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Multi-objective Windy Postman Problem in a Fuzzy Transportation Network

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ABSTRACT
Researchers have become increasingly captivated by the windy postman problem (WPP), a major combinatorial optimisation problem with several practical applications. It is crucial to take the experts' belief levels into account when modelling such a real-world application since these applications frequently involve uncertain aspects. A fuzzy set is one of the tools that might be regarded as appropriate for modelling such human perspectives. Applying fuzzy set theory to a multi-objective windy postman problem is the focus of this study. Maximising the overall profit and minimising the transportable time of the route visited by a postman are the objectives of the problem. In an effort to solve the fuzzy multi-objective windy postman problem (FMWPP), we have developed a chance-constrained programming model (CCPM). Subsequently, the epsilon-constraint method, a classical multi-objective solution methodology, is used to solve the deterministic transformation of the relevant CCPM. Moreover, the model is solved using two multi-objective genetic algorithms (MOGAs): fast Pareto genetic algorithm (FastPGA) and nondominated sorting genetic algorithm II (NSGAII). To demonstrate the proposed model, a numerical example is presented. We conclude by comparing the performance of the MOGAs on four randomly generated FMWPP instances.

KEYWORDS
transportation; windy postman problem; epsilon constraint method; multi-objective genetic algorithms; performance metrics.

1. INTRODUCTION

Among the significant classical combinatorial optimisation problems, M. K. Kwan [1] introduced the Chinese postman problem (CPP). The basic premise of CPP is that a postman sets out from the post office on a mission to deliver all the essential products by way of the city's many streets and lanes. Every street in a city must be crossed at least once by the postman, who must travel the bare minimum distance to deliver the mail. This situation is defined as follows and is thought of as a network optimisation problem. Let $\mathcal{N}$ be an undirected connected network. $V_{\mathcal{N}}$ and $E_{\mathcal{N}}$ are respectively considered as the set of all vertices and the set of all edges in $\mathcal{N}$. The objective is to find the smallest closed walk of $\mathcal{N}$ such that every edge in $E_{\mathcal{N}}$ is traversed at least once in the network. The shortest closed walk is explored by considering the symmetrical nature of the costs and time attached to each edge. In other words, the cost and time required for travelling any given edge

e_{ij} in either direction in the network is the same. On the other hand, there are cases where the related edge costs of the network problem start to skew, turning it into a windy postman problem. The windy postman problem (WPP) was introduced by E. Minieka [2] and is concerned with finding the minimum cost of a closed tour that passes through every edge at least once of an undirected network. In this problem, the cost of traversing through an edge e_{ij} in one direction (c_{ij}) can differ from the cost of traversing through it in the other direction (c_{ji}). Among the many interesting variants of the ancient Chinese postman problem, the windy postman problem (WPP) stands out. Both directions of an edge's traversal on a WPP network incur different costs. Several scholars [3, 4] have studied the problem, which is deemed NP-complete [5]. A. Corberán et al. [6] subsequently introduced a new set of facet-inducing inequalities for the WPP. Afterwards, a variant of WPP called the min-max version with multiple vehicles was suggested by Benavent et al. [7]. The goal of this task is to find the longest tour with the minimum possible length. They looked at the associated polyhedron and suggested an integer linear programming (ILP) formulation. Following that, a metaheuristic for the min-max form of the windy rural postman problem with k vehicles was proposed by Benavent et al. [8]. Later, Benavent et al. [9] described various new facet-inducing inequalities derived from the WRPP and applied them to the min-max windy rural postman problem. The periodic edge routing problem [10], the time-dependent windy rural postman problem [11], hierarchical WPP with variable service prices [12], etc., have all been the subject of further investigation on WPP. Considering the existing literature, there is a dearth of published works in the multi-objective domain, which are discussed hereafter. Typically known as the $makespan$ in the scheduling theory, the bi-objective capacitated arc routing problem (CARP) was developed by Lacomme et al. [13]. Here, the objective is to minimise the overall routing cost as well as the distance of the longest journey. A multi-objective genetic algorithm (MOGA) was used by the authors to explore a solution for the problem. The multi-objective CARP was solved by Grandinetti et al. [14] using the ε-constraint technique in their subsequent work. This method minimised the overall transportation cost, $makespan$ and the total vehicles needed to service all the required routes. Using a hybrid multi-objective simulated annealing technique, Rabbani et al. [15] demonstrated the capacitated WPP with multiple vehicles and found a solution to the problem.

All of the studies that take WPP model variants into account do so in a crisp context with well-defined associated model parameters. But mostly in real-world contexts, where certain circumstances are modelled as network optimisation problems, inevitably contain some degree of indeterminacy (uncertainty). Uncertainty in real-world network problems is inevitable and arises due to the manifestation of certain unknowable aspects, such as inadequate or missing information, a lack of proof and incorrect assessments. Consequently, classical network models are no longer applicable in such cases. Therefore, to depict the unknown or indeterminate phenomena, we need a suitable tool.

Previous research has shown that probability theory can be used to represent unknown events relevant to network optimisation. The idea of a random graph, where the edges and vertices are chosen at random with equal chances, was initially put up by P. Erdős and A. Rényi [16] in this context. The study on the connectedness of random graphs was reported by E. Gilbert [17] in the same year. The problem for a random network was resolved by Tan et al. [18] within the framework of CPP. On the other hand, probability theory is not the right tool to use for every ambiguous occurrence. The computed probability distribution approaches the long-run cumulative frequency, especially with a big enough sample size. When this occurs, inferential probability distributions are a suitable basis for representing unknown events in probability theory. On the flip side, probability theory loses its legitimacy as a method for representing uncertainty when the sample size is insufficient or none are available. In real-world network problems, it is common for factors which are poorly defined or nebulous, like trip time, fuel cost, toll tax and distance between cities. Inadequate data, lack of historical evidence, incorrect interpretations, etc., often lead to what is known as non-stochastic uncertainty in these parameters. As a result, we cannot avoid asking subject-matter experts to rate the likelihood of each event's occurrence. Fuzzy set theory is one such approach to handling human ambiguity, which is characterised as non-stochastic uncertainty. Wang and Wen [19] investigated a time-bound CPP with an imprecise time frame in this paradigm. Subsequently, the hierarchical CPP utilising fuzzy time parameters was suggested by Sökmen and Yilmaz [20]. Within the context of uncertainty theory [21], various network problems can be observed in the literature (Majumder [22], Samanifar et al. [23, 24]). Recently, Samanifar et al. [25] also put forward the WPP with unknown parameters under an uncertain paradigm.

Considering the above-mentioned literature, so far as we are aware, no multi-objective WPP has been investigated in the aforementioned research on the windy postman problem in either crisp/deterministic or uncertain settings. Here, we consider a weighted connected fuzzy transportation network, where the associated edge weights are represented as fuzzy numbers. A postman (courier person) is supposed to traverse this network for the shipment of the articles(goods). Henceforth, we have introduced a fuzzy multi-objective windy postman problem (FMWPP) on a transportation network in this article. To understand the application of FMWPP, we present a suitable example. A newspaper delivery person in a scenic, hilly tourist town is tasked with delivering newspapers to residences, hotels and cafés located along winding, steep roads. A network with edges (streets) and nodes (delivery locations) is a good metaphor for the delivery area. The goal is to find a closed delivery route that goes through every street at least once, all the while trying to maximise total *revenue* and minimise total *travel time*, which are two competing (conflicting) and uncertain objectives. The associated parameters, *revenue* and *travel time,* are both asymmetric because their values change depending on the direction of travel and are uncertain because of things like traffic, weather and fluctuating delivery volumes. For instance, if the delivery person serves hotels and villas on the way uphill, he may earn more money, but if he returns downhill, he may earn less or no money at all. The variables that affect *travel time* include, but are not limited to, weather (such as fog or rain), the number of tourists, steep gradients (longer times uphill) and the direction of travel. In light of the above, it is reasonable to assume that both the *revenue* and the *travel time* will be subject to some degree of uncertainty. Therefore, fuzzy variable becomes a good alternative to describe these parameters. One way to look at this problem is as an application of FMWPP where the delivery person aims to traverse each street at least once, such that it maximises the total fuzzy *revenue* by favouring more profitable directions, while also minimising total fuzzy *travel time* by favouring faster and more efficient paths. Subsequently, we have summarised the important contributions of our proposed problem below.

— FMWPP with profit maximisation and time minimisation is proposed;
— With all input parameters treated as fuzzy variables, the FMWPP is formulated as a chance-constrained programming model (CCPM);
— The deterministic model transformation is then resolved using the epsilon-constraint method [26], a classical multi-objective technique. Following this, the deterministic model of FMWPP is solved using two MOGAs: FastPGA [27] and nondominated sorting algorithm II (NSGAII) [28]. Here, it is to be mentioned that to the best of our knowledge, FastPGA has yet to be applied to any multi-objective network problem as observed in the literature.

Chance-constrained programming [29, 30] can be used to deal with optimisation problems associated with fuzzy parameters. The core idea underlying this powerful modelling method is to optimise the critical value of the fuzzy objective function(s) with a user-specified confidence (chance) level(s) while taking into consideration distinctive chance restrictions. In a chance-constrained programming model (CCPM), a decision-maker provides a confidence level α, the value of which is assumed to be selected from the closed interval $[0,1]$. In the subsequent sections, we use the idea of CCPM to develop our suggested model, FMWPP.

The outline of the rest of the article is mentioned as follows. In Section 2, we present the essential and fundamental concepts related to our proposed study. We illustrate the model's formulation in Section 3, and then present the deterministic transformation of the model in Section 4. Section 5 delves into the methodologies that are used to solve the proposed model. Section 6 provides a discussion of the necessary results of the proposed FMWPP. Lastly, Section 7 presents the findings of our investigation.

2. RUDIMENTARY CONCEPTS

In the parts that follow, we will review some basic ideas connected to fuzzy variables, credibility theory and triangular fuzzy variables (TFV) in order to explain the fuzzy multi-objective windy postman problem.

L.A. Zadeh [31] was the first to propose the idea of a fuzzy set. Accordingly, L.A. Zadeh [32] put forth the idea of a possibility measure as a way to quantify a fuzzy event. But a self-duality feature for a possibility measure does not exist. Later, the credibility measure was proposed by B. Liu and Y. Liu [33] to address the shortcomings of the possibility measure. Afterwards, credibility theory was established by B. Liu [34] as a mathematical subfield that investigates the credibility-based behaviour of fuzzy phenomena.

Assume Θ is a non-empty set, the power set of Θ is denoted as $\mathcal{P}(\Theta)$, and Pos is a measure of possibility. The triplet, $\Theta, \mathcal{P}(\Theta),$ and Pos constitute what is known as a possibility space. Consequently, a fuzzy variable is a function that maps the set of real numbers to a possibility space $(\Theta, \mathcal{P}(\Theta), Pos)$. Consider μ as the membership function of a fuzzy variable η. Then, we may define the possibility (Pos), necessity (Nec) and credibility (Cr) of a fuzzy event $\{\eta \geq q\}$ as mentioned below:

$$Pos\{\eta \geq q\} = \sup_{x \geq q} \mu(x);$$

$$Nec\{\eta \geq q\} = 1 - \sup_{x < q} \mu(x);$$

$$Cr\{\eta \geq q\} = \frac{1}{2}(Pos\{\eta \geq q\} + Nec\{\eta \geq q\}).$$

Definition 2.1 [35]: The expression $\psi(x) = Cr\{\theta \in \Theta | \eta(\theta) \leq x\}$ defines the credibility distribution $\Psi: \mathfrak{R} \rightarrow [0, 1]$ of a fuzzy variable ψ. Furthermore, for every α that falls inside the interval $[0, 1]$, the existence and uniqueness of the inverse function $\psi^{-1}(\alpha)$ indicate that the credibility distribution ψ is regular. The term used to describe the inverse of η is the inverse credibility distribution [36].

For example, a TFV η which is represented by a triplet (q_1, q_2, q_3) with $q_i \in \mathfrak{R}, \forall i = 1,2,3$ and $q_1 < q_2 < q_3$. Accordingly, the membership function and the credibility distribution of η are shown respectively in *Eqs. (1), (2)* and *(3)*.

$$\mu(\eta_x) = \begin{cases} \dfrac{x - q_1}{q_2 - q_1}; & if\ q_1 \leq x \leq q_2; \\ \dfrac{x - q_2}{q_3 - q_2}; & if\ q_2 < x \leq q_3; \\ 0 & ;\ otherwise. \end{cases} \tag{1}$$

$$\psi(\eta \leq x) = \begin{cases} 0 & ;\ if\ x < q_1; \\ \dfrac{x - q_1}{2(q_2 - q_1)}; & if\ q_1 \leq x < q_2; \\ \dfrac{x + q_3 - 2q_2}{2(q_3 - q_2)}; & if\ q_2 \leq x < q_3; \\ 1 & ;\ if\ x \geq q_3. \end{cases} \tag{2}$$

$$\psi(\eta \geq x) = \begin{cases} 1 & ;\ if\ x < q_1; \\ \dfrac{2q_2 - q_1 - x}{2(q_2 - q_1)}; & if\ q_1 \leq x < q_2; \\ \dfrac{q_3 - x}{2(q_3 - q_2)}; & if\ q_2 \leq x < q_3; \\ 0 & ;\ if\ x \geq q_3. \end{cases} \tag{3}$$

In addition, as seen in *Eqs. (4)* and *(5)*, the inverse credibility distributions of the TFV, denoted as η, for a predetermined confidence level β are calculated respectively following *Eqs. (2)* and *(3)*.

$$\psi^{-1}(\alpha) = \begin{cases} (1 - 2\beta)q_1 + 2\beta q_2 & ;\ if\ 0 \leq \beta \leq 0.5; \\ (2 - 2\beta)q_2 + (2\beta - 1)q_3; & if\ 0.5 < \beta \leq 1. \end{cases} \tag{4}$$

$$\psi^{-1}(\alpha) = \begin{cases} 2\beta q_2 + (1 - 2\beta)q_3 & ;\ if\ 0 \leq \beta \leq 0.5 \\ (2\beta - 1)q_1 + (2 - 2\beta)q_2; & if\ 0.5 < \beta \leq 1. \end{cases} \tag{5}$$

Following the ideas put out by Y. Liu and J. Gao [37] on independent fuzzy variables and regular credibility distributions, J. Zhou et al. [36] demonstrated a significant operational law for such variables as mentioned below.

Theorem 2.1 [35]: The independent fuzzy variables $\eta_1, \eta_2, \eta_3 \dots \eta_n$ are defined by their respective regular credibility distributions $\psi_1, \psi_2, \psi_3, \dots, \psi_n$. A fuzzy variable $\eta = f(\eta_1, \eta_2, \eta_3 \dots \eta_n)$ which is strictly increasing with respect to $\eta_1, \eta_2, \eta_3 \dots \eta_m$ and strictly decreasing with respect to $\eta_{m+1}, \eta_{m+2}, \eta_{m+3} \dots \eta_n$ has the inverse credibility distribution:

$$\psi^{-1}(\alpha) = f\left(\psi_1^{-1}(\alpha), \psi_2^{-1}(\alpha), \dots \psi_m^{-1}(\alpha), \psi_{m+1}^{-1}(1-\alpha), \psi_{m+2}^{-1}(1-\alpha), \dots \psi_n^{-1}(1-\alpha)\right). \tag{6}$$

3. FUZZY MULTI-OBJECTIVE WINDY POSTMAN PROBLEM

This section presents the model formation of the proposed FMWPP using the chance-constrained programming technique.

3.1 The proposed model

Consider an interconnected network $\mathcal{N} = (Q_{\mathcal{N}}, \mathcal{W}_{\mathcal{N}})$, where $Q_{\mathcal{N}} = \{v_1, v_2, \dots, v_m\}$ and $\mathcal{W}_{\mathcal{N}}$ are finite sets of vertices and edges of $\mathcal{N}$, respectively. There are n pairs of vertices in the set $E_{\mathcal{N}}$, denoted as $\langle v_i, v_j \rangle$, such that each pair represents an edge e_{ij}. Each e_{ij} is associated with a non-negative finite quantities ω_{ij}, also referred to as the weight of e_{ij}. Furthermore, the associated weight ω_{ij} of e_{ij} will differ from e_{ji}'s associated weight ω_{ji} when traversed from v_j to v_i.

We have examined a fuzzy multi-objective windy postman problem for a transportation network $\mathcal{N}$ in this article, where the associated parameters to an edge are represented as a triangular fuzzy number. A street in the suggested problem is represented by an edge, e_{ij}, and a vertex is the point where the streets connect. As a result, the edge represented by e_{ij} joins the neighbouring vertices v_i and v_j. When traveling down a street e_{ij}, a courier company's delivery employee or postman receives compensation for navigating each lane of the road in order to deliver items to clients on time. The postman (courier person) is responsible for covering all transportable outlays incurred during the trip, including fuel costs, toll taxes, congestion fees, vehicle maintenance costs, etc. Furthermore, because each street has two lanes, traffic congestion may cause the time needed to travel each of the lanes, or e_{ij} and e_{ji} of a street, to vary within a given day. Hence, the associated parameters to an edge are most likely to be asymmetric in nature. More specifically, at any one time, one lane of the street may have more traffic than the other. When driving on a road like this, one may experience varying travel expenses (such as gasoline costs for the vehicle, congestion fees, toll taxes, etc.) and delivery times to the recipient. Accordingly, the courier distribution person will attempt to make the most of the overall earnings from the whole commission amount after accounting for all potential road travel charges, such as fuel costs, congestion fees, toll taxes, vehicle overhaul costs, etc. Concurrently, the delivery person endeavours to reduce the overall travel time spent on the trip. The shortest closed walk (tour) of $\mathcal{N}$ is used to optimise the delivery person's total overall profit and journey time, making sure that every edge e_{ij} of $\mathcal{W}_{\mathcal{N}}$ is walked at least once. In actuality, we treat all related parameters – that is, commission amount, travel expense and travel time – as triangular fuzzy variables in the proposed FMWPP, as explained below:

- $\eta_{C_{ij}}$: fuzzy commission amount that the delivery person receives for passing a street e_{ij};
- $\eta_{E_{ij}}$: fuzzy travel expenditure incurred by the courier person throughout their journey through e_{ij};
- $\eta_{T_{ij}}$: fuzzy duration of time it takes for the delivery person to traverse via e_{ij}.

There are two goals to the suggested model: first, to maximise the postman's earnings from the tour; and second, to minimise the overall travel time of the trip. The model is developed by using the concept of CCPM.

The proposed FMWPP is presented below in Model (7):

$$\begin{cases} Max\ \bar{Z}_1 \\ Min\ \bar{Z}_2 \\ subject\ to: \\ \psi\left\{\left[\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\left(\eta_{C_{ij}}x_{ij}+\eta_{C_{ji}}x_{ji}\right)-\left(\eta_{E_{ij}}x_{ij}+\eta_{E_{ji}}x_{ji}\right)\right)\right]\geq \bar{Z}_1\right\}\geq \alpha_1; \\ \psi\left\{\left[\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\eta_{T_{ij}}x_{ij}+\eta_{T_{ji}}x_{ji}\right)\right]\leq \bar{Z}_2\right\}\geq \alpha_2; \\ \sum_{j=1}^{n}x_{ij}-\sum_{k=1}^{n}x_{ki}=0, \qquad i=1,2,\dots,n; \\ x_{ij}+x_{ji}\geq 1,\ e_{ij}\in\mathcal{W}_{\mathcal{N}}; \\ x_{ij}\in\{0,1\},\ e_{ij}\in\mathcal{W}_{\mathcal{N}}; \\ \eta_{C_{ij}}\neq\eta_{C_{ji}}, \eta_{E_{ij}}\neq\eta_{E_{ji}}, \eta_{T_{ij}}\neq\eta_{T_{ji}}. \end{cases} \tag{7}$$

Here, $\alpha_1$and α_2 regulate the chance levels at which the first and second constraints hold good. The crucial values relating to overall profit and total elapsed time connected with a postman trip, which correspond to the first and second constraints, respectively, are represented by the objectives $\bar{Z}_1$ and $\bar{Z}_2$. The first constraint establishes the optimistic α_1 – estimated total profit linked to the postman's α_1 – tour. In addition, the second constraint establishes the α_2 – pessimistic estimated overall time needed to complete the postman's associated α_2 – route. Furthermore, each vertex v_i is associated with a closed walk determined by the third constraint, and each edge is traversed at least once, as confirmed by the fourth constraint. According to the proposed Model (7), both objectives are optimised while optimising a path (a closed walk that visits each edge at least once). For each edge e_{ij}, we have a decision variable x_{ij} that can take on the values 0 or 1. The optimal path of Model (7) includes e_{ij} if and only if x_{ij}=1, else it discards e_{ij}.

4. DETERMINISTIC TRANSFORMATION

In this section, based on the credibility theory, we propose the deterministic equivalent model of the FMWPP as presented in Model (7).

Theorem 4.1: Let $\eta_{C_{ij}}$, $\eta_{E_{ij}}$ and $\eta_{T_{ij}}(i,j = 1,2,\dots m)$ be the independent triangular fuzzy variables (TFVs), where $\eta_{C_{ij}} = (C_{ij1}, C_{ij2}, C_{ij3})$, $\eta_{E_{ij}} = (E_{ij1}, E_{ij2}, E_{ij3})$ and $\eta_{T_{ij}} = (T_{ij1}, T_{ij2}, T_{ij3})$, $i,j = 1,2,\dots m$. Then the deterministic equivalent of FMWPP as presented in Model (7) is given by:

— when $0 \le \alpha_1 \le 0.5$ and $0 \le \alpha_2 \le 0.5$:

$$\begin{cases} Max\ \bar{Z}_1 = \begin{bmatrix} 2\alpha_1 \left\{ \sum_{i=1}^{m} \sum_{j=1, j\neq i}^{m} \left(\left(\eta_{C_{ij2}} x_{ij} + \eta_{C_{ji2}} x_{ji} \right) - \left(\eta_{E_{ij2}} x_{ij} + \eta_{E_{ji2}} x_{ji} \right) \right) \right\} + \\ (1-2\alpha_1) \left\{ \sum_{i=1}^{m} \sum_{j=1, j\neq i}^{m} \left(\left(\eta_{C_{ij3}} x_{ij} + \eta_{C_{ji3}} x_{ji} \right) - \left(\eta_{E_{ij3}} x_{ij} + \eta_{E_{ji3}} x_{ji} \right) \right) \right\} \end{bmatrix} \\ Min\ \bar{Z}_2 = \begin{bmatrix} (1-2\alpha_2) \left\{ \sum_{i=1}^{m} \sum_{j=1, j\neq i}^{m} \left(\eta_{T_{ij1}} x_{ij} + \eta_{T_{ji1}} x_{ji} \right) \right\} + \\ 2\alpha_2 \left\{ \sum_{i=1}^{m} \sum_{j=1, j\neq i}^{m} \left(\eta_{T_{ij2}} x_{ij} + \eta_{T_{ji2}} x_{ji} \right) \right\} \end{bmatrix} \\ subject\ to: \\ \sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{n} x_{ki} = 0, \quad i = 1,2,\dots,n; \\ x_{ij} + x_{ji} \ge 1, e_{ij} \in \mathcal{W}_{\mathcal{N}}; \\ x_{ij} \in \{0,1\}, e_{ij} \in \mathcal{W}_{\mathcal{N}}; \\ \eta_{C_{ij1}} \neq \eta_{C_{ji1}}, \eta_{C_{ij2}} \neq \eta_{C_{ji2}}, \eta_{E_{ij1}} \neq \eta_{E_{ji1}}, \eta_{E_{ij2}} \neq \eta_{E_{ji2}}, \eta_{T_{ij1}} \neq \eta_{T_{ji1}}, \eta_{T_{ij2}} \neq \eta_{T_{ji2}}. \end{cases} \tag{8}$$

— when $0.5 < \alpha_1 \le 1$ and $0.5 < \alpha_2 \le 1$:

$$\begin{cases} Max\ \bar{Z}_1 = \begin{bmatrix} (2\alpha_1 - 1) \left\{ \sum_{i=1}^{m} \sum_{j=1, j\neq i}^{m} \left(\eta_{C_{ij1}} x_{ij} + \eta_{C_{ji1}} x_{ji} \right) - \left(\eta_{E_{ij1}} x_{ij} + \eta_{E_{ji1}} x_{ji} \right) \right\} + \\ (2-2\alpha_1) \left\{ \sum_{i=1}^{m} \sum_{j=1, j\neq i}^{m} \left(\left(\eta_{C_{ij2}} x_{ij} + \eta_{C_{ji2}} x_{ji} \right) - \left(\eta_{E_{ij2}} x_{ij} + \eta_{E_{ji2}} x_{ji} \right) \right) \right\} \end{bmatrix} \\ Min\ \bar{Z}_2 = \begin{bmatrix} (2-2\alpha_2) \left\{ \sum_{i=1}^{m} \sum_{j=1, j\neq i}^{m} \left(\eta_{T_{ij2}} x_{ij} + \eta_{T_{ji2}} x_{ji} \right) \right\} + \\ (2\alpha_2 - 1) \left\{ \sum_{i=1}^{m} \sum_{j=1, j\neq i}^{m} \left(\eta_{T_{ij3}} x_{ij} + \eta_{T_{ji3}} x_{ji} \right) \right\} \end{bmatrix} \\ subject\ to: \\ \sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{n} x_{ki} = 0, \quad i = 1,2,\dots,n; \\ x_{ij} + x_{ji} \ge 1, e_{ij} \in \mathcal{W}_{\mathcal{N}}; \\ x_{ij} \in \{0,1\}, e_{ij} \in \mathcal{W}_{\mathcal{N}}; \\ \eta_{C_{ij2}} \neq \eta_{C_{ji2}}, \eta_{C_{ij3}} \neq \eta_{C_{ji3}}, \eta_{E_{ij2}} \neq \eta_{E_{ji2}}, \eta_{E_{ij3}} \neq \eta_{E_{ji3}}, \eta_{T_{ij2}} \neq \eta_{T_{ji2}}, \eta_{T_{ij3}} \neq \eta_{T_{ji3}}. \end{cases} \tag{9}$$

Proof: Since $x_{ij} \in \{0,1\}$ for $i, j = 1,2, \dots m$, it follows from the extension principle (L.A. Zadeh [31, 32]) that the objective functions $\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\left(\eta_{C_{ij}}x_{ij} + \eta_{C_{ji}}x_{ji}\right) - \left(\eta_{E_{ij}}x_{ij} + \eta_{E_{ji}}x_{ji}\right)\right)$ and $\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\eta_{T_{ij}}x_{ij} + \eta_{T_{ji}}x_{ji}\right)$ are also triangular fuzzy numbers. Here, $\boldsymbol{x}$ is a $\frac{m!}{(m-2)!}$ −dimension decision vector such that every $x_{ij} \in \boldsymbol{x}$. Furthermore, since $\eta_{C_{ij}}$, $\eta_{E_{ij}}$ and $\eta_{T_{ij}}(i, j = 1,2, \dots m)$ are the independent triangular fuzzy variables, then it follows from Theorem 2.1, and Eqs. (4) and (5), the chance-constraints $\psi\left\{\left[\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\left(\eta_{C_{ij}}x_{ij} + \eta_{C_{ji}}x_{ji}\right) - \left(\eta_{E_{ij}}x_{ij} + \eta_{E_{ji}}x_{ji}\right)\right)\right] \geq \bar{Z}_1\right\} \geq \alpha_1$ and $\psi\left\{\left[\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\eta_{T_{ij}}x_{ij} + \eta_{T_{ji}}x_{ji}\right)\right] \leq \bar{Z}_2\right\} \geq \alpha_2$ are equivalent to:

$$2\alpha_1\left\{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\left(\eta_{C_{ij2}}x_{ij} + \eta_{C_{ji2}}x_{ji}\right) - \left(\eta_{E_{ij2}}x_{ij} + \eta_{E_{ji2}}x_{ji}\right)\right)\right\} + (1-2\alpha_1)\left\{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\left(\eta_{C_{ij3}}x_{ij} + \eta_{C_{ji3}}x_{ji}\right) - \left(\eta_{E_{ij3}}x_{ij} + \eta_{E_{ji3}}x_{ji}\right)\right)\right\}$$

and

$$(1-2\alpha_2)\left\{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\eta_{T_{ij1}}x_{ij} + \eta_{T_{ji1}}x_{ji}\right)\right\} + 2\alpha_2\left\{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\eta_{T_{ij2}}x_{ij} + \eta_{T_{ji2}}x_{ji}\right)\right\};$$

— when $0 \leq \alpha_1 \leq 0.5$ and $0 \leq \alpha_2 \leq 0.5$.

And

$$(2\alpha_1 - 1)\left\{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\eta_{C_{ij1}}x_{ij} + \eta_{C_{ji1}}x_{ji}\right) - \left(\eta_{E_{ij1}}x_{ij} + \eta_{E_{ji1}}x_{ji}\right)\right\} + (2-2\alpha_1)\left\{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\left(\eta_{C_{ij2}}x_{ij} + \eta_{C_{ji2}}x_{ji}\right) - \left(\eta_{E_{ij2}}x_{ij} + \eta_{E_{ji2}}x_{ji}\right)\right)\right\}$$

and

$$(2-2\alpha_2)\left\{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\eta_{T_{ij2}}x_{ij} + \eta_{T_{ji2}}x_{ji}\right)\right\} + (2\alpha_2 - 1)\left\{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\left(\eta_{T_{ij3}}x_{ij} + \eta_{T_{ji3}}x_{ji}\right)\right\}.$$

— when $0.5 < \alpha_1 \leq 1$ and $0.5 < \alpha_2 \leq 1$:

Therefore, considering both the above-mentioned conditions, it directly follows Models (8) and (9).

5. SOLUTION METHODOLOGIES

The solution methodologies employed in this study to resolve the proposed problem are detailed in the following subsections.

5.1 Epsilon-constraint method

Y. Haimes et al. [26] proposed the epsilon-constraint method to deal with MOOP. By optimising a single objective and converting the others into constraints with user-defined values, this approach simplifies problems with numerous objectives into optimisation problems with a single objective. Efficiently solving problems in the non-convex region of the objective space is also achieved using this method. A two-objective function multi-objective optimisation problem (MOOP) is presented in Model (10), so that we can discuss the approach:

$$\begin{cases} Max\ Z_1(\boldsymbol{x}) \\ Min\ Z_2(\boldsymbol{x}) \\ subject\ to: \\ \quad g_u(\boldsymbol{x}) \geq 0, u = 1,2, \dots, U; \\ \quad h_w(\boldsymbol{x}) = 0, w = 1,2, \dots, S; \\ \quad \boldsymbol{x} \geq 0. \end{cases} \tag{10}$$

A q – dimensional decision vector $\boldsymbol{x}$, a set of inequality constraints $g_u(\boldsymbol{x})$ and a set of equality constraints $h_w(\boldsymbol{x})$ are all part of the Model (10), which also includes the objective functions $Z_1(\boldsymbol{x})$ and $Z_2(\boldsymbol{x})$. Therefore, Model (10) can be turned into a compromise single objective optimisation problem using the epsilon-constraint method as shown below in Model (11):

$$\begin{cases} Max\ Z_1(\boldsymbol{x}) \\ subject\ to: \\ \quad Z_2(\boldsymbol{x}) \le \epsilon_2; \\ \quad g_u(\boldsymbol{x}) \ge 0, u = 1,2,\dots,U; \\ \quad h_w(\boldsymbol{x}) = 0, w = 1,2,\dots,S; \\ \quad x \ge 0, \end{cases} \tag{11}$$

where ϵ_2 signifies the value that lies somewhere between the two extremes of $Z_2(\boldsymbol{x})$, and not necessarily the one that is propinquity to zero.

5.2 Multi-objective genetic algorithm

For optimisation problems with multiple objectives, an MOGA can generate several nondominated solutions in a single run. An attractive alternative for investigating several MOOP solutions is an MOGA, which may simultaneously search numerous unexplored areas of the convex, non-convex and discontinuous solution spaces of numerous complicated MOOPs, all at once. On top of that, MOGAs do not necessitate a decision maker to rank the objectives. With these features, MOGAs are well-suited to solve MOOPs. A MOGA's goal is to improve its nondominated solution generation over generations so that it approaches Pareto optimality. A nondominated front in the objective space is effectively created by these nondominated solutions. The first nondominated front of the solutions is the optimal one, and it corresponds to the Pareto front (PF). In order to solve difficult multi-objective optimisation problems, the multi-objective genetic algorithm (MOGA) put out by C. Fonseca and P. Flaming [38] has been attracting enormous interest. There are a plethora of MOGAs in the literature, including those by H. Eskandari et al. [27], K. Deb et al. [28], Zitzler and Thiele [39], A. Nebro et al. [40], and many more. In this study, two MOGAs, FastPGA [27] and NSGAII [28], are employed to solve the deterministic models of the proposed FMWPP.

Fast Pareto genetic algorithm

For large-dimensional search spaces, H. Eskandari et al. [27] developed the fast Pareto genetic algorithm, also known as FastPGA. This is a population-based multi-objective genetic algorithm. The FastPGA architecture includes a new method for fitness assignment and ranking strategy. The rapid dissemination of the Pareto optimum solution set is guaranteed by an elitist operator. Additionally, an operator for population regulation is also introduced that may dynamically adjust the population size as needed, up to a maximum that the user specifies, which is eventually the size of the set of nondominated solutions. Algorithm convergence behaviour and computing effort are both enhanced by the population regulation operator. We highlight the functioning concept of FastPGA in the paragraph that follows.

An initial population of solution vector, P_0, is generated at random, the solutions of which are then evaluated with respect to the m objective functions. Subsequently, for a particular generation t, a selection operator is used to choose pairs of solutions P_t' from prior populations P_{t-1}. Next, a new population of offspring, O_t is created by using crossover and mutation operators. In order to create a composite population CP_t, the solutions of O_t are evaluated and then merged with the solutions of the prior population P_{t-1}. The solutions of CP_t are then ranked based on the novel ranking strategy [27] using their fitness values into two distinct groups.

Using this ranking approach, the nondominated solutions are ranked first. This means that no solution is better than these solutions when all objective functions are considered at the same time. The second rank is used to identify all dominated solutions. In order to reproduce a solution, these ranks are utilised to assess its fitness:

— We can assess the fitness of the first-rank nondominated solutions by comparing them to each other and assigning a fitness value. Following the recommendation of K. Deb et al. [28], these fitness values are calculated using the crowding distance approach, which has been shown to help maintain diversity among the nondominated solutions in the Pareto optimal front;

— Each dominated solution in the second rank is compared to all other solutions and assigned a fitness value depending on the number of solutions it dominates. Any dominated solution x_i is considered in the fitness assignment together with the dominating solution. By giving every solution x_i a strength value $\boldsymbol{St}(\boldsymbol{x_i})$, we may determine how many solutions it dominates in the composite population $\boldsymbol{CP_t}$.

For each dominated solution, $\boldsymbol{x_i}$, the fitness value is calculated by taking the total strength values of all solutions it dominates and subtracting them from the sum of all solutions it is dominated by. When calculating its fitness value, FastPGA takes into account the solutions that are dominating and dominated with regard to a given solution $\boldsymbol{x_i}$. This approach facilitates a lower likelihood of two solutions having the same fitness value, and essentially provides more information on Pareto dominance and niching relations among the composite population's solutions. Following the calculation of fitness values for all alternate solutions in $\boldsymbol{CP_t}$, the solutions are compared, leading to one of three possible outcomes. Two solutions, ranked differently, are chosen in the first scenario. Here, we favour the solution that has the lower rank. In the second case, the fitness values of the two chosen solutions are different, but their ranks are the same. Here, the solution with having larger fitness value is preferred. Finally, it is possible for two solutions to have both identical ranks and fitness values, with a random preference given to any one of them.

To guarantee that the nondominated (non-inferior) solutions are passed down to next generations, an elitism operator with relatively high intensity is used successively after the ranking of the solutions of the population P_t is accomplished. To do this, all of the solutions from the prior generation's population, P_{t-1} are intermingled with the offspring population O_t. Based on the number of nondominated solutions found in the composite population CT_t, the combination of P_{t-1} with the created offspring O_t allows for the preservation of superior solutions and the discarding of inferior solutions in the next generation. With a large and static population size, the number of nondominated solutions tends to grow with each generation, which means that early generations experience a low elitism intensity. Too much elitism intensity could lead to premature convergence, while too little might make convergence excessively slow and computationally costly. Consequently, FastPGA makes use of a regulatory operator to continuously fine-tune the population size up to a user-specified upper limit.

Every successively created population in each generation goes through the same procedure until the termination condition is met.

Nondominated sorting genetic algorithm II

An elitist multi-objective genetic algorithm, the nondominated sorting genetic algorithm II (NSGAII), was developed by K. Deb et al. [28]. The intent is to keep the fittest candidates for the next population, which will enhance the convergence of the algorithm. The procedure begins by creating a population of random solutions represented as P_0 of size N. In a given generation t, the individuals of the parent population P_t undergo changes in genetics using genetic operators, including selection, crossover and mutation in order to produce a new population C_t with the same number of potential solutions as the parent population. A population S_t of size $2N$ is generated by combining the parent and progeny populations (generated through mating the parent) in order to guarantee elitism. The following are the two procedures that NSGAII undertakes for selecting N nondominated solutions from S_t for the next generation:

— **Ordering by rank**: The process of assigning a rank to each member in S_t frontifies them. Different nondominated fronts N_1, N_2,..., N_l are frontified to the solutions in S_t according to their ranks. Any set of nondominated solutions of rank k is represented by each N_k, where k is an element of the set $\{1,2,\dots,l\}$. Solutions that have the same rank belong to the same nondominated front. It is usually better to choose solutions with lower nondomination ranks. To rephrase, p is better than q if there exist two solutions p and q in S_t and $p_{rank} < q_{rank}$. The solutions from N_1 are initially taken into account in order to construct the following population, P_{t+1}. If N_1 is less in size than N, then all of the solutions of N_1 are added to P_{t+1}. We rank the remaining solutions of P_{t+1} from subsequent nondominated fronts. Starting with the second front's solutions (N_2) and moving them to P_{t+1}, we continue with the third front's solutions (N_3) and continue in this manner until no more solutions from a non-inferior front (N_k) can be entirely interleaved into P_{t+1};

— **Ranking according to the crowding distance**: According to the method suggested by K. Deb et al. [28], if every solution of N_k does not fit in P_{t+1}, the solutions are arranged in descending order based on the crowding distance ($i_{distance}$) values. In N_k, p is favoured over q if p has a better crowding distance than q, that is, if $p_{rank} = q_{rank}$ and $p_{distance} > q_{distance}$. This is especially true if p and q are the two nondominated solutions. Filling up P_{t+1} with solutions from N_k having the comparatively larger $i_{distance}$ are essentially selected.

The individuals of P_{t+1} will replace P_t for the next generation as the formation of P_{t+1} is accomplished. This process keeps going until the termination condition of the algorithm, such as the maximum number of generations or function evaluations, is attained.

6. NUMERICAL ILLUSTRATION

This section provides a numerical example to demonstrate our suggested model, FMWPP. Here is a brief overview of the planned FMWPP. Let us consider a network that is not directed in any particular direction or is an undirected network; each node is a vertex, and each edge is a street that connects two nodes. Beginning the journey from the office of a courier service, a postman or delivery person will be traversing all the streets at least once to deliver the items to their final destinations before coming back to the office again. In exchange for crossing a street, the postman gets a certain amount in commission. But he is the one who has to make the payments for the ride, which includes things like gas refilling, repairs, tolls and upkeep. On top of that, the postman is always looking for ways to cut corners on his trip. So, when the delivery person prepares the tour, he keeps two goals in mind that need optimisation. First, he wants to make as much money as possible from the courier service's commission and the money he spends on gas and other transportation costs. Second, he wants to do the tour in as little time as possible.

The precise values of the related factors (commission amount, trip cost and travel length) are often hard to estimate since the postman plans the trip ahead of time and the features associated with a postman's tour are quite dynamic. Some of the factors that impact a postman's travel expenditures are fuel price, toll tax, vehicle running cost and driveway costs. All of these elements are constantly changing and evolving. The amount of money a postman gets to spend on gas and the number of packages that need to be delivered to a certain address are two factors that directly affect his travel budget. Just like how traffic conditions on the streets might change over time, the amount of time it takes to go somewhere is largely dependent on him. This uncertainty over the tour's estimated profit – which is dependent on commission amount, travel cost and trip time – is typical among decision-makers (the postman, in this case). Fuzzy variables are used to express the values of these factors so that they may be dealt with logically.

Using an undirected connected network $\mathcal{N}$ (cf. *Figure 1*), we can optimise the postman's route so that it maximises profit while decreasing journey time. Here, there are asymmetrical parameters connected with each edge e_{ij}. The postman must travel eighteen streets at least once, each of which is represented by an edge e_{ij} and connects a pair of vertices (v_i, v_j) in $\mathcal{N}$. There are three fuzzy parameters:

— The fuzzy commission amount;
— The fuzzy trip cost;
— The fuzzy journey time;

which are associated with each e_{ij} of $\mathcal{N}$. *Table 1* displays the related fuzzy parameters of $\mathcal{N}$ as fuzzy variables.

We used the epsilon-constraint approach to find the solutions of the deterministic equivalent models (cf. Model (8) and Model (9)) of Model (7) of the proposed FMWPP. Merely, to keep things simple, we set both the α_1 and α_2 confidence levels of Model (8) to 0.4. On the other hand, Model (9) takes α_1 and α_2 to be 0.8. Accordingly, *Table 2* displays the compromise solutions produced by Model (8) and Model (9), together with the related tours of the delivery person for network $\mathcal{N}$.

We examine two MOGAs, FastPGA and NSGAII, to produce a set of non-inferior solutions for Models (8) and (9). In *Table 3*, we can perceive the allied parameters of the MOGAs for both models.

Table 4 displays the nondominated solutions, whereas *Figure 2* provides visual representations of them. It can be shown that FastPGA produces more distinct nondominated solutions than NSGAII in this case. Moreover, while solving Model (8) and Model (9) for $\mathcal{N}$, it is noted from *Table 4* and *Figure 2* that the solutions provided by the epsilon-constraint approach are further generated by the MOGAs. In *Table 4*, these solutions are reported in bold.

Models (8) and (9) are subsequently solved at various confidence levels for sensitivity analysis. In this case, for the purpose of keeping things simple, we assign the confidence levels α_1 and α_2 to the identical values, thus $\alpha_1 = \alpha_2 = \alpha$. Model (8) is solved for α_1, α_2 falling within the interval $[0,0.5]$. However, Model (9) is solved for α_1, α_2 falling within the interval $(0.5, 1]$. *Table 5* and *Figure 3* show the results of the models at various confidence levels.

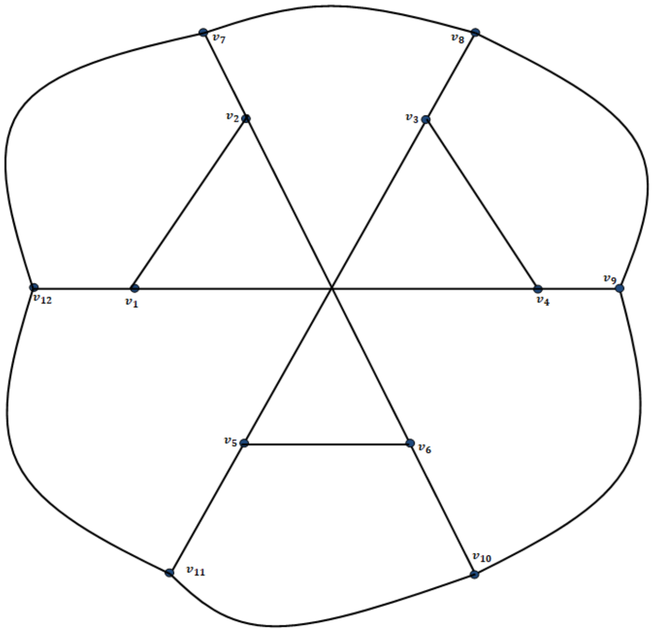

Figure 1 – A connected undirected network

Table 1 – Specifics on the network $\mathcal{N}'s$ commission, travel costs and time spent on travelling

Edges (e_{1j})	Commission amount	Expense incurred	Time
e_{12}	$\mathcal{T}$(132.6,143.2, 149.6)	$\mathcal{T}$(34.2, 37.7, 40.2)	$\mathcal{T}$(73.2, 77.2, 79.7)
e_{21}	$\mathcal{T}$(137.2,144.9, 151.6)	$\mathcal{T}$(27.4, 32.5, 36.4)	$\mathcal{T}$(73.4, 75.7, 78.4)
e_{14}	$\mathcal{T}$(122.4,138.5, 146.8)	$\mathcal{T}$(28.4,30.2, 32.3)	$\mathcal{T}$(65.3, 68.3, 69.8)
e_{41}	$\mathcal{T}$(122.4,138.5, 146.8)	$\mathcal{T}$(27.3, 29.8, 33.4)	$\mathcal{T}$(67.3, 69.4, 72.3)
e_{112}	$\mathcal{T}$(137.6,145.7, 156.3)	$\mathcal{T}$(24.4, 27.5, 29.7)	$\mathcal{T}$(68.3, 72.5, 79.9)
e_{121}	$\mathcal{T}$(127.6,137.7, 142.3)	$\mathcal{T}$(25.6, 29.9, 31.7)	$\mathcal{T}$(73.4, 77.6, 82.3)
e_{26}	$\mathcal{T}$(134.2,152.3, 161.5)	$\mathcal{T}$(32.5, 37.5, 39.3)	$\mathcal{T}$(71.2, 74.4, 77.3)
e_{62}	$\mathcal{T}$(126.2,132.6, 141.7)	$\mathcal{T}$(28.4, 34.6, 37.8)	$\mathcal{T}$(69.7, 74.3, 78.4)
e_{27}	$\mathcal{T}$(113.2,123.3, 132.3)	$\mathcal{T}$(29.4, 33.5, 37.3)	$\mathcal{T}$(72.6, 76.6, 79.2)
e_{72}	$\mathcal{T}$(117.9,122.5, 131.4)	$\mathcal{T}$(34.2, 37.6, 40.3)	$\mathcal{T}$(69.4, 75.4, 77.9)
e_{34}	$\mathcal{T}$(103.4,107.4, 114.3)	$\mathcal{T}$(31.9, 36.7, 39.4)	$\mathcal{T}$(73.2, 76.5, 79.2)
e_{43}	$\mathcal{T}$(121.6, 129.4, 134.5)	$\mathcal{T}$(33.4, 35.8, 39.2)	$\mathcal{T}$(71.3, 76.5, 77.9)
e_{35}	$\mathcal{T}$(111.3, 119.7, 123.6)	$\mathcal{T}$(34.5, 37.5, 39.6)	$\mathcal{T}$(73.5, 77.4, 79.7)
e_{53}	$\mathcal{T}$(117.6, 121.6, 127.2)	$\mathcal{T}$(37.3, 39.2, 41.2)	$\mathcal{T}$(72.2, 75.5, 78.8)
e_{38}	$\mathcal{T}$(109.6, 129.7, 134.5)	$\mathcal{T}$(29.6, 35.5, 38.9)	$\mathcal{T}$(73.4, 76.4, 79.8)
e_{83}	$\mathcal{T}$(102.2, 109.7, 114.5)	$\mathcal{T}$(32.4, 36.7, 39.8)	$\mathcal{T}$(75.1, 78.3, 81.2)
e_{49}	$\mathcal{T}$(121.3, 129.3, 135.9)	$\mathcal{T}$(32.5, 37.7, 39.2)	$\mathcal{T}$(70.2, 74.2, 78.6)
e_{94}	$\mathcal{T}$(118.7, 122.6, 129.8)	$\mathcal{T}$(34.1, 36.3, 40.3)	$\mathcal{T}$(72.1, 75.5, 77.8)
e_{56}	$\mathcal{T}$(109.4, 113.7, 117.9)	$\mathcal{T}$(31.2, 35.4, 37.8)	$\mathcal{T}$(70.2, 74.9, 78.9)
e_{65}	$\mathcal{T}$(113.5, 117.6, 119.3)	$\mathcal{T}$(29.3, 32.7,. 35.4)	$\mathcal{T}$(67.3, 73.3, 76.4)
e_{511}	$\mathcal{T}$(118.1, 129.7, 133.3)	$\mathcal{T}$(27.4, 34.5, 38.2)	$\mathcal{T}$(75.3, 79.7, 82.3)

Edges (e_{1j})	Commission amount	Expense incurred	Time
e_{115}	$\mathcal{T}$(124.6, 128.9, 133.6)	$\mathcal{T}$(32.9, 36.7, 39.9)	$\mathcal{T}$(76.2, 80.1, 84.2)
e_{610}	$\mathcal{T}$(127.9, 134.6, 138.9)	$\mathcal{T}$(33.7, 38.8, 43.2)	$\mathcal{T}$(79.2, 83.8, 85.2)
e_{106}	$\mathcal{T}$(124.6, 129.8, 134.7)	$\mathcal{T}$(36.3, 38.7, 41.3)	$\mathcal{T}$(67.4, 73.2, 78.7)
e_{78}	$\mathcal{T}$(125.7, 128.6, 136.9)	$\mathcal{T}$(29.4, 34.2, 36.1)	$\mathcal{T}$(73.4, 77.5, 81.7)
e_{87}	$\mathcal{T}$(102.7, 110.7, 114.4)	$\mathcal{T}$(29.7, 35.1, 37.8)	$\mathcal{T}$(77.4, 80.3, 86.5)
e_{712}	$\mathcal{T}$(105.6, 115.9,125.3)	$\mathcal{T}$(34.5, 38,5, 40.1)	$\mathcal{T}$(78.4, 79.6, 81.9)
e_{127}	$\mathcal{T}$(112.6, 117.6, 124.4)	$\mathcal{T}$(31.3, 35.4, 37.8)	$\mathcal{T}$(79.4, 81.4, 83.7)
e_{89}	$\mathcal{T}$(106.6, 112.3, 118.4)	$\mathcal{T}$(35.6, 37.8, 39.7)	$\mathcal{T}$(78.9, 81.2, 83.8)
e_{98}	$\mathcal{T}$(114.6, 119.3, 125.3)	$\mathcal{T}$(34.8, 37.9, 40.2)	$\mathcal{T}$(76.8, 79.2, 81.7)
e_{910}	$\mathcal{T}$(123.4, 129.5, 130.4)	$\mathcal{T}$(29.9, 35.8, 38.6)	$\mathcal{T}$(68.3, 73.5, 76.7)
e_{109}	$\mathcal{T}$(129.4, 134.3, 139.7)	$\mathcal{T}$(31.4, 35.6, 38.9)	$\mathcal{T}$(73.6, 78.9, 82.5)
e_{1011}	$\mathcal{T}$(117.8, 124.5, 128.3)	$\mathcal{T}$(32.8, 37.3, 39.8)	$\mathcal{T}$(76.3, 79.1, 83.7)
e_{1110}	$\mathcal{T}$(107.4, 113.2, 117.3)	$\mathcal{T}$(29.7, 35.7, 38.6)	$\mathcal{T}$(78.5, 81.6, 84.4)
e_{1112}	$\mathcal{T}$(121.6, 126.8, 134.5)	$\mathcal{T}$(34.1, 36.9, 37.8)	$\mathcal{T}$(72.9, 75.8, 78.9)
e_{1211}	$\mathcal{T}$(115.8, 119.3, 123.4)	$\mathcal{T}$(31.6, 36.7, 38.1)	$\mathcal{T}$(73.8, 78.7, 82.7)

Table 2 – Compromise solutions of $\mathcal{N}$

Objective functions	Epsilon-constraint method	
	Model (8) with α_1 and α_2 set as 0.4	Model (9) with α_1 and α_2 set as 0.8
$\bar{Z}_1$	1909.52	2021.66
$\bar{Z}_2$	1806.58	1872.20
Optimized tour	$e_{21}-e_{14}-e_{41}-e_{112}-e_{1211}-e_{1112}-e_{127}-e_{72}-e_{26}-e_{65}-e_{53}-e_{34}-e_{49}-e_{98}-e_{87}-e_{78}-e_{83}-e_{35}-e_{511}-e_{1110}-e_{109}-e_{910}-e_{106}-e_{62}$	$e_{21}-e_{14}-e_{41}-e_{112}-e_{1211}-e_{1112}-e_{127}-e_{72}-e_{27}-e_{78}-e_{83}-e_{38}-e_{89}-e_{94}-e_{43}-e_{35}-e_{56}-e_{65}-e_{511}-e_{1110}-e_{109}-e_{910}-e_{106}-e_{62}$

Table 3 – Values for the associated parameters of NSGAII and FastPGA

Parameter settings	FastPGA and NSGAII
Initial size of the population	100
Maximum population size	100
Crossover probability	0.9
Mutation probability	$1/n$, where n is the total edges of the network $\mathcal{N}$
Crossover type	single point
Mutation operator	bit flip mutation
Selection operator	binary tournament
Solution evaluations	25000

Table 4 – Nondominated solutions of $\mathcal{N}$ when executed by FastPGA and NSGAII

Confidence level (α_1, α_2)							
0.4				**0.8**			
FastPGA		**NSGAII**		**FastPGA**		**NSGAII**	
$\bar{Z}_1$	$\bar{Z}_2$	$\bar{Z}_1$	$\bar{Z}_2$	$\bar{Z}_1$	$\bar{Z}_2$	$\bar{Z}_1$	$\bar{Z}_2$
1909.52	1806.58	1909.52	1806.58	2021.66	1872.20	2021.66	1872.20
1965.62	1807.78	1965.62	1807.78	1973.14	1874.34	1973.14	1874.34
2211.10	1818.54	2211.10	1818.54	2279.20	1883.50	2279.20	1883.50
2182.80	1817.34	2182.80	1817.34	2216.26	1884.04	2216.26	1884.04
2420.00	1971.52	2420.00	1971.52	2402.10	2040.98	2402.10	2040.98
2391.70	1970.32	2391.70	1970.32	2490.20	2042.18	2490.20	2042.18
2588.72	2123.86	2588.72	2123.86	2671.52	2202.76	2671.52	2202.76
2437.10	1971.70	2437.10	1971.70	2670.08	2197.70	2997.04	2518.46
2736.20	2295.74	2736.2	2295.74	2815.72	2357.88	2825.40	2375.14
2735.34	2277.14	2735.34	2277.14	2821.22	2368.90	3047.18	2834.66
3009.04	2736.42	3009.04	2736.42	2997.04	2518.46	–	–
2901.90	2432.48	–	–	2825.40	2375.14	–	–
2934.42	2496.14	–	–	3047.18	2834.66	–	–
3024.18	2784.34	–	–	–	–	–	–

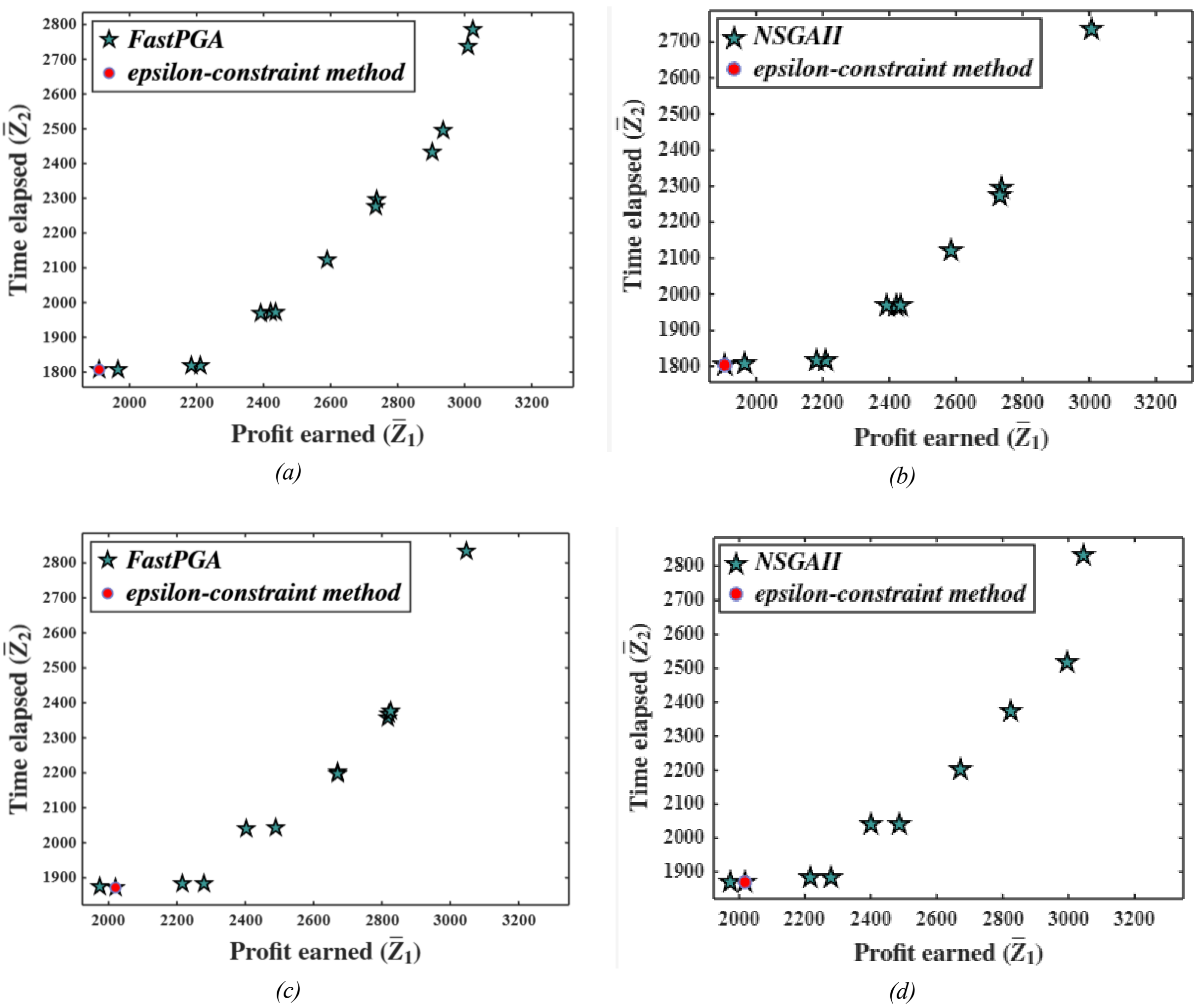

Figure 2 – Graphical representation of the nondominated solutions generated by: a) FastPGA at the chance level 0.4; b) NSGAII at the chance level 0.4; c) FastPGA at the chance level 0.8; d) NSGAII at the chance level 0.8

Table 5 – Sensitivity analysis of Model (8) and Model (9) at different confidence levels when solved by the epsilon-constraint method

Confidence level (α_1, α_2)	$\bar{Z}_1$	$\bar{Z}_2$
0.005	1969.92	1731.36
0.1	1955.08	1749.52
0.2	1939.46	1768.64
0.3	1923.84	1787.76
0.4	1909.52	1806.58
0.5	1892.40	1825.00
0.6	1935.62	1841.40
0.7	1978.64	1856.80
0.8	2021.66	1872.20
0.9	2064.68	1887.60
0.995	2105.55	1902.23
1.0	2107.70	1903.00

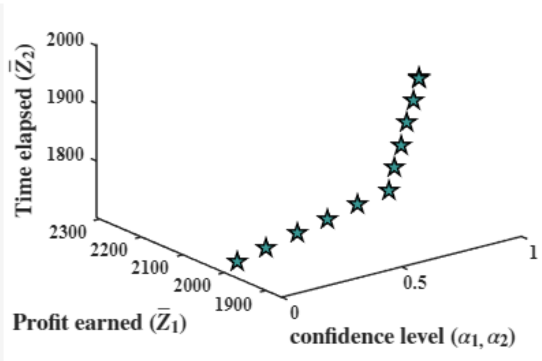

Figure 3 – Graphical illustration of the sensitivity analysis of Models (8) and (9) for $\mathcal{N}$

We have used Models (8) and (9) to find the deterministic transformation of the associated CCPM for four randomly generated instances of FMWPP for simulation purposes. The problem instances are designated as $FMWPP\ \#i$, where i =1,2,3 and 4 with the number of vertices are 10, 20, 30 and 40, respectively. Each of these instances, has undirected $\binom{i}{2}$ edges, where $i = 10, 20, 30$ and 40. Here, for each $FMWPP\ \#i$ instance, three asymmetric parameters are associated with an edge. Subsequently, these parameters are:

— The commission amount

— The trip cost of the postman;

— Time elapsed to complete the trip.

Each of these parameters is represented as a triangular fuzzy number to capture the uncertain phenomena associated with it. These instances can be accessed from the below mentioned link *https://drive.google.com/drive/folders/1wWAO1ILwk0tv5acWjEP_RrKeDy6SVlc6?usp=drive_link*.

We have taken into account two performance metrics – hypervolume (HV) [39] and inverted generational distance (IGD) [41] – in order to compare the performance of the MOGAs, FastPGA and NSGAII. One can gauge both the convergence and diversity with these metrics. In order to simulate FastPGA and NSGAII on our instances, we have employed the jMetal4.5 framework [42]. Because these algorithms are inherently random, we execute them all 100 times, taking into account 250 generations in each run. All the other parameters listed in *Table 3* are taken as being identical for both the MOGAs. The Pareto front (PF) of ideal solutions is rarely accessible for problems in the actual world. Consequently, we build a reference front using all the solutions of the first front of both FastPGA and NSGAII to approximate the PF. Afterwards, the reference front is used to assess the performance metric values for each algorithm run. Here, for each performance metric, we find the *mean* and standard deviation (*sd*) to get a sense of the related data's central tendency and variability, respectively.

We solve each of the four fuzzy instances $FMWPP\ \#i$, where $i = 1,2,3,4$ using Model (8) and Model (9), with the former model having a confidence level of 0.4 and the latter model of 0.8. So, we set the confidence levels of Model (8) to 0.4 for all four instances, and in *Table 6* we provide the *mean* and standard deviation of all four fuzzy instances of $FMWPP\ \#i$, where $i = 1,2,3,4$. Similarly, after solving Model (9) using MOGAs and setting the confidence levels to 0.8, *Table 7* displays the means and standard deviations for all four cases. The bolded values in these tables represent the best options. The results show that FastPGA produces a better $mean$ of the HV than NSGAII, considering all four instances (cf. *Tables 6 and 7*). On the other hand, in every instance, the sd values of the HV produced by NSGAII outperform those of FastPGA. Furthermore, taking IGD into account, we perceive that FastPGA is a better MOGA than NSGAII in all four instances, both in terms of $mean$ and sd.

Based on the data reported in *Tables 6 and 7*, visual representations of the HV and IGD are depicted in *Figures 4 and 5*. Here, we can observe the *mean* value differences of HV and IGD as produced by FastPGA and NSGAII, for 100 observations on the deterministic models of the four fuzzy instances at two distinct confidence levels (0.4 and 0.8). According to Gardner and Altman [43], the Gardner–Altman plots are employed for this particular purpose. The 'X' and 'Y' in the x-axis markers of *Figure 4* and *Figure 5*, respectively, represent the data vectors generated by the FastPGA and the NSGAII. At both the confidence levels of 0.4 and 0.8, it is clear from examining all of the HV *mean* values in *Figure 4* that the 'X' *mean* is significantly greater than the 'Y' mean. However, as shown in *Figure 5*, the *mean* values of 'X' are lower than those of 'Y' according to the IGD measure, which is ideally expected.

Table 6 – Mean and sd of HV and IGD after 100 executions of FastPGA and NSGAII at the confidence level of 0.4

MOGAs	Uncertain instances	HV		IGD	
		mean	sd	mean	sd
FastPGA	FMWPP #1	**6.242E-01**	7.937E-02	**4.836E-04**	**3.198E-04**
	FMWPP #2	**7.205E-01**	8.370E-02	**3.377E-04**	**1.961E-04**
	FMWPP #3	**7.079E-01**	9.260E-02	**4.396E-04**	**2.419E-04**
	FMWPP #4	**7.566E-01**	8.486E-02	**5.341E-04**	**2.206E-04**
NSGAII	FMWPP #1	6.048E-01	**5.231E-03**	7.200E-04	4.641E-04
	FMWPP #2	6.426E-01	**1.616E-02**	7.178E-04	4.368E-04
	FMWPP #3	6.176E-01	**3.048E-02**	8.058E-04	4.446E-04
	FMWPP #4	6.583E-01	**1.997E-02**	1.037E-03	4.677E-04

Table 7 – Mean and sd of HV and IGD after 100 executions of FastPGA and NSGAII at the confidence level of 0.8

MOGAs	Uncertain instances	HV		IGD	
		mean	sd	mean	sd
FastPGA	FMWPP #1	**7.223E-01**	1.137E-01	**6.844E-04**	**3.891E-04**
	FMWPP #2	**7.261E-01**	7.934E-02	**4.847E-04**	**2.628E-04**
	FMWPP #3	**7.148E-01**	8.917E-02	**6.021E-04**	**3.057E-04**
	FMWPP #4	**7.625E-01**	8.051E-02	**6.916E-04**	**2.843E-04**
NSGAII	FMWPP #1	6.765E-01	**5.554E-02**	1.024E-03	5.792E-04
	FMWPP #2	6.485E-01	**1.519E-02**	9.864E-04	5.353E-04
	FMWPP #3	6.042E-01	**1.067E-01**	1.084E-03	5.122E-04
	FMWPP #4	6.395E-01	**9.457E-02**	1.172E-03	4.873E-04

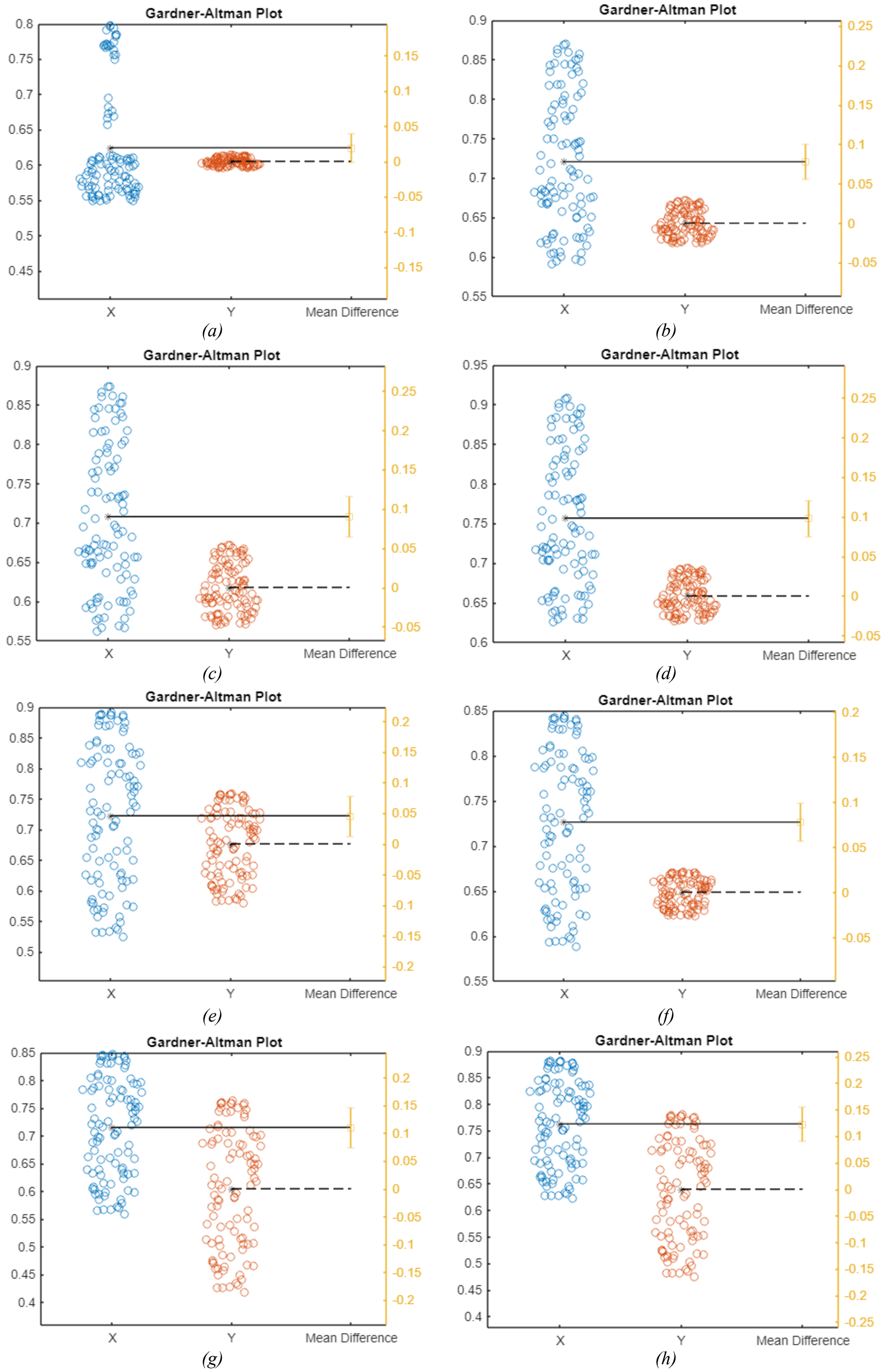

Figure 4 – For the fuzzy instances: a) FMWPP #1, b) FMWPP #2, c) FMWPP #3, d) FMWPP #4 at a chance (confidence) level of 0.4; e) FMWPP #1, f) FMWPP #2, g) FMWPP #3, h) FMWPP #4 at a confidence level of 0.8, the Gardner–Altman plots for the HV produced by the MOGAs

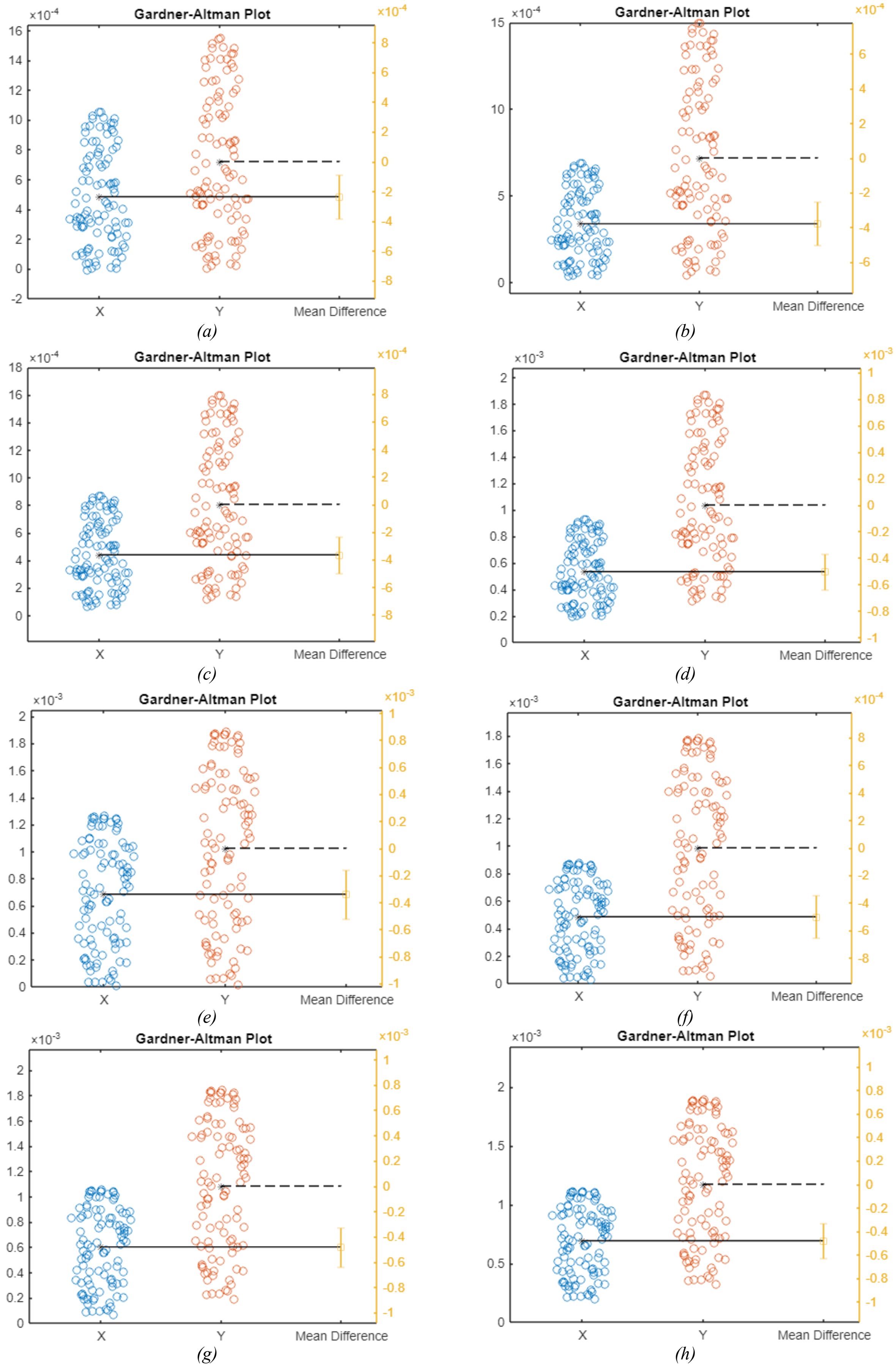

Figure 5 – For the fuzzy instances a) FMWPP #1, b) FMWPP #2, c) FMWPP #3, d) FMWPP #4 at a chance (confidence) level of 0.4; e) FMWPP #1, f) FMWPP #2, g) FMWPP #3, h) FMWPP #4 at a confidence level of 0.8, the Gardner–Altman plots for the IGD produced by the MOGAs

Besides, we examine a two-stage hypothesis test on the HV and IGD performance indicators corresponding to the four FMWPP instances. In this case, to test the null hypothesis (h_0), the p −value is set as $\prec_p^+$for the two-sample parametric $t - test$. The value of $\prec_p^+$, is accepted in this case if h_0 is refused. However, using the two-way nonparametric Wilcoxon signed rank test, we determine the p-value, $\prec_p^-$, that is associated with h_0 when it is accepted. Since we are unsure of the distributions underlying the hypervolume and inverted generational distance, we have performed a two-stage hypothesis test that incorporates both parametric and non-parametric testing. Subsequently, h_0 considered during the test is mentioned below:

- h_0: FastPGA and NSGAII produce comparable performance metrics.

The results of the hypothesis testing, which are carried out at a significance level of 1%, are reported in *Table 8*. The FastPGA⋆NSGAII column in this table ends with one of the two possibilities.

— FastPGA $\prec_p^+$ or $\prec_p^-$ NSGAII: Either $\prec_p^+$ or $\prec_p^-$ do not accept h_0, and FastPGA is considerably better than NSGAII at the 1% threshold of significance;

— FastPGA $\succ_p^+$ or $\succ_p^-$ NSGAII: Either $\succ_p^+$ or $\succ_p^-$ do not accept h_0, and FastPGA is noticeably worse than NSGAII at the 1% threshold of significance.

The $p -$values in *Table 8* show that FastPGA outperforms NSGAII for hypervolume and inverted generational distance at the 1% threshold of significance, with respect to both the chance levels 0.4 and 0.8. Moreover, at the confidence level of 0.4, and for the instances FMWPP #1 and FMWPP #3, FastPGA emerges as superior compared to NSGAII for HV by conducting t-test. Similar analysis can be done for the instance FMWPP #2 for the same performance metric at the 0.8 confidence level. Subsequently, for all the remaining instances, considering both HV and IGD, FastPGA becomes significantly better than NSGAII, at all the chance levels, only after the Wilcoxon signed rank test is conducted.

Table 8 – Statistical significance of the hypothesis testing of hypervolume (HV) and inverted generational distance (IGD) generated by the MOGAs with respect to the four FMWPP instances

Uncertain instances	Confidence level			
	0.4		0.8	
	HV	*IGD*	*HV*	*IGD*
	FastPGA ⋆ NSGAII	FastPGA ⋆ NSGAII	FastPGA ⋆ NSGAII	FastPGA ⋆ NSGAII
FMWPP #1	$\prec^+_{0.0072}$	$\prec^-_{4.1235E-05}$	$\prec^-_{3.7246E-04}$	$\prec^-_{2.2363E-06}$
FMWPP #2	$\prec^-_{8.0087E-17}$	$\prec^-_{1.5022E-13}$	$\prec^+_{3.4604E-18}$	$\prec^-_{7.8768E-15}$
FMWPP #3	$\prec^+_{3.6084E-17}$	$\prec^-_{1.0032E-11}$	$\prec^-_{1.3511E-13}$	$\prec^-_{6.3671E-14}$
FMWPP #4	$\prec^-_{4.4266E-23}$	$\prec^-_{1.6816E-18}$	$\prec^-_{4.8603E-19}$	$\prec^-_{4.0073E-15}$

7. CONCLUSIONS

So far as we are aware, there is no multi-objective windy postman problem under any uncertain paradigm, which is what sets our proposed study apart from others. The findings of a fuzzy multi-objective windy postman problem (FMWPP) model with chance constraints are shown in this study. Both the overall revenue and the total amount of time elapsed during the voyage of the postman or article delivery person to travel along all the streets (edges) in the network are the objective functions that the model is trying to improve. Following this, the deterministic transformations of the model are solved at two distinct confidence levels utilising the traditional epsilon-constraint method and the two multi-objective genetic algorithms (FastPGA and NSGAII). Furthermore, a real-world numerical example is used to calibrate the model. Subsequently, we have considered four randomly generated larger FMWPP instances to evaluate the MOGAs' performance on the performance metrics, HV and IGD. Here, it is observed that for all four random instances, FastPGA emerges as superior to NSGAII with respect to both the performance metrics, as shown in *Tables 6 and 7*.

Our long-term goal in future is to broaden the scope of our research to include not only the many-objective fuzzy windy postman problem, but also its variants under type-2, intuitionistic fuzzy and neutrosophic fuzzy domains, including the many-objective mixed Chinese postman problem, the many-objective rural postman problem and the many-objective hierarchical windy postman problem.

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Multi-objective Windy Postman Problem in a Fuzzy Transportation Network

Abstract

Researchers have become increasingly captivated by the windy postman problem (WPP), a major combinatorial optimisation problem with several practical applications. It is crucial to take the experts' belief levels into account when modelling such a real-world application since these applications frequently involve uncertain aspects. A fuzzy set is one of the tools that might be regarded as appropriate for modelling such human perspectives. Applying fuzzy set theory to a multi-objective windy postman problem is the focus of this study. Maximising the overall profit and minimising the transportable time of the route visited by a postman are the objectives of the problem. In an effort to solve the fuzzy multi-objective windy postman problem (FMWPP), we have developed a chance-constrained programming model (CCPM). Subsequently, the epsilon-constraint method, a classical multi-objective solution methodology, is used to solve the deterministic transformation of the relevant

CCPM. Moreover, the model is solved using two multi-objective genetic algorithms (MOGAs): fast Pareto genetic algorithm (FastPGA) and nondominated sorting genetic algorithm II (NSGAII). To demonstrate the proposed model, a numerical example is presented. We conclude by comparing the performance of the MOGAs on four randomly generated FMWPP instances.

Keywords
transportation; windy postman problem; epsilon constraint method; multi-objective genetic algorithms; performance metrics.