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Mid-Term Revenue Risk Assessment Model for Railway Public-Private Partnership Projects Based on Copula-NPVaR

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ABSTRACT

Public-private partnership (PPP) proves to be an effective approach to solving the financing problems of railway construction projects. However, there is a lack of studies on how to control its high revenue risk on financial returns caused by great uncertainties regarding revenue and cost during a long concession period for both the public and private sectors. Few studies focused on the mid-term risk assessment model, and there is still no well-developed mid-term risk assessment mechanism. To fill this research gap, this paper develops a mid-term revenue risk assessment model combining copula and net present value (NPV)-at-risk method. The copula-NPVaR (net present value at risk) model is applied in a numerical case study based on a railway construction project. The results indicate that by means of a copula, the simulated NPVs, under the more reasonable stochastic assumptions of random variables, will approach the real value. This model can be a useful tool to assess the profitability and financial sustainability of the project in the operation period, providing the stakeholders with more reliable and quantitative reference on the revenue risk. This study offers both theoretical foundations and practical suggestions to practitioners that will ultimately promote the adaptive and agile management of railway PPP projects.

KEYWORDS

railway public-private partnership projects; revenue risk; mid-term risk assessment; infrastructure finance.

1. INTRODUCTION

The financing issue of transportation infrastructures remains a controversial topic in developing countries. As railway networks expand, government fiscal investment cannot fulfil the huge funding demands, particularly amid rising global public debt, which reached $97 trillion and is growing twice as fast in developing nations [1]. Consequently, the PPP (public-private partnership) mode is widely promoted in railway and other infrastructures to attract private investment and expertise, improving both funding sustainability and service quality. Driven by favourable policies before 2019, China's PPP market has achieved rapid growth with a total investment value of 2.3 trillion USD by 2022. However, the profitability problems emerge when most of the projects go into the mid-term stage of the operation period [2].

PPP is a long-term partnership where the private sector is responsible for financing and providing public products or services for reasonable revenues [3]. As the operator of projects, the private sector shares significant revenue risk, particularly in railway PPP projects, which involve huge initial investment, long concession periods and uncertainties in demand and cost [4]. For instance, several railway lines in Zhejiang Province, China (Hanghuang Line, Jinliwen Line and Jiujingqu Line), have experienced unexpected increases in operational costs, while their annual revenue has remained relatively stable. This trend highlights their limited profitability and financial vulnerability [5]. Therefore, research has increasingly focused on assessing revenue risk for railway transport PPP projects to support informed decision-making.

Due to the long concession period, PPP infrastructure projects require a life-cycle risk management framework to ensure value-for-money and balance of interest between stakeholders [6]. Over 80% of the PPP projects in China were initiated after 2014, most remaining in the preliminary stage and confronted with unforeseen revenue risk due to forecasting uncertainties [7]. To objectively reflect the service quality, efficiency and emerging risks, periodic evaluations are essential for informed PPP agreement adjustments [8]. Mid-term revenue risk assessments help project managers gauge the financial status of the project, yet a key challenge is modelling potential revenue risk. The absence of such models will undermine the recognition of emerging revenue risk and the implementation of risk mitigation or allocation mechanisms.

This paper aims to propose a mid-term revenue risk assessment model and apply it based on a real-world railway project case. The results may provide a guideline and foundation for the establishment of a standard mid-term revenue risk assessment framework, which paves the way for adaptive and agile project management.

2. LITERATURE REVIEW

2.1 Life-cycle risk management for PPP projects

Numerous studies emphasised adopting life-cycle approaches in risk management for PPP. Akomea-Frimpong et al. [9] reviewed 49 studies on financial risk management in public-private partnership projects from 1995 to 2019. Le et al. [10] and Liu et al. [11] highlighted the unique, dynamic and complex nature of risk factors in PPP projects, attributable to their substantial initial investments and long concession periods. Conversely, the absence of a life-cycle perspective in PPP management will result in producing unsatisfactory service quality and more unexpected risks [12]. Zou et al. [6] proposed a life-cycle framework aiming to balance the interests of stakeholders involved. This dynamic process encompasses risk identification, analysis, allocation and monitoring, from feasibility analysis continuing through the operation and transfer phases. Risk assessments play a crucial role in informing other key aspects. For instance, dynamic risk allocation mechanisms, such as minimum revenue guarantees (MRG) and maximum revenue caps (MRC), rely on dynamic risk assessment to be effectively designed [13]. Additionally, assessment results can support the application of cooperative game-theoretic solutions, such as Shapely and Owen, which can be applied to analyse the behavioural dynamics to achieve the optimal risk-reward allocation [14, 15].

Risk assessment from a life-cycle perspective involves not only ex-ante assessment but also mid-term and dynamic assessment. Mid-term assessment is particularly crucial in periods involving negotiation, renegotiation and risk allocation adjustments to achieve optimal value for money. Due to the long concession periods of transportation PPP projects, multiple mid-term assessments should be conducted in response to the unexpected risks. According to the policy issued by the Ministry of Finance in China, the monitoring institutions of PPP projects are supposed to carry out mid-term assessments every 3 to 5 years, analysing the operation situation, verifying the realisation of value for money and assessing the unexpected risks. However, there is a lack of a well-developed mid-term risk assessment model that clarifies the content, method and requirement [11]. As for large-scale urban rail transit PPP projects, the financial distress is mainly driven by inaccurate forecasts on cash flow, operation expenditure and maintenance cost, defined as revenue risk [16]. Therefore, an efficient mid-term revenue risk assessment model is needed to evaluate and monitor the potential loss of railway PPP projects caused by revenue risk.

2.2 Revenue risk assessment models

The revenue risk of railway PPP projects is defined as the probability of actual revenue below forecasted driven by multiple factors such as insufficient traffic flow, operation cost overrun, debt and the rate of return [17]. Generally, the discounted cash flow (DCF) model and the non-discounted cash flow model are commonly employed to evaluate project revenue risk. The former one stresses the time value of money and calculates the net present value (NPV) using a discount rate, while the latter focuses on the payback period or rate of return [4]. Given that the concession period of a railway PPP project is typically longer than twenty years, DCF models prove to be a suitable choice for risk assessment [18]. However, since traditional DCF models failed to adequately reflect the market volatility, researchers investigated stochastic models to incorporate the uncertainty. Two main stochastic tools are applied in revenue risk assessment for railway PPP projects: NPV-at-risk and real option.

NPV-at-risk (NPVaR) was proposed by Ye and Tiong [19] to portray the risk profile via NPV within a confidence interval, which was an application of financial value-at-risk (VaR) into the project management domain. Based on the prototype, Zhu et al. [20] established a stochastic model for the BOT scheme, accounting for the impact of the risk attitude of stakeholders. Kumar et al. [21] proposed a standard financial risk analysis model based on NPVaR to assess 30 real-world PPP projects in India. However, these models failed to utilise the operating data available for mid-term assessment. Namely, the stochastic assumptions within these models were mainly based on empirical hypotheses from previous project cases, which may not accurately reflect the real-time situation of the project being appraised.

Real option theory is another feasible tool, especially for projects involving risk-sharing mechanisms like minimum revenue guarantee (MRG) and maximum revenue cap (MRC). These forms of government support or obligations can be interpreted as real options since they are triggered when some pre-specified conditions are met. Real option methodology is well-suited for valuation by incorporating the typical uncertainty of PPP project performance through managerial flexibility assessment [22]. However, existing real option models typically assume traffic revenue follows a geometric Brownian motion and treat different variables as independent. This assumption may be unrealistic due to the lack of operational data, making accurate parameter estimation challenging [23]. The relevance between variables does exist and can be derived in a statistical method if enough operational data are available in the concession period. Specifically, the maintenance and operation costs will probably increase when increasing traffic flow brings more revenue. The accuracy of simulation results will be undermined if this relationship is neglected in revenue risk assessment.

2.3 Copula-based method

A copula is a function used to describe the dependency between random variables. Wang et al. [24] developed a copula-based system for water security risk assessment, which is especially efficient in analysing multi-factor coupled safety risk. Ullah and Akbar [25] applied the copula function to measure the upper tail dependence of drought characteristics, which is highly important in extreme condition measuring. In project financing fields, compared with previous studies on a single project investment, the copula helps to assess the joint default risk. Chakkalakal et al. [26] assessed the risk-return characteristics of transport infrastructure portfolios from the perspective of institutional investors based on the t-Copula-CVaR (conditional value at risk) model, and the results supported the use of this novel risk assessment tool, incorporating non-normal distributions to represent the multivariate dependence structures. The major advantage of the copula-based model is that it circumvents the restrictive stochastic assumptions of NPVaR models and real option models [27], and it generates a joint probability distribution in multivariate models. To clarify the research gap, a summary of revenue risk assessment models for transport infrastructure projects is presented in *Table 1*. This paper proposes a copula-NPVaR model for mid-term revenue risk assessment of railway PPP projects, which answers two questions: how to make use of cash flow information derived from the previous period in mid-term assessment and how to build a reasonable joint stochastic assumption of multivariate structure in the simulation.

Table 1 – A summary of revenue risk assessment models

Model category & stochastic assumption	**Representative authors**	**Remarks**
NPV-at-risk *Stochastic assumption*: Multiple random variables, such as traffic volume, operation cost and toll rate, are set to be independent, or a covariance between jointly distributed random variables.	Ye and Tiong (2000) [19]	NPV-at-risk method and a better decision for privately financed infrastructure projects investment.
	Zhu et al. (2016) [20]	A stochastic BOT model which considers the impact of risk and risk attitudes of investors to get a more reasonable concession interval.
	Kumar et al. (2018) [21]	NPV-at-risk model applied to 30 practical BOT projects to identify the critical risk and discuss the risk mitigation mechanism.
Real option *Stochastic assumption:* Traffic flow is thought to be the main factor contributing to the volatility of the assessment model.	Biancardi et al. (2024) [22]	An optimum upper and lower boundary of MRG and MRC based on the real option method.

Model category & stochastic assumption	Remarks
Copula-NPVaR (applied in this paper) *Stochastic assumption:* A copula function is built according to the real cash flow information obtained in the operation period to present the joint distribution of income and cost.	A mid-term assessment model making use of cash flow information gained in the operation period and applying a copula to reveal the relationship between variables.

3. MODEL ESTABLISHMENT

3.1 Model framework

This subsection introduces the framework of the mid-term revenue risk assessment model proposed in this paper, clarifying the objective, methodology, input and output of the model.

The model aims to evaluate the revenue risk level in a mid-term assessment by utilising the cash flow information of cash inflows and outflows generated from the project appraised. Since the evaluation mainly focuses on predicting the NPV under the influence of the uncertainty and fluctuation of cash flow and without risk allocation mechanisms like MRG and MRC, NPV-at-risk could be an intuitive method to realise the goal. Different from the experience-based stochastic assumptions in previous studies, for mid-term assessment, the probability distribution assumptions of random variables can be derived from the practical data during the operation period. A copula is an important tool in this model to produce a joint distribution of relevant variables and make the stochastic assumption more reasonable.

Consequently, the assessment model is designed to be a discounted cash flow model with relevant random variables, and a Monte Carlo simulation can be applied to work out the distribution of NPV and NPV-at-risk. The above-discussed model framework is illustrated in *Figure 1*.

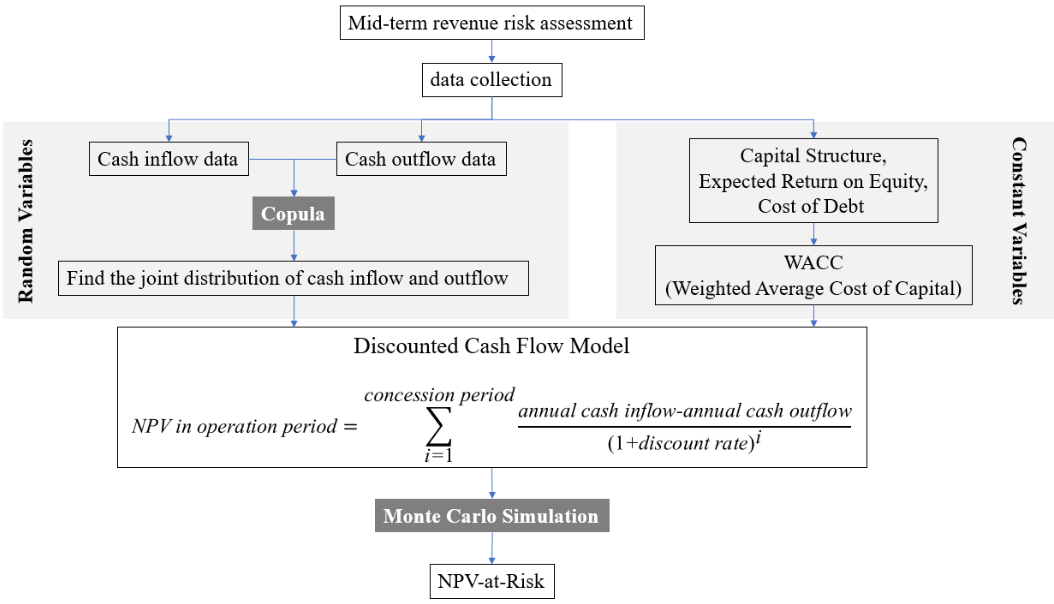

Figure 1 – Mid-term revenue risk assessment model framework

3.2 Discounted cash flow model

An unfavourable feature of transportation projects compared to other infrastructures is the great uncertainty regarding the cost and income [5], on which the model in this paper focuses. The discounted cash flow (DCF) model is applied to obtain the net present value (NPV) of the operation period as shown in *Equation 1*. In China, a special purpose vehicle (SPV) of a railway PPP project gets over 70% of the financing from bank lenders, and the rest is equity provided by stakeholders. To take into account both the rate of return of stakeholders and the debt interest, the weighted average cost of capital ($WACC$) can be used as the discount rate for the project as expressed in *Equation 2* [28]:

$$NPV_O = \sum_{t=1}^{T} \frac{R_t - C_t}{(1 + WACC)^t} \tag{1}$$

$$WACC = P_E \times R_E + P_D \times R_D \times (1 - T_C) \tag{2}$$

where NPV_O is the net present value generated from all the operating activities; the length of concession period is T; the annual income in year t is assumed to be R_t, while the annual operating cost is C_t; P_E is the percentage of financing that is equity and P_D is that of debt; R_E refers to the cost of equity, in other words, the expected rate of return of private sectors; R_D is the cost of debt, the interest rate; T_C is the corporate tax rate.

The costs in the construction period are not considered because they are sunk costs from the perspective of mid-term assessment in the operation period. The annual income of railway construction projects consists of transportation income and diversified business income [29]. However, high-revenue business activities such as property development are strictly regulated in China. As a result, revenue from diversified business activities generally constitutes a small proportion of the total income. Thus, the annual income R_t can be regarded as a variable entirely dependent on the traffic flow. As for the annual cost of railway, the main divisions include labour cost, facility expenditure, maintenance expenditure and interest payment. Since the interest payment is incorporated into the WACC, there is no need to consider it again. The remaining part of the cost has been found to vary closely with the demand [30], which is consistent with the cognition that in any business, the cost of production will be influenced by the variations of the volume. Therefore, the sequences of R_t and C_t should be interdependent random sequences here to represent both their volatility and relationship.

3.3 NPV-at-risk

NPV-at-risk ($NPVaR$) is a valuation of project investment risk, which calculates the probable minimal NPV given a confidence level [19]. In this paper, $NPVaR_{1-\alpha}$ (NPV-at-risk at the confidence level of $1-\alpha$) is mathematically defined as the infimum of the set of NPV, the cumulative distribution function value of which is larger than α;

$$NPVaR_{1-\alpha} = \inf\{x \epsilon \mathbb{R}, F_{NPV}(x) > \alpha\} \tag{3}$$

where F_{NPV} is the cumulative distribution function of NPVs. Then, the key task is to build the distribution function of NPV based on the cash flow data in the operation period.

3.4 Copula

In order to get the NPV distribution, stochastic assumptions on the random variables in *Equation 1* should be made. The relationship between annual income and cost has been discussed above, so a joint distribution of them should be established. This section introduces copula theory, which is applied to obtain the joint distribution of annual income and cost.

According to Sklar's Theorem, if $H_{X,Y}(x,y)$ is a joint distribution function with marginal distributions $F_X(x)$ and $G_Y(y)$, then there exists a copula function $C(u,v)$ such that for all u,v in $\mathbf{I}^2$ ($[0,1]^2$).

$$H_{X,Y}(x,y) = C\{F_X(x), G_Y(y)\} \tag{4}$$

Given that $F_X(x)$ and $G_Y(y)$ are continuous functions, the uniqueness of $C(u,v)$ can be established. However, in practical scenarios, determining this unique copula function becomes highly challenging. Fortunately, researchers have introduced various copula functions that can effectively replace $C(u,v)$, offering more practical and manageable alternatives.

Among families of copulas, Archimedean copulas are more popular and widely used in applications because of their simple form and nice properties [31]. Archimedean copulas are copulas that can be expressed in terms of:

$$C(u,v) = \varphi^{[-1]}(\varphi(u) + \varphi(v)) \tag{5}$$

where φ is a strictly decreasing, continuous function and is known as the generator of an Archimedean copula. $\varphi^{[-1]}$ is the "pseudo inverse" defined as:

$$\varphi^{[-1]}(t) = \begin{cases} \varphi^{-1}(t), & 0 \le t \le \varphi(0) \\ 0, & \varphi(0) \le t \le \infty \end{cases} \tag{6}$$

The generators φ of the Archimedean family usually have only one parameter θ in the function, and we can obtain the value of θ through the Kendall rank correlation coefficient, also known as Kendall's tau (τ). Kendall's τ is a measure of the rank correlation between two quantities. If a set of observations of joint random variables X and Y are obtained, the Kendall's τ of this sample is defined as:

$$\tau_s = P\left((x_i - x_j)(y_i - y_j) > 0\right) - P\left((x_i - x_j)(y_i - y_j) < 0\right) \tag{7}$$

Given pairs $(x_i,\ y_i)$ and $(x_j,\ y_j)$ in observations, if $(x_i - x_j)(y_i - y_j) > 0$, they are defined as concordant pairs, and if $(x_i - x_j)(y_i - y_j) < 0$, then they are discordant pairs. There are totally $\binom{n}{2} = \frac{n(n-1)}{2}$ ways of picking 2 pairs out of n. Therefore, τ_s can be obtained from *Equation 8*;

$$\tau_s = \frac{2(c-d)}{n(n-1)} \tag{8}$$

where c means the number of concordant pairs and d is the number of discordant pairs.

Above is the measure of observations, and also, for an Archimedean copula function with a generator φ_θ, its Kendall rank correlation coefficient τ_θ can be obtained from:

$$\tau_\theta = 1 + 4\int_0^1 \frac{\varphi_\theta(u)}{\varphi'_\theta(u)} du \tag{9}$$

4. NUMERICAL CALCULATION WITH SIMULATION

4.1 Random variables generation

Lu et al. investigated the growth pattern of traffic volume based on data from California transportation projects and found that both compound growth and linear growth functions can model the growth trends of traffic volume [32]. Since the volatility of train ticket prices in China is strictly controlled by the government, the rate can be regarded as a constant variable, and thus we can treat the annual income as linear growth or compound growth. Also, we assume that the annual cost has the same growth pattern as annual income. At first, to simplify the calculation, we assume the actual growth is consistent with a linear growth pattern and verify this assumption through the following steps. Let R_t^r and C_t^r be the actual annual income and cost in the year t in the operation period. Then, obtain the first-order difference as $\Delta R_t^r = R_{t+1}^r - R_t^r$ and $\Delta C_t^r = C_{t+1}^r - C_t^r$. ΔR_t^r and ΔC_t^r should be a stationary sequence due to the linear growth assumption of R_t^r and C_t^r, which means their values do not depend on time. In the real case simulation, the ADF (augmented Dickey-Fuller) test can be applied to verify the stationarity of this sample. If the sample passes the stationary test, then, $(\Delta R, \Delta C)$ can be the bivariate random vector to be generated in the following step. The following steps are based on the situation that the first-order differences of income and cost are stationary.

Bivariate Archimedean copula can be employed to generate relevant random variables [33]. This simulation procedure was based on the following theorem:

(U_1, U_2) is a bivariate random vector with uniform marginals. The joint distribution function of u_1, u_2 is of the Archimedean copula form $C(u_1, u_2) = \varphi^{[-1]}(\varphi(u_1) + \varphi(u_2))$ with a particular generator φ. The joint distribution function of S and T can be characterised by:

$$H(s,t) = P(S \le s, T \le t) = s \times K_C(t) \tag{10}$$

where S and T are independent variables and S is uniformly distributed on $(0,1)$.

Suppose $(\Delta R, \Delta C)$ has a bivariate distribution function based on a two-dimensional Archimedean copula with generator φ and their marginal distributions are F_R and F_C which can be obtained according to the actual data in the operation period. The simulation procedure is as follows:

1) Generate independent random variables s and w that follow a uniform distribution $U(0,1)$;
2) Let $t = K_C{}^{-1}(w)$;
3) Let $u = \varphi^{-1}[s\varphi(t)]$ and $v = \varphi^{-1}[(1-s)\varphi(t)]$;
4) And the desired simulated value $(\Delta R, \Delta C) = (F_R{}^{-1}(u), F_C{}^{-1}(v))$.

4.2 Choice of a copula

Suppose it is year n in operation period and the first-order difference pairs $(\Delta R_1^r, \Delta C_1^r)$, $(\Delta R_2^r, \Delta C_2^r)$, …, $(\Delta R_{n-1}^r, \Delta C_{n-1}^r)$ of n years are available. Let Kendall's tau of the sample equal that of one copula, $\tau_\theta = \tau_s$, and then the value of θ, and the generator φ_θ is determined.

Given this φ_θ, the simulated pairs $(\Delta R_1^s, \Delta C_1^s)$, $(\Delta R_2^s, \Delta C_2^s)$, …, $(\Delta R_{n-1}^s, \Delta C_{n-1}^s)$ of n years in operation period can be obtained through the above procedure in Section 4.1. To measure the accuracy of simulation, we compare the net present value based on simulated data $NPV_{[1,n]}^s$ and that of practical data $NPV_{[1,n]}^r$ for these n years.

$$NPV_{[1,n]}^s = \sum_{t=1}^{n} \frac{R_t^s - C_t^s}{(1+d)^t} \tag{11}$$

$$R_t^s = R_1^r + \sum_{i=1}^{t-1} \Delta R_i^s \tag{12}$$

$$C_t^s = C_1^r + \sum_{i=1}^{t-1} \Delta C_i^s \tag{13}$$

$$NPV_{[1,n]}^r = \sum_{t=1}^{n} \frac{R_t^r - C_t^r}{(1+d)^t} \tag{14}$$

Repeat the simulation process for N times, and then $NPV_{[1,n],1}^s$, $NPV_{[1,n],2}^s$,…, $NPV_{[1,n],N}^s$ are obtained.

4.3 Calculation of NPV-at-risk

Once the copula is determined, the first-order difference pairs of the rest of operation period, $(T-n)$ years, can be generated and noted as $(\Delta R_{n+1}^s, \Delta C_{n+1}^s)$, $(\Delta R_{n+2}^s, \Delta C_{n+2}^s)$, … , $(\Delta R_T^s, \Delta C_T^s)$. Then a simulated NPV of mid-term assessment in year n, noted as $NPV_{mid,n}^S$, can be worked out as:

$$NPV_{mid,n}^s = NPV_{[1,n]}^r + NPV_{[n+1,T]}^s \tag{15}$$

$$NPV_{[n+1,T]}^s = \sum_{t=n+1}^{T} \frac{R_t^s - C_t^s}{(1+d)^t} \tag{16}$$

where NPV_n^r, R_t^s, C_t^s is shown in *Equations (12), (13)* and *(14)*. With the help of computer programming, repeat this process and get a large number of $NPV_{mid,n}^S$.

5. APPLICATION OF THE COPULA-NPVAR: A NUMERICAL EXAMPLE

5.1 Data processing

This section presents a case study of a Chinese railway construction project to validate the proposed assessment method. The construction and operation of Intercity railway Line H in Province Z, which links the capital city and a prefecture-level city, are carried out under a PPP agreement. This railway line spans 50 kilometres, operates at a speed of 120 km/h, and completes a whole journey in less than 1 hour. It consists of 12 stations, including critical places such as high-speed railway hubs and a university campus, serving approximately 100,000 daily commuters. However, regional urban integration, strongly promoted by local authorities, has led to the development of alternative transport infrastructure, including an intercity highway and a high-speed railway. These competing routes have diverted a portion of the passenger flow, impacting the operational revenue of Line H. Under the PPP agreement, government subsidies are included in the revenue but cannot be utilised to cover operational shortfalls. Consequently, any misestimation of revenue or costs may lead to heightened operational risks. Given the potential financial stress, it is necessary to carry out a mid-term risk assessment to evaluate the project's financial sustainability during the operation period. The project's financing structure consists of 30% equity and 70% debt, with a debt interest rate of 5% and the rate of return for the investors is set to be 8%. Considering a corporate tax rate of 25%, the weighted average cost of capital ($WACC$) is calculated as 5%.

The first step is to select the time point (year n) for the mid-term assessment. n should not be too small, or the appraiser cannot obtain enough data for an accurate assessment. Given that a railway concession is usually for a period of 25 years, setting the time point in the 8th operation year to the 10th operation year ($n \approx T/3$) is a proper choice. In this case, we collect operating data of ten years from Line H to carry out the mid-term risk assessment. *Table 2* shows the operating revenue R_t^r and cost C_t^r from 2013 to 2023, and the data come from the annual reports of the operator. Data perturbation and scaling techniques have been applied to anonymise and desensitise the original dataset.

Assume that both annual revenue and cost have a linear growth pattern and obtain their first-order difference. ADF test in MATLAB for series ΔR_t and ΔR_t shows that they are stationary sequences. According to *Equations (7)* and *(8)*, we obtain the value of Kendall's tau of the two series $\tau_s = 0.4667$. Then, we can estimate the value of parameter θ for all the copulas functions: θ(Clayton)=1.75, θ(Frank) = (2.43, 5.06), θ(Ali-Mikhail-Haq) = 0.774, θ(Gumbel-Hougaard) = 1.875.

Table 2 – Operating data of intercity railway line H from 2013 to 2023 (desensitised data)

Year	2013	2014	2015	2016	2017	2018	2019
Operating revenue ($R_t^r \times 10^4 CNY$)	69912.8	73479.3	85017.3	79702.3	75600.4	67003.1	73913.3
Operating cost ($C_t^r \times 10^4 CNY$)	20921.7	21766.94	32652.58	28571.01	27399.6	27901.01	32694.4
Year	**2020**	**2021**	**2022**	**2023**			
Operating Revenue	79863.3	77814.2	81546.7	77890			
Operating Cost	43208.9	34934.6	32727.9	33051.8			

5.2 Choice of copula function

Once the parameter θ is determined, the generator φ_θ is obtained. The next step is to generate a bivariate distribution $(\Delta R_t^s, \Delta C_t^s)$ through the above instructions. The marginal distribution F_R and F_C of ΔR_t and ΔC_t are empirical distributions of them based on data from 2013 to 2023. When random numbers u and v are generated, the corresponding ΔR_t^s, ΔC_t^s can be worked out through the empirical distribution F_R and F_C. However, since the empirical distribution functions are staircase functions with limited data points, linear interpolation is applied here to get simulated points as shown in *Figure 2*. For instance, if $u = 0.55$, the corresponding $\Delta R = 0.0759$ can be obtained through linear interpolation of the existing points (-0.2049,0.5) and (0.3567,0.6).

Given the generated $(\Delta R_t^s, \Delta C_t^s)$, the simulated NPV of these ten years $NPV_{[1,10],i}^s$, average number M and degree of deviation D are worked out in turn. *Figure 3* illustrates the simulated $NPV_{[1,10],i}^s$ scatter grams with different copulas and without copula, and *Table 3* specifies their M and D.

The NPV of practical data in this ten-year operation period, discounted at 5%, equals to 3.8898 billion CNY. According to *Table 3* and *Figure 3*, the simulations based on Frank copula and Gumbel-Hougaard copula show the best accuracy in average level of simulated numbers, because their M values are closer to $NPV_{[1,n]}^r$. Besides, the simulations based on the Clayton copula and the Gumbel-Hougaard copula show the minimal degree of deviation with minimal D.

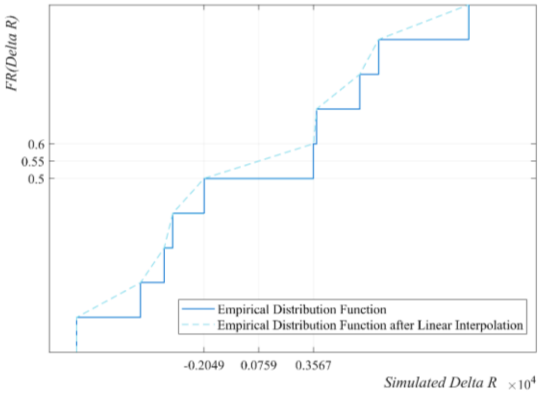

Figure 2 – Obtaining ΔR_t^s through linear interpolation

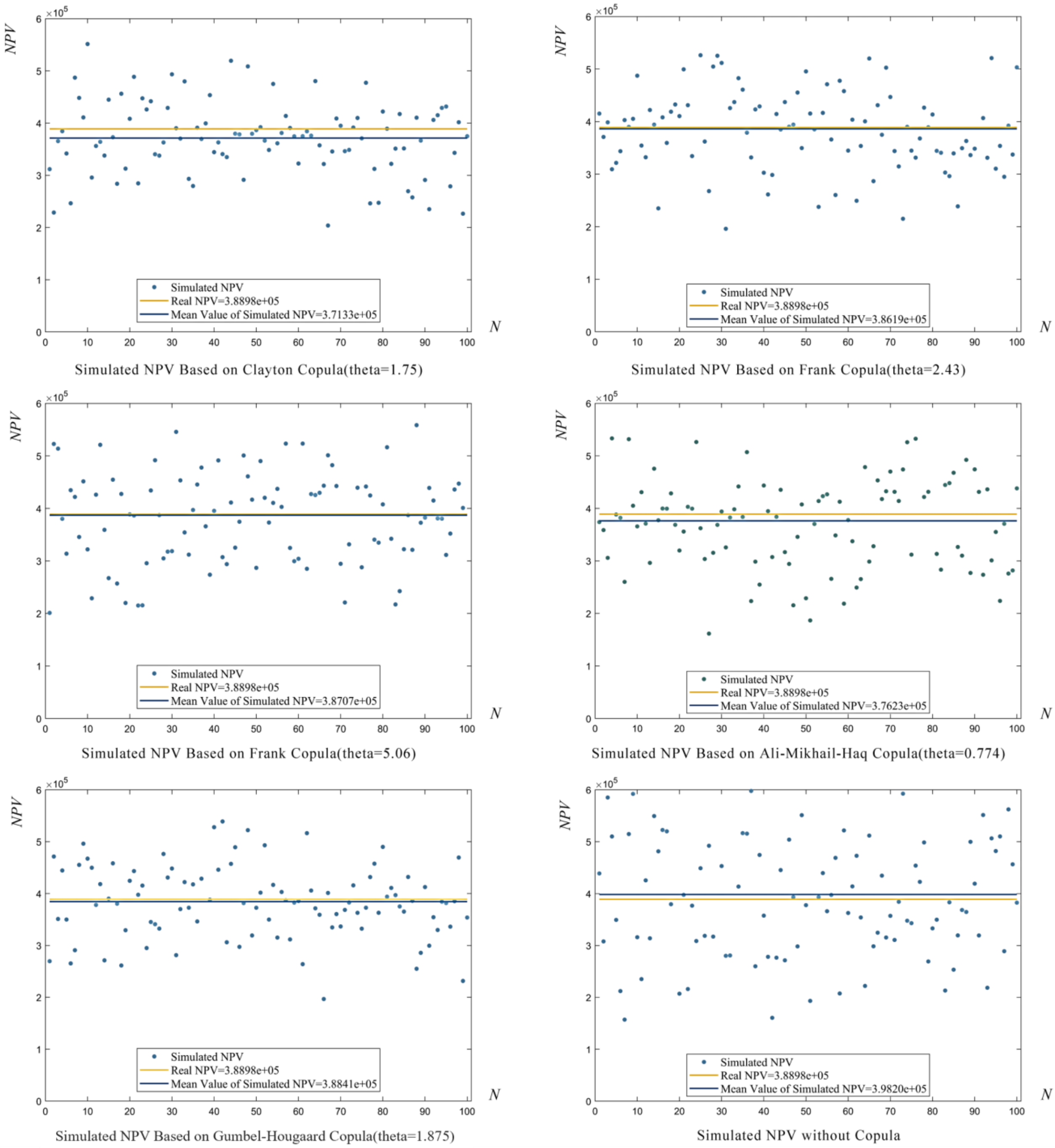

Figure 3 – Scattergrams of simulated NPV (100 trials) with different copulas and without copulas

The simulation without a copula is presented as a control group, in which ΔR and ΔC are treated as independent variables. All the simulation results with copula show an average NPV lower than real NPV, while that of control group is the only one larger than $NPV_{[1,n]}^{r}$. This is mainly because if the relevance of annual revenue and cost is considered, the distribution of NPV follows a fat-tailed pattern which is not the case for independent variables. Besides, the simulated NPV in the control group has a larger degree of statistical dispersion than NPV generated with copulas.

Therefore, simulations with copulas proved to be a better way to get the distribution of NPV in accordance with actual conditions. According to the M and D in *Table 3*, the Gumbel-Hougaard copula is chosen as the best way to describe the relevance of annual cost and revenue.

Table 3 – Accuracy of simulation with different copulas and without copulas

Copula	$M(\times 10^4 CNY)$	D
Clayton ($\theta = 1.75$)	3.7133e+05	0.0864
Frank ($\theta = 2.43$)	3.8619e+05	0.1017
Frank ($\theta = 5.06$)	3.8707e+05	0.1274
Ali-Mikhail-Haq ($\theta = 0.774$)	3.7623e+05	0.1226
Gumbel-Hougaard ($\theta = 1.875$)	3.8441e+05	0.0901
N/A	3.9820e+05	0.1814

5.3 Calculation of NPVaR

Finally, the simulated NPV are generated. *Figure 4* is the probability distribution and cumulated distribution of simulated NPV based on 1,000 trials (N=1,000). The NPVaRs at different confidence levels are marked on the axis.

The purpose of this mid-term assessment is to provide a quantified reference for renegotiations or other dynamic adjustments in the operation period, because of risk sharing. NPVaR measures the minimal possible value at a certain confidence level based on the practical operating data. For instance, 6.1647 billion CNY is a very pessimistic estimate of NPV_o at the confidence level of 0.95. Given an expected NPV_o of 7 billion CNY, $NPVaR_{0.95}$ implies that the risks in operation period may cause 0.8353 billion CNY loss in NPV. If the public sector and private sector share the risk equally, each of them should take a loss of 0.41765 billion CNY. In the same way, NPVaR can also measure the possible excess profit of this PPP project, as a quantified reference of MRC. This situation happens when the confidence level is assigned as less than 0.5, where $NPVaR_\alpha$ represents a very optimistic estimate of NPV_o. As shown in *Table 3*, given the expected NPV_o of 7 billion CNY, $NPVaR_{0.05}$ implies an excess profit of 0.8818 billion CNY, and both sectors can get 0.4409 billion CNY extra profit based on 1:1 sharing ratio.

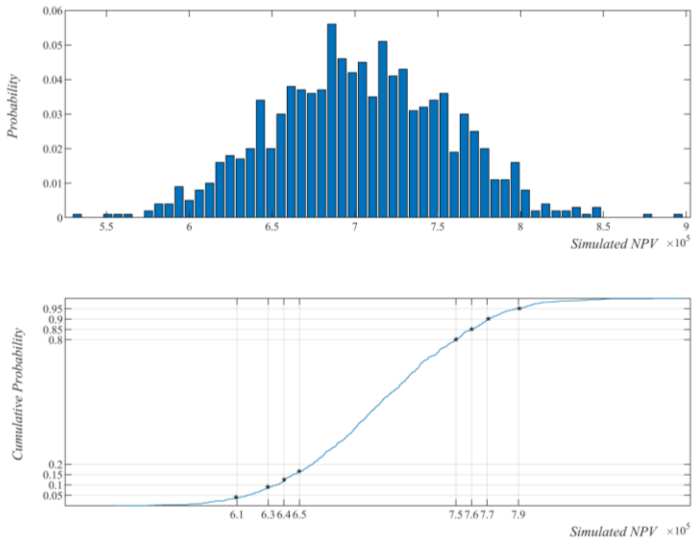

Figure 4 – Simulation results based on the Gumbel-Hougaard copula

6. DISCUSSIONS

Numerous existing studies have focused on the revenue risk assessment model for ex-ante analysis, but the improved model for mid-term assessment has been scarcely addressed due to the lack of awareness of life-cycle management and efficient analytic tools. Therefore, new assessment models should be established to make use of the advantage of mid-term assessment: the project produces practical data to help appraisers make more realistic and reasonable assumptions in the assessment model, rather than experience-based hypotheses. The copula-NPVaR model proposed in this paper provides a method to present the interdependent variables in a joint distribution and produces more accurate simulation results. Based on the numerical simulation results in a case study, some suggestions and remarks for the implementation of better mid-term revenue risk assessment are summarised as follows:

1) The relationship between interdependent variables cannot be neglected in the mid-term assessment model. For example, the operating ratio (percentage of revenue spent on operating expenses) of Indian Railways remained between 97% and 107% from 2018 to 2023, indicating a strong historical correlation between revenue and cost [34]. Underestimating operating expenditure can impose significant financial strain on the railway sector. Simulation results in this paper indicate that treating annual revenue and costs as independent variables, as in the control group, leads to more dispersed NPV distributions with extreme values. Specifically, the pessimistic estimation will probably show NPV at a lower level, and the optimistic estimation shows a larger NPV, which misleads the stakeholders into overestimating the revenue risk.
2) The revenue floor and cap in MRG & MRC should remain adaptable during the operation period. The NPV distribution simulated in the mid-term assessment may considerably deviate from that based on ex-ante assumptions. If we cannot distinguish which sector is responsible for this deviation, it is unfair to still apply the previous revenue floor and cap. For instance, if the project gets more revenue mainly due to the high quality of service provided by the private sector, a low cap may strip the revenue from it, and if the project gets less revenue mainly because of poor management, then a high floor makes the public sector undertake unnecessary revenue risk. The mid-term assessment provides a quantified reference for stakeholders to know about how much loss or abnormal profit they would have.
3) A data analysis software may help with the implementation of the mid-term assessment. Some steps within this simulation model, such as the random variable generation and the choice of copula function, involve complex calculations. Therefore, a data analysis software, comprising data encapsulation, is supposed to be developed to simplify the operation of appraisers.
4) The copula-NPVaR model can also be applied to mid-term revenue risk assessment or project evaluation in any other fields marked by long concession periods and interdependent cash flow divisions. If enough operating data are available, appraisers can use this model to re-evaluate the revenue risk.

7. CONCLUSIONS

Mid-term revenue risk assessment is crucial for life-cycle risk management of railway PPP projects. Among various risk assessment methods, NPV-at-risk analysis is intuitive and practical, but fails to employ the operating data in mid-term assessment. This paper proposed an improved model, applying copula functions to establish the bivariate distribution of revenue and cost based on project cash flow data. The main contribution lies in making more reasonable stochastic assumptions on interdependent variables in NPV-at-risk simulations. The results of the case study show that the copula-NPVaR model reduces statistical dispersion of simulated NPVs, providing more realistic and quantifiable risk insights.

This study is among the first attempts to model variable relationships in NPV simulation model via mathematical tools, offering a more scientific way for mid-term revenue assessments. However, it is limited to a bivariate copula and divides the cash flow simply into inflow and outflow without differentiating their components. Since certain cost and revenue divisions may be more interdependent, future research should explore multi-criteria relationships via multivariate copula or other data analysis tools.

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