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Evaluating the Efficiency of Urban Public Transport in Belgrade – A Data Envelopment Analysis Approach

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ABSTRACT

In an era marked by rapid urban development, the efficiency of public transport (PT) systems has taken on unprecedented significance. The main model for assessing the efficiency of PT systems is the data envelopment analysis (DEA). This study leverages DEA to conduct a comprehensive evaluation of 12 PT lines in Belgrade, to optimise resource allocation. The first DEA model developed by Charnes, Cooper and Rhodes (CCR) under the assumption of a constant returns to scale production technology. By employing the input-oriented CCR DEA model, this research provides a comprehensive understanding of the efficiency landscape in Belgrade's PT system. The primary hypothesis is that PT lines exhibiting characteristics such as reduced resource utilisation, a smaller fleet of engaged vehicles, lower vehicle frequency, decreased operating costs, and yet achieving a higher volume of transported passengers, are likely to be more efficient. The results of this analysis highlight three exceptional lines – Lines 23, 31 and 95 – all of which achieved an efficiency score of 1.000. Additionally, the study employs super-efficiency analysis, identifying Line 31 as the "most efficient". These findings validate the initial hypothesis and have far-reaching implications for stakeholders, providing invaluable insights into potential strategies for fortifying Belgrade's PT system.

KEYWORDS

public transport; efficiency measurement; data envelopment analysis; input-oriented CCR model; super-efficiency.

1. INTRODUCTION

In the era of urbanisation and increasing environmental concerns, the efficiency and effectiveness of urban public transport systems have taken centre stage in discussions surrounding sustainable urban development. The current impetus behind measuring the performance of these systems is rooted in the need for continual enhancement to attract users, especially in a competitive landscape with financial constraints [1]. With that in mind, performance measurement in the public transport system is gaining significant momentum nowadays since organisations need to continuously improve their performance to attract more users while designing and delivering services in an increasingly competitive environment with tight financial constraints [2]. The efficiency evaluation of public transport systems is essential for planning urban transport systems, including prioritising public transport vehicles [3] and managing the entire system [4]. As cities worldwide strive to strike a balance between economic growth, environmental sustainability and the constraints of limited existing infrastructure, the effectiveness of public transport systems is a central consideration for both policymakers and researchers [5-6].

The data envelopment analysis (DEA) has gained traction in evaluating various systems by comparing multiple inputs and outputs to determine relative performance and has been extensively used as a comparison tool for benchmarking exercises in the public transport sector [7]. DEA is a non-parametric method used for performance evaluation in operational research and economics and is based on the original work from Farrell [8]. Subsequent development led to the establishment of the CCR model by Charnes, Cooper and Rhodes [9], and the BCC model by Banker, Charnes and Cooper [10]. DEA has gained prominence as an effective tool for measuring the relative efficiency of decision-making units, such as different elements of public transport systems. Unlike parametric methods that require predefined functional forms, DEA evaluates efficiency by comparing units based on their relative use of inputs to produce outputs [11-13]. This method enables the identification of efficient units that serve as benchmarks for less efficient units, shedding light on potential areas of improvement. In a time of evolving urban dynamics and an increasing focus on sustainable urban development, the insights provided by DEA offer a valuable framework to enhance the design and operation of urban public transport systems, aligning them with economic and environmental aspirations.

With cities like Belgrade grappling with the challenges of population growth, traffic congestion and environmental degradation, the role of an efficient public transport system becomes paramount. With that in mind, this paper aims to investigate the efficiency of the public transport lines in the City of Belgrade through the lens of the input-oriented CCR DEA model, to optimise the allocation of resources within the public transport system. The urban public transport system in Belgrade is one of the greatest transport systems in Southeastern Europe, and it consists of four subsystems: bus, trolleybus, tramway and urban-suburban rail subsystems. The line network consists of more than 252 regular lines in daily traffic. Tram subsystem, with 10 lines, trolleybus subsystem with 6 lines, urban bus subsystem with 158 lines and suburban bus subsystem with 74 lines and urban-suburban rail subsystem, with 4 lines. In the City of Belgrade, the local government provides the public transport service based on a public service contract with the public operator in the system, or based on a public-private partnership contract with private operators. All lines covered in this study are operated by the public operator in the system, “GSP” Beograd.

Lines selected for analysis and measuring efficiency are urban lines that represent major transport corridors and make a dominant share, in the sense of all departures and total gross transport work, in the urban public transport system in the City of Belgrade. By employing this model, authors aim to provide a comprehensive assessment of the city’s public transport efficiency, considering a range of inputs such as the number of working vehicles, number of departures, line capacity and operational costs, and one output, the number of transported passengers. Through this analysis, the authors endeavour to contribute to the understanding of factors influencing the efficiency of urban public transport systems and provide insights for policymakers and stakeholders to enhance the performance of Belgrade’s transport system. The main hypothesis within this paper is an assumption that lines in the system that engage fewer resources, a smaller number of engaged vehicles, smaller vehicle frequency, smaller operating costs, etc., and have a larger number of transported passengers, are more efficient.

After the introduction, the rest of the paper is structured as follows. Section 2 includes a review of relevant literature related to the implementation of DEA for measuring the efficiency of public transport systems, focusing on papers that apply DEA for measuring public transport lines efficiency. Section 3 presents the methodology and the DEA model used in this paper, while Section 4 presents the input and output data used in the model. Section 5 contains results derived from the previously defined model, and finally, Section 6 contains concluding considerations and presents potential directions for further research.

2. LITERATURE REVIEW

Previous applications of DEA within the public transport system have predominantly focused on performance evaluation. This method has been widely used in the public transport sector for over two decades, providing valuable insights into operational efficiency, resource allocation and service effectiveness. Numerous studies have applied DEA to assess the efficiency of public transport companies and systems, identifying key factors influencing performance. The following table presents a summary of the most relevant research in this field, highlighting the scope of each study and its key findings.

Table 1 – Overview of studies on DEA application for public transport lines efficiency

Study	Country/ region	Scope	Key findings
Chu et al. & Obeng [14], [15]	USA	Developed a composite measure for the efficiency and effectiveness of public bus transport agencies.	Highlighted the need to distinguish between efficiency and effectiveness measures, especially for public agencies.
Obeng [15]	USA	Examined the impact of technical inefficiencies in public transport using DEA.	The study evaluates the impact of subsidies on efficiency by incorporating them into the production frontier, using labour, fuel and fleet size as inputs and vehicle miles as output.
Nolan [16]	USA	Assessed the technical efficiency of mid-sized bus transport agencies using DEA combined with regression analysis.	Identified key determinants of cost efficiency in U.S. transport agencies.
Viton [17]	USA	Evaluated the efficiency of multi-mode bus transport systems, assessing service expansion potential without additional resources.	Found that around 80% of bus systems were technically efficient, with only minor inefficiencies. A later study indicated minor overall improvement in efficiency (1988-1992).
Viton [18]	USA	Investigated the assertion of declining productivity in U.S. bus transport and analysed a piecewise-linear best-practice production frontier using DEA for multi-modal bus transport data between 1988 and 1992.	Efficiency is assessed using both static and dynamic measures of productivity change, with the main outcome indicating a minor overall improvement in bus transport efficiency during the specified period.
Boilé [19]	USA	Assessed 23 public transport systems using DEA to identify inefficiencies in resource allocation.	Identified technical and scale inefficiencies, providing a method to benchmark efficient and inefficient systems.
Karlaftis [20]	USA	Investigated the relationship between efficiency and effectiveness across 256 transport systems over five years using DEA.	Found a positive correlation between efficiency and effectiveness, highlighting their interdependence.
Boame [21]	Canada	Applied bootstrap DEA to estimate technical efficiency in Canadian urban transport systems.	Identified efficiency variations and emphasised the importance of robustness in DEA analysis.
Tsamboulas [22]	Europe	Evaluated performance under different regulatory frameworks using DEA. Compared publicly vs privately operated systems.	Found that privately operated systems were more efficient, while publicly operated ones were more effective.
Odeck [23]	Norway	Investigated the impact of fuel consumption and workforce on efficiency using DEA in the Norwegian public bus transport system.	Estimated a potential for about 21% input savings in the Norwegian bus sector.
Margari et al. [24]	Italy	Examined public transport system efficiency within a DEA-based framework by considering regulatory, environmental factors and statistical noise. Combined DEA with SFA to analyse inefficiency measures and exogenous effects.	Found that managerial inefficiencies played a minor role, while regulatory policies and environmental factors were more critical.
De Borger et al. [25]	Norway	Extended bootstrapping methods to non-parametric convex cost frontiers when studying Norwegian bus operators.	Showed significant biases in the original DEA results, stressing the need for robustness testing.
Von Hirschhausen & Cullmann [26]	Germany	Applied bootstrapping DEA to 179 German bus transport companies.	Revealed a low average technical efficiency among the studied companies.
Hawas et al. [27]	UAE	Assessed Al Ain Public Bus Service using DEA, considering cost reduction and performance enhancement scenarios.	Suggested route adjustments to enhance performance while maintaining operational efficiency.
Hahn et al. [28]	South Korea	Developed a network-based DEA model for Seoul's bus companies.	Recommended policies like expanding transport systems and eco-friendly vehicles through tax incentives.
Vaidya [29]	India	Evaluated 26 urban transport organisations using DEA, grouping criteria into operations, finance and accident-based factors.	Produced a Transportation Efficiency Number (TEN) for overall performance measurement.

Study	Country/ region	Scope	Key findings
Roháčová [30]	Slovakia	Applied DEA to optimise urban public transport in Banská Bystrica.	Proposed efficiency improvements and vehicle allocation strategies for inefficient routes.
Carvalho et al. [31]	Brazil	Analysed public transport systems in 21 cities using DEA spanning from 2005 to 2010.	The evaluation covers three key scenarios: infrastructure efficiency, service effectiveness, and the trade-off between efficiency and effectiveness.
Venkatesh & Kushwaha [32]	India	Investigated Indian State Transport Undertakings using DEA with a cost-oriented approach.	Observed both short and long-term efficiency trends and the importance of cost minimisation strategies.
Fitzová & Matulová [33]	Czech Republic & Slovakia	Analysed urban public transport in the Czech Republic and Slovakia using DEA.	Found that higher fares, subsidies, network density, investment, tram systems and driver proportion, while integration had a less significant impact.
Sharma et al. [34]	India	Used the DEA to assess efficiency in the Jaipur metro system.	By evaluating travel times, the research offers insights for modifications to enhance operational effectiveness.
Liu et al. [35]	China	Evaluated bus public transport benefits using a unique approach involving the dynamic network DEA model, cross-efficiency evaluation and Shannon entropy aggregation.	Found limited effectiveness in benefit distribution, stressing the need for improved service effectiveness.

While a considerable portion of existing DEA literature concentrates on comparing public transport agencies, fewer studies investigate individual lines.

Sheth et al. [36] introduced a combined DEA and goal programming approach for evaluating bus route performance, considering both passenger and operator viewpoints. The study emphasises the need for performance measurement systems that consider multiple goals, encompassing the perspectives of providers, consumers and society. This framework is designed to incorporate various factors, such as provider costs, consumer satisfaction, environmental impacts and societal targets. Simulated data were used for analysis, and they were generated after consulting transportation professionals and using public domain databases. The data were applied to 60 bus routes, descriptive statistics were computed for the variables used in the analysis, and efficiency scores and deviations were calculated using the proposed model's formulation. Their simulation-based study offered insights into optimising routes.

Barnum et al. [37] examined 46 bus routes in a United States urban transport agency using DEA, emphasising the connection between input-output relationships. Inputs are the resources that supply the transport service, while outputs are the variables that measure the use and the quality of the transport service. After DEA implementation, best-practice routes are efficient ones, and their input and output proportions can serve as benchmarks for inefficient routes. Routes with high adjusted scores can be compared to these benchmarks to identify areas for improvement.

Lao & Liu [38] extended DEA's applicability by integrating Geographic Information Systems (GIS) to evaluate 24 fixed bus routes of a United States operator. Their study assessed both operational efficiency and spatial effectiveness. The researchers utilised GIS to create demographic profiles for each bus line's service corridor and applied DEA to compute operational efficiency and spatial effectiveness scores based on costs and benefits. Interestingly, no clear positive or negative relationships between operational efficiency and spatial effectiveness were found among the bus lines. By employing DEA models, the study offers comprehensive insights into bus line performance from both managerial and geographic perspectives. Comparing operational efficiency and spatial effectiveness scores enables the identification of top-performing and underperforming bus lines, leading to recommendations for expansion, subsidies or redesign.

Hahn et al. [39] employ a DEA model to estimate the efficiency of Seoul's bus routes using data from the first half of 2008. Each route is treated as a decision-making unit (DMU) producing passenger numbers and revenue based on input variables such as the number of vehicles, stops, travel distance, intervals and management fees. The results indicate that among the 18 evaluated routes, 2 DMUs operate under constant returns to scale (CRS) while the remaining 16 operate under increasing returns to scale (IRS). Inefficient DMUs are identified and can improve efficiency by benchmarking against more efficient counterparts.

Georgiadis et al. [2] in their paper focus on evaluating the performance of individual bus lines in the Thessaloniki public transport network using DEA. The study considers factors like traffic conditions, depot locations and population density to analyse efficiency and operational effectiveness. The study introduces a clustering methodology for bus lines and emphasises considering various factors to enhance performance. The results suggest ways to improve performance through strategic service design modifications. However, the analysis acknowledges challenges in fully isolating management issues from external factors.

Zhu et al. [40] introduce an independent efficiency measurement model for bus routes, aiming to address heterogeneous outputs in DEA. It incorporates new operating environment factors using a three-stage DEA approach to assess managerial efficiency while accounting for external effects and statistical noise. The goal was to measure managerial efficiency while accounting for the operating environment's impact. Findings showed the proposed framework's effectiveness, providing decision support for producers and regulators. The analysis, based on 39 bus routes in Jiangyin City, China, shows that the new model is effective, providing insights into efficiency and operating environment impacts on bus route performance.

Güner & Coşkun [41] conducted a multidimensional analysis of public transport performance, focusing on both operational and service efficiencies. The study proposes a non-radial DEA approach that considers mathematical interdependencies between operational and service efficiency models. This methodology is applied to 30 public transport bus routes from the Sakarya City Metropolitan Municipality, effectively providing optimal efficiency scores and proportional changes for interrelated input variables, while also evaluating independent input variables separately. This approach offers a more comprehensive performance analysis, allowing for monitoring both operational and service efficiencies, optimising resources, and generating applicable input targets for inefficient routes.

Cazuza de Sousa Júnior et al. [42] assess the public transport system in Brazil's Metropolitan Region of Fortaleza using DEA and the Malmquist index. Inputs like operating time, fleet age and mileage, along with outputs like fare revenue and passengers, were considered based on data from automated fare collection systems. The analysis focused on 15 out of 19 municipalities with regular public transport service, served by 7 companies. The analysis provides insights into route performance, aiding effective diagnosis and understanding of operations. Also, this approach allows for a more detailed analysis of the operational performance than those studies where DMUs are the operators or the transportation systems.

However, despite these advancements, the literature predominantly focusing on public transport agency comparisons highlights a gap in research on individual bus line evaluations. Even [2] and [43] highlight that the number of studies employing DEAs to treat routes as DMUs in the realm of public transport is limited. Only a minor fraction (1.6%) of research has evaluated route performance, with the majority concentrating on operators (76%) and systems (21%). Notably, a significant portion of route performance analyses emerged after 2010 [42]. This study seeks to address this gap by applying the input-oriented CCR DEA model to assess the efficiency of key public transport lines in the City of Belgrade, thereby contributing to the existing body of knowledge on public transport efficiency assessment.

3. METHODOLOGY

Measuring the efficiency of public transport lines in a certain urban area can represent a challenging task. With that in mind, selecting a suitable model for evaluation represents the key to the comprehensive analysis. In this study, the authors adopt the DEA to rigorously evaluate the efficiency of public transport lines within the City of Belgrade. DEA is a widely accepted non-parametric technique utilised for comparative efficiency assessment across multiple units with shared objectives using linear programming.

The decision to employ DEA is underpinned by several compelling factors. DEA is particularly well-suited for assessing the efficiency of complex systems, such as public transport systems, enabling the consideration of multiple inputs and outputs simultaneously. This approach provides a comprehensive and multi-dimensional evaluation of performance, accommodating the multifaceted nature of public transport operations, which involve various resources, service outputs and objectives. One significant advantage of DEA is its non-parametric nature, which refrains from imposing restrictive assumptions on the relationships between inputs and outputs. This flexibility proves invaluable in situations where these relationships are intricate, nonlinear or not well-understood. By avoiding these assumptions, DEA allows us to capture the true operational characteristics of public transport lines without bias. DEA also excels at making relative comparisons, a particularly useful feature when assessing public transport lines. It empowers us to assess the efficiency of multiple units or entities within the same system, enabling the identification of performance disparities, highlighting areas for improvement, and establishing benchmarks for best practices. Through DEA, we can determine which lines operate efficiently and which may require optimisation efforts. Moreover, the DEA's input-oriented approach directs our focus toward resource efficiency. Within the context of public transport, resource allocation is a critical consideration. DEA's methodology allows us to concentrate on the judicious use of resources, including vehicles, operational costs, etc. This emphasis on resource minimisation while maintaining service quality aligns with our goal of contributing to the development of a cost-effective, environmentally friendly and sustainable public transport system for the benefit of Belgrade's residents. With that in mind, our approach employs an input-oriented CCR DEA model, which prioritises the minimisation of resource inputs while ensuring consistent output levels. The choice of an input-oriented model aligns with our primary objective, to optimise the allocation of resources within the public transport system. The described model is shown below:

$$e_{j_o} = max \sum_r u_r \, y_{rj_o} \tag{1}$$

with constraints:

$$\sum_i v_i \, x_{ij_o} = 1 \tag{2}$$

$$\sum_r u_r \, y_{rj} - \sum_i v_i \, x_{ij} \leq 0 \qquad \forall\, j = 1, \dots, n \tag{3}$$

$$u_r, v_i \geq \varepsilon \qquad \forall\, r, i \tag{4}$$

where the above-mentioned variables are:

e_{j_o} – efficiency of the decision-making unit j_o;
j – decision-making units counter;
i – input counter;
r – output counter;
n – number of decision-making units;
x_{ij} – input i of decision-making units j;
y_{rj} – output r of decision-making units j;
x_{ij_o} – input i of decision-making units j_o, which efficiency we measure;
y_{rj_o} – output r of decision-making units j_o, which efficiency we measure;
v – input weight coefficient;
u – output weight coefficient;
ε – a very small positive number.

The DEA input-oriented CCR model aims to measure the efficiency of a specific decision-making unit (j) by comparing its output-to-input ratio with those of other units. The unit's efficiency score is determined by how effectively it utilises inputs to generate outputs while considering weight coefficients and constraints to ensure fairness and meaningful efficiency assessment. The value for e_{j_o} in the first equation tends to the maximum value, which is 1. With that in mind, if this value is equal to 1, the analysed unit is considered efficient, while if it is smaller than 1, the unit is considered relatively inefficient compared to others.

4. INPUT AND OUTPUT DATASETS

Upon establishing the model to be employed for assessing efficiency, the next critical step in our analytical journey is the definition of data groups that will form the basis of our comprehensive analysis. These data groups represent the foundational components of the study, laying the groundwork for a meticulous examination of public transport line efficiency in Belgrade. At the outset of the analysis, a crucial step entails defining the specific public transport lines that will be encompassed within the efficiency examination. For this paper, the authors chose a total of 12 lines to assess their relative efficiency. These lines represent the DMUs for the proposed DEA model. The analysis includes 8 bus lines (Lines 16, 18, 23, 31, 56, 65, 88 and 95), 2 trolleybus lines (Lines 29 and 40) and 2 tram lines (Lines 7 and 9). As previously highlighted, the chosen set of public transport lines offers a comprehensive representation of Belgrade's public transport system. These lines encompass a substantial portion of both the total passenger volume carried and the extent of network coverage within the urban area. *Figure 1* illustrates the spatial distribution and positioning of the analysed public transport lines within the City of Belgrade.

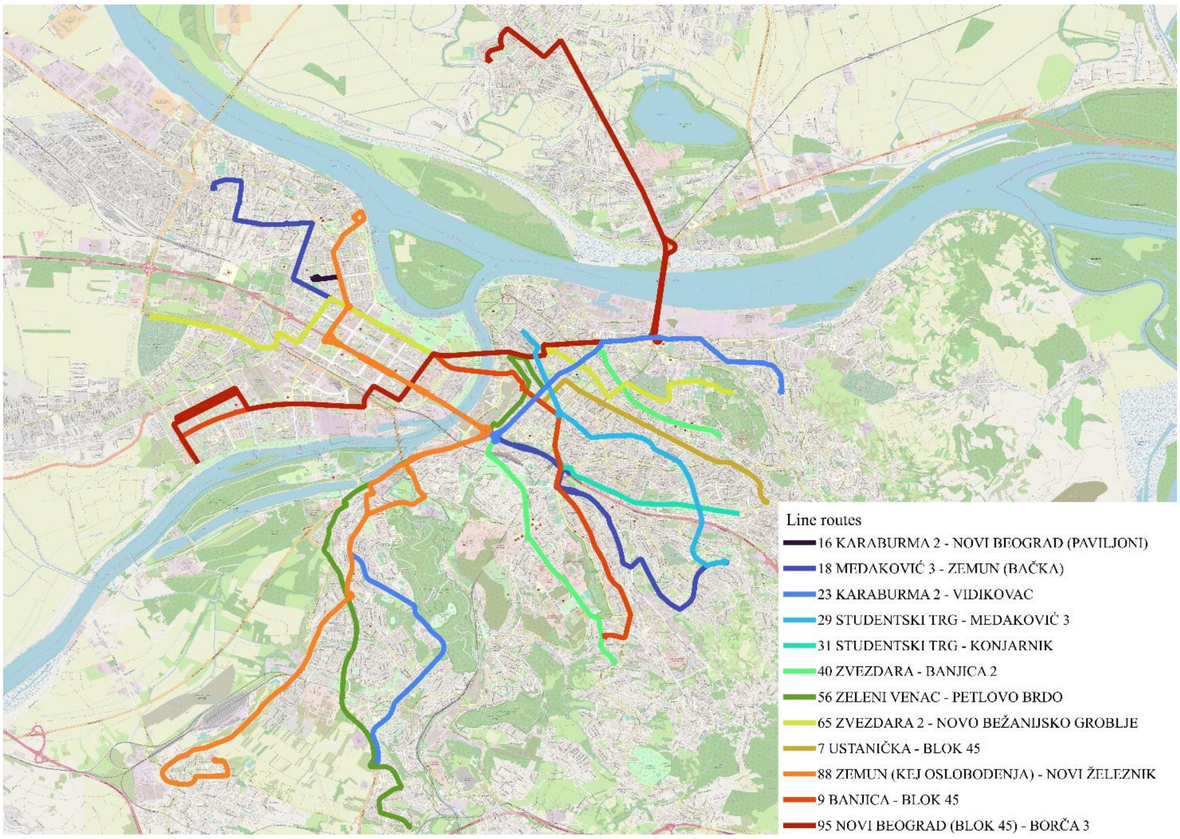

Figure 1 – Spatial distribution and positioning of public transport lines within the City of Belgrade

The efficiency analysis for each of the proposed DMUs necessitates the meticulous definition of data categories. Every DEA model comprises two crucial groups of data: input data and output data. Notably, the model employed within this paper aligns with the input-oriented CCR model, characterised by multiple inputs and a single output. In this particular case, the authors employed four input and output parameters in the analysis. Input parameters used in the analysis are:

— Number of working vehicles (N);
— Number of departures (n_{dep});
— Line capacity (C);
— Operational costs (T).

Number of working vehicles

The number of vehicles dedicated to a particular public transport line is a foundational input parameter. It directly influences a line's capacity to efficiently cater to passenger needs. A higher number of working vehicles can potentially lead to more frequent and reliable services, and a higher number of transported passengers on that particular line, positively impacting efficiency. The number of vehicles can only be determined per line if there is no refitting between the lines. Data regarding the number of vehicles running on selected lines are sourced directly from the monitoring and control system of SfPT of the City of Belgrade.

Number of departures

Departure frequency is a crucial input factor. It determines the regularity and availability of public transport services, which, in turn, can significantly affect passenger convenience and satisfaction. More frequent departures enhance a line's potential to efficiently meet passenger demand. Ideally, more departures on the line lead to more passengers transported. Like in the case of "Number of working vehicles", the data on the number of departures for the selected lines are also collected from the SfPT monitoring and control system.

Line capacity (sets/day)

Line capacity, expressed in sets per day, offers insights into a line's ability to accommodate passengers effectively. It quantifies the maximum passenger load that a specific line can handle within a given day, revealing the line's operational capability. A higher line capacity not only signifies efficient resource utilisation but also indicates the potential to serve a greater number of passengers effectively, making it a key factor in optimising public transport services. Like in the case of previous parameters, the data regarding line capacity are also collected from the SfPT.

Operational costs (RSD per day)

Operational costs denote the financial resources necessary for the daily operation and maintenance of the public transport system. These costs encompass a spectrum of expenditures, including fuel, maintenance, labour, personnel, administrative activities and more. A deep comprehension of operational costs is vital for assessing the financial intricacies of public transport services, ensuring sustainability, and optimising resource allocation efficiency. In the context of DEA analysis, line efficiency is indicated by the ability to transport more passengers at lower operational costs. For this paper, the operational costs parameter was calculated by multiplying the cost per kilometre by the number of kilometres a line covers in a day. The data regarding the number of kilometres a line covers in a day were collected from the SfPT as well, while values for cost per kilometre have been sourced from the "Contract on the implementation of performing the utility activities of city transport of passengers in the territory of the City of Belgrade" between SfPT and public operator in the public transport system in the City of Belgrade, "GSP" Belgrade. The contract foresees different values for cost per kilometre for different types of vehicles (solo bus, articulated bus, tram, trolleybus, etc.

As the idea is to analyse the efficiency of each line separately, the selected input parameters best describe the differences between the selected lines. Other potential input parameters, such as operating hours, demographic characteristics or residential density, were not selected since the difference in their values is negligible. Operating hours for all urban lines in the City of Belgrade start around 4 AM and end around midnight, so the total operating hours per line are in correlation with the number of vehicles. As represented by the spatial distribution, all lines serve central urban areas with high residential density and the same demographic characteristics. Considering this, even if we were able to present these characteristics separately for each line, we believe that the differences in their values would be negligible and therefore not useful for the model. The daily vehicle mileage is not listed as a separate input parameter since it is already accounted for within the operational cost parameter. In Belgrade's public transport system, operational costs are calculated as the unit cost value, defined for different vehicle types, multiplied by the number of vehicles and the daily distance travelled per vehicle.

Unlike input parameters, the defined model has only one output parameter, the number of transported passengers (passengers/day). **The number of transported passengers (P)** is a crucial output parameter in evaluating the performance and efficiency of public transport lines. It quantifies the core objective of any public transportation system to efficiently move people from one location to another. The higher the number of transported passengers, the more effectively a public transport line fulfils its primary purpose. This output parameter directly reflects the quality of service provided to the public and is a key indicator of the success of a public transport system. In the context of DEA analysis, a line's efficiency is often measured by its ability to transport a greater number of passengers, showcasing its effectiveness in meeting the city's transportation needs. The data on the number of transported passengers represent the results of the research on passenger counting conducted by the authors in October and November 2022. As part of this research, the authors conducted a systematic passenger count on a sample of vehicles on the mentioned lines (each departure of a vehicle in the sample is counted). To facilitate subsequent analysis and the forecasting of passenger numbers for the entire operational period of the lines, the authors have developed a specialised data processing tool known as the public transport demand analysis tool (PTD). This software enables comprehensive analysis and calculates various parameters regarding public transport systems. *Table 2* shows the input and output data for each of the DMUs.

Table 2 – Input and output data for each of the DMUs

DMUs	Input				Output
	N [-]	n_{dep} [-]	C [sets/day]	T [RSD/day]	P [-]
7	20	252	70,013.99	1,677,592.15	41,776
9	19	256	60,890.02	1,571.069.69	35,696
16	20	384	59,840.02	1.090.556.20	39,441
18	21	279	42,400.00	1,279,612.34	39,736
23	25	323	52,480.00	1,511,769.08	56,870
29	28	497	64,875.00	1,376,432.68	34,439
31	18	381	64,480.03	759,405.46	44,622
40	21	227	28,690.02	794,161.76	18,823
56	18	323	51,680.01	1,235,834.28	29,460
65	18	283	45,280.02	1.038,373.96	33,838
88	22	310	45,760.01	1,521,606.77	38,149
95	31	340	54,079.99	1,134,366.52	52,611

5. RESULTS AND DISCUSSION

Following the establishment of the model and definition of all sets of input and output data and their values, it is possible to enter the analysis phase. To determine the relative efficiency of the observed lines, a total of 12 linear programming assignments have been set up and solved. The software package Excel Solver was used to solve these assignments. *Figure 2* shows the efficiency scores for each of the lines.

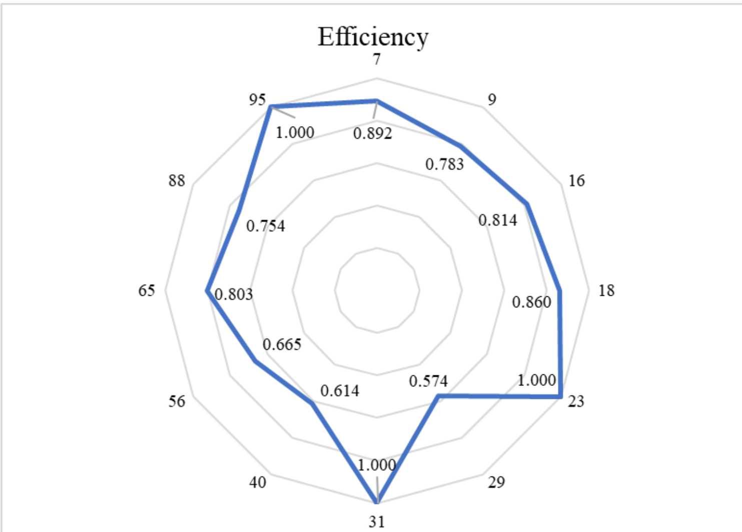

Figure 2 – Efficiency scores for each DMU

Lines with efficiency scores equal to 1.000 are considered highly efficient, operating at optimal levels. These include Lines 23, 31 and 95. On the other hand, lines with efficiency scores below 1.000 but higher than 0.700 fall into the moderately inefficient category, while lines with efficiency scores of 0.700 or below belong to the highly inefficient category. Lines 29 and 40 have relatively low-efficiency scores of 0.574 and 0.614, respectively, indicating room for improvement in their resource utilisation and operational effectiveness. Line 56 also falls into the less efficient category with an efficiency score of 0.665. Additionally, Line 88 (efficiency score: 0.754), Line 9 (efficiency score: 0.783), Line 65 (efficiency score: 0.803), Line 16 (efficiency score: 0.814), Line 18 (efficiency score: 0.860) and Line 7 (efficiency score: 0.892), fall into the moderately efficient range but may still benefit from improvements in resource allocation.

DEA generates reports such as the answer and sensitivity reports, which can suggest adjustments to certain parameters to improve efficiency. While DEA can identify inputs that may be reduced to enhance performance, it does not account for the potential interdependence among parameters. In complex systems like public transport, modifying one parameter often directly affects others. For example, decreasing the number of working vehicles or departures might improve technical efficiency by reducing input usage, but it could also lead to a decline in the number of passengers. As such, expert judgment is crucial when interpreting DEA results and defining interventions for individual routes. This ensures that decisions are both data-driven and sensitive to the interconnected dynamics of the system.

Moderately inefficient lines require targeted adjustments to improve their performance while maintaining service quality. Interventions for these lines primarily involve optimising resource allocation. This includes reducing the number of working vehicles to better match passenger demand and adjusting departure frequencies to avoid unnecessary trips. Since operational costs in Belgrade's public transport are determined by a predefined methodology based on vehicle type and mileage, cost reductions cannot be directly influenced. However, increasing efficiency through better vehicle utilisation and scheduling can indirectly contribute to cost-effectiveness by reducing unnecessary operational expenditures. Highly inefficient lines require more substantial interventions, as they often suffer from severe resource underutilisation or insufficient passenger demand. For these lines, reducing the number of working vehicles and optimising schedules is essential. Additionally, since operational costs are directly tied to vehicle mileage, cost efficiency can be improved by modifying routes to reduce total mileage, which requires strategic changes to where and how the line operates. This may involve rerouting certain sections, eliminating inefficient route extensions or consolidating overlapping services. In cases where a line has significantly low passenger demand, increasing efficiency requires service attractiveness improvements, such as better connectivity, enhanced comfort and targeted marketing efforts to boost ridership.

The efficiency or inefficiency of the analysed public transport lines can be understood by considering specific parameters within the DEA model. Efficient lines, like Lines 23, 31 and 95, achieve high-efficiency scores by effectively balancing the number of working vehicles and departures with their line capacity and operational costs. This indicates optimal resource allocation and cost management. Inefficient lines, such as Lines 29, 40 and 56, exhibit lower efficiency scores due to imbalances in these parameters. They tend to have either excessive operational costs or suboptimal resource utilisation in relation to their capacity, highlighting areas for potential improvement in cost control and resource allocation. For example, Line 29 can become more efficient by optimising the number of working vehicles. Consider reducing the fleet size to match passenger demand more effectively. Additionally, focus on cost reduction measures, such as improving fuel efficiency or renegotiating maintenance contracts, to lower operational costs while maintaining service quality. Line 40 has the lowest number of transported passengers from all observed lines by far, and this is its biggest concern. Improving service quality and attractiveness to passengers can be a viable approach. This could involve optimising schedules, maintaining vehicles for comfort and cleanliness, and implementing effective marketing and promotion strategies. By providing a more appealing and efficient service, Line 40 may increase its passenger numbers, ultimately contributing to improved overall efficiency and financial sustainability. Line 56 can work towards greater efficiency by examining its scheduling practices. Consider adjusting departure schedules to optimise resource utilisation and reduce operational costs.

After our initial analysis, we have identified three efficient lines within the public transport system, specifically Line 23, Line 31 and Line 95. Now, to further refine our assessment and uncover the pinnacle of operational excellence, we turn to super-efficiency measurement using DEA. This advanced analysis allows us to distinguish which among the already efficient lines can be classified as "super-efficient", surpassing the norm in resource allocation and cost management. This investigation will unveil invaluable insights into the highest-performing line, potentially offering crucial benchmarks for optimising the broader transportation network. In the context of our super-efficiency analysis using DEA, it is important to note that super-efficiency is achieved when the constraints applied to the specific DMU under evaluation effectively "turn off". This unique status signifies that the DMU is no longer subject to the constraints that apply to others, underscoring its exceptional performance and efficiency within the framework. *Figure 3* shows the super-efficiency scores for the already efficient lines.

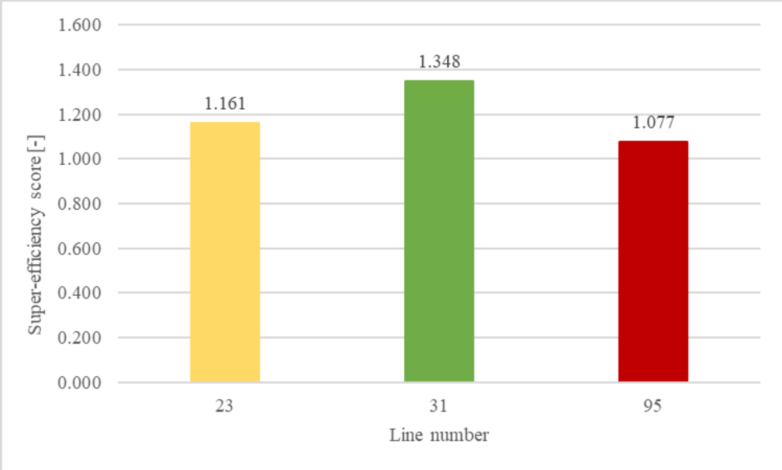

Figure 3 – Super-efficiency scores for each of the efficient DMUs

From *Figure 3,* it can be concluded that Line 31 is super-efficient. The outstanding super-efficiency score of 1.348 achieved by Line 31 not only underscores its remarkable performance but also strongly affirms the expert opinion that Line 31 is considered the "most efficient" among all 12 lines under analysis. Line 31's success can be attributed to a harmonious synergy between resource allocation, scheduling and cost management. It effectively balances the number of working vehicles, departures, line capacity and operational costs to maximise passenger transport efficiency. As the top-performing line, Line 31 can serve as a valuable benchmark and source of best practices for other lines in the public transport system seeking to emulate its remarkable performance and further enhance their operational efficiency.

6. CONCLUSION

In an era marked by urbanisation and growing environmental concerns, the efficiency and effectiveness of urban public transport systems hold paramount importance in the pursuit of sustainable urban development. The assessment of these systems' performance has gained significant momentum, driven by the need for continuous improvement in the face of competition and financial constraints. Measuring the efficiency of public transport systems is essential in contemporary urban planning and transportation management, as cities strive to balance economic growth with environmental sustainability. This study uses the DEA methodology to evaluate the efficiency of public transport lines in Belgrade, with a specific focus on optimising resource allocation within the city's public transport system.

DEA, renowned for its versatility in evaluating complex systems, served as a robust analytical tool in this study. The input-oriented CCR DEA model was selected for its ability to prioritise resource minimisation while maintaining consistent output levels. Twelve prominent public transport lines in Belgrade, spanning various transportation modes, were subjected to rigorous analysis. Four key input parameters, number of working vehicles, departures, line capacity and operational costs, were selected with a pivotal output parameter, the number of transported passengers. This comprehensive framework captured the intricate dynamics of resource allocation, cost management and service effectiveness within the public transport system.

The study yielded significant insights into the efficiency of Belgrade's public transport lines. Notably, Lines 23, 31 and 95 emerged as exemplars of efficiency, achieving perfect scores of 1.000. These lines demonstrated a remarkable ability to strike an optimal balance between resource inputs and operational costs, thereby exemplifying their proficiency in passenger transport efficiency. Conversely, several lines exhibited varying levels of inefficiency, indicating the need for strategic improvements in resource allocation, scheduling and cost management. Lines 29 and 40, with efficiency scores of 0.574 and 0.614, respectively, presented opportunities for enhancing their operational effectiveness. Moreover, Line 56, with an efficiency score of 0.665, also showed potential for optimisation. In contrast, Line 88, Line 9, Line 65, Line 16, Line 18 and Line 7 fell into the moderately efficient range, suggesting room for improvement in their resource allocation and cost-efficiency strategies. Notably, the introduction of super-efficiency analysis unveiled Line 31 as a remarkable standout, further underscoring its exceptional performance and offering valuable benchmarks for other lines to aspire to. This research effectively validated the primary hypothesis: lines that judiciously allocated resources while attracting higher passenger numbers exhibited enhanced efficiency. Such lines excelled in optimising resource allocation, leading to reduced operational costs while simultaneously improving passenger transport effectiveness. To analyse the overall system efficiency, it is necessary to examine the remaining lines and group them according to efficiency levels. Having results for each individual line would allow us to categorise them based on the required interventions, thereby identifying priority interventions at the system level.

In conclusion, this study underscores the pivotal role of efficient public transport systems in the context of urban development and environmental sustainability. By harnessing the power of DEA, this research not only provides valuable insights into Belgrade's public transport system but also furnishes a methodological blueprint for assessing and enhancing urban public transport efficiency in similar settings. The findings underscore the imperative of resource optimisation and cost-effective service delivery, setting a compelling agenda for future research and policy interventions in the realm of urban transportation.

As urbanisation continues to shape the global landscape, urban planners, policymakers and researchers must collaborate to develop transportation systems that efficiently serve growing populations while minimising their environmental footprint. This study represents a stepping stone towards achieving this goal, offering a data-driven approach to enhance public transport systems in cities worldwide. Future research endeavours in this domain should further explore strategies to optimise resource allocation, reduce operational costs and bolster passenger transport efficiency, ultimately fostering more sustainable and liveable urban environments. Efficiency improvement in public transport systems involves optimising resource allocation and cost management. Strategies typically revolve around matching the number of vehicles and departures with passenger demand, reducing operational costs through measures like improving fuel efficiency or renegotiating maintenance contracts, and fine-tuning scheduling practices to minimise idle time and underutilisation of resources. Overall, enhancing efficiency aims to strike a better balance between resource allocation and cost management while ensuring the effective provision of passenger transport services.

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