06:02	06:32 06:37	07:05 07:45	...	08:38 08:40	09:09 09:29	09:59 10:04	10:34 10:36	...

Through the optimisation of the train timetable for the Yagan-Huzhuobuqi section under the double-track reconstruction of the Lin-Ha Railway, the proposed model successfully achieved the accurate layout of train paths and maintenance windows for this section. A comparison of the solution efficiency under the two scenarios is presented in *Table 9*. When station arrival and departure track capacity constraints are considered, the number of constraints and variables increases significantly. Without considering station capacity constraints, the model includes 37,218 variables and 97,835 constraints, whereas with station capacity constraints, the number of variables increases to 399,548 and the number of constraints to 1,761,975. The solution time also increases from 956 seconds to 5,223 seconds. The variation of the solution GAP value over time is shown in *Figure 7*. As can be observed, the GAP value decreases rapidly in the early stages, dropping below 50% after 420 seconds, below 15% after 500 seconds, and further accelerating after 900 seconds. Ultimately, the GAP value reduces to 0% after 5,247 seconds. The “Gap-Time” results demonstrate that the Gurobi solver is capable of obtaining the optimal solution for the model.

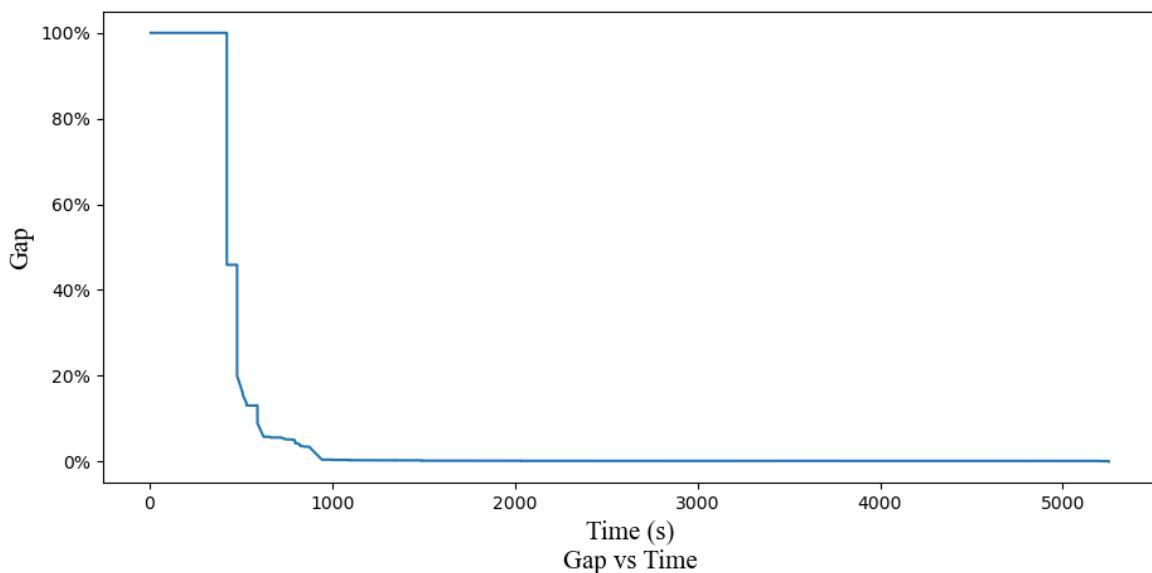


Figure 7 – “Gap-Time” relationship during the solution process

Table 9 – Comparison of solution efficiency

	Optimisation objectives	Number of variables	Number of constraints	Gap value	Computation time
A	15933.0	37218	97835	0.00%	956.265s
B	15933.0	399548	1761975	0.00%	5223.463s

## 5. CONCLUSION

This paper focuses on the optimisation of train timetables under the mixed passenger-freight train operation mode and draws the following conclusions. (1) An integrated optimisation model for train timetables and maintenance windows under the mixed passenger-freight operation mode was established. The model can provide an overall optimal train timetable solution and improve timetable formulation efficiency. (2) The proposed model incorporates station arrival and departure track capacity constraints and linearises these constraints to ensure that the solutions align better with actual operational conditions, thereby enhancing train operation efficiency. (3) A real-world case study was solved using the commercial solver Gurobi, which demonstrated that the optimal solution can be obtained within a relatively short time. This validates the correctness and effectiveness of the proposed model. However, since the problem size increases with the number of stations and trains, future research will primarily focus on developing efficient algorithms to address issues related to slow solution times or failure to obtain optimal solutions. Additionally, attention will be given to solving the instability of the train timetable caused by unforeseen disruptions.

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