Traffic & Transportation

Optimisation of High-Speed Railway Freight Transport Service Plan in Inter-Modal Transport Based on Extended Time-Space Network

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Original Scientific Paper
Submitted: 18 Mar 2025
Accepted: 23 May 2025
Published: 29 Jan 2026

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Publisher:
Faculty of Transport and Traffic Sciences, University of Zagreb

ABSTRACT

The air-rail inter-modal transport is a feasible choice to enlarge the freight service scope of high-speed railway. Essentially, optimising the service plan for high-speed rail express under inter-modal mainly involves determining the train and flight trips, and space-time route selections for each batch of express shipment from the origin to the destination. We construct an extended space-time network to capture the transport and transfer space-time attributes of the serviced express shipments. A multi-commodity flow model is then established with a series of practical constraints. The Lagrangian relaxation algorithm is designed to decompose the original problem into a shortest path problem of a single-batch express shipment in a multi-dimensional network. The sub-problem is solved by the dynamic programming method, and a heuristic algorithm based on sub-gradient sorting is designed to ensure the feasibility of the dual solutions. In order to compare the performance of the traditional solver method with that of the LR algorithm, a nonlinear mixed-integer programming model was constructed in the appendix and solved by using the DICOPT solver. Taking the Shanghai-Kunming corridor as an example, the experimental results demonstrate that the LR algorithm can obtain high-quality solutions within a relatively short time, while the traditional solver method has certain limitations. Furthermore, the inter-modal transport is validated with a significant advantage in expanding the service scope of express demand. The research results are of great theoretical significance for the rational allocation of transportation resources and enhancement of the quality and efficiency of high-speed rail express services.

KEYWORDS

high-speed rail freight transport; service plan; inter-modal; space-time extended network; Lagrangian relaxation algorithm.

1. INTRODUCTION

With the acceleration of China's industrialisation process, the upgrading of residents' consumption and the implementation of the national double-carbon target, the demand scale and proportion of high-value-added freight transportation are gradually expanding. In addition, modern logistics has extremely high requirements for cost control, service quality, delivery speed and environmental friendliness. The traditional logistics mode has encountered bottlenecks in tapping potential and improving quality. With the rapid development of logistics modernisation, the development of high-speed railway in China has made remarkable achievements. By the end of 2024, the operating mileage of China's high-speed railway had exceeded 48,000 kilometres. The coverage of the high-speed railway network has gradually expanded. More and more attention has been paid to the development of freight transport by utilising the rich capacity of high-speed passenger transport.

The high-speed rail express is aimed at the premium express logistics market, emphasising punctuality and high-quality services, and providing rapid transportation services for all kinds of customers. The high-speed rail express is of great significance in reducing logistics operational costs, unlocking potential economic benefits, and creating environmentally friendly logistics. The government places significant emphasis on the development of high-speed rail express services and has clearly outlined in several key policy documents the need for their strong promotion.

Currently, the high-speed rail express service primarily operates on specific high-speed railway corridors. To further extend the service coverage of the high-speed rail express, the Civil Aviation Administration of China and China National Railway Group have signed a strategic agreement on air-rail inter-modal transport, promoting the establishment of a new air-rail integrated express service system. Some international aviation hubs such as Zhengzhou, Chongqing and Chengdu will be developed an efficient air-rail inter-modal logistics ports.

Although the national policies support the development of new air-rail inter-modal express delivery, there are currently no actual implementation cases in China. The inter-modal transport has been relatively well-developed in some European countries, yielding notable results and accumulating a wealth of operational experience. The inter-modal transport offers significant advantages in ensuring the timeliness of express delivery, achieving the rational allocation of transportation resources, and meeting the express demands of regions with underdeveloped transport network structures. With the surge in demand for high-speed rail express services in China, exploring new inter-modal and transfer modes for express delivery has important practical significance.

This study focuses on optimising the high-speed railway freight transport service plan under inter-modal transport conditions, aiming to expand the scope of high-speed rail express services and improve delivery timeliness. The rest of this paper is organised as follows. Section 2 provides a review of the relevant literature. Section 3 provides a detailed description and assumptions for our problem and defines necessary mathematical symbols to facilitate the construction of the subsequent mathematical model. Section 4 constructs the extended space-time network to capture the express delivery in the serviced network; in addition, a multi-commodity flow model is established in this section. Section 5 provides a Lagrangian relaxation algorithm framework to decompose the original problem into a series of easy-to-solve sub-problems. Section 6 reports the results of a computational study that tests the performance of the proposed mathematical model and decomposition algorithm. The paper is concluded in Section 7. In addition, the traditional nonlinear mixed-integer programming model for our problem is proposed in the last section.

2. LITERATURE REVIEW

At present, the high-speed rail express is still in the exploration stage, most of the researches are mainly focused on the high-speed rail express organisation mode, carrying equipment, regulations and systems, and operating procedures. He et al. [1] analysed the feasibility of high-speed railway passenger and freight co-transportation. Zhou et al. [2] discussed the operation process, service mode and development strategy of high-speed rail express. Bi et al. [3] analysed the adaptability of high-speed railway express delivery to the high-speed railway network according to the capacity utilisation ratios of various high-speed railway lines. Chen et al. [4] used the four-stage forecasting method to predict the demand for high-speed rail express. LI et al. [5] developed a location model based on minimising total cost with coverage constraints to study the location planning problem of express transfer hubs in high-speed rail networks. Pazour et al. [6] proposed an uncapacitated network design model considering highway traffic and transit times for realising the freight distribution. Based on the operational characteristics of various high-speed railway express modes, Zhen et al. [7] have taken into consideration the integrated optimisation problems, such as whether high-speed trains of various high-speed railway express modes can transport the freights of specific types, the amounts of freights transported by each HSR express mode, and the arrangement of capacity resources of each mode.

Recently, a series of research has also focused on the optimisation of high-speed railway express service plans from the perspective of transportation organisation and operation. Gao et al. [8] comprehensively optimised the high-speed rail express schedule and demand assignment plan oriented by the time-varying high-speed rail express demand. In addition, another paper of this study team focused on constructing an optimisation model for the special express train line planning by introducing the alternative sets [9]. Li et al. [10] concentrated on the problem of high-speed freight train line planning under large-scale network conditions, and constructed an integer programming model with 0-1 variables and designed an integer Benders decomposition-based branch-and-cut algorithm. Xu et al. [11] focused on the train capacity allocation problem for the mixed transportation pattern of passenger and freight in high-speed railway systems. Li et al. [12]

proposed a sharing-carriage and sharing-train mode to fully exploit the remaining capacity of the existing railway schedule. Then, a space-time network was constructed, and a Benders decomposition approach was proposed to decompose the problem into two sub-problems. Qi et al. [13] jointly optimised the train timetable and stop plan with time-dependent passenger and freight demands on high-speed railway.

Many scholars have recently paid attention to the research related to the organisation of express delivery in the subway, and have achieved rich research results. The operation theory of metro express is similar to that of high-speed rail express, and related studies can provide theoretical support for the optimisation of the high-speed rail express operation plans. Ozturk et al. [14] investigated the organisation of freight transportation by inserting freight trains into the passenger train timetable while considering multiple freight and delivery situations. Sahli et al. [15] studied the problem of reserving carriages for freight transportation during off-peak periods with considering only freight allocation. Di et al. [16] presented a joint optimisation approach of flow control and carriage arrangement in a metro-based underground logistics system, aiming to minimise the weighted sum of the operation cost and the total delay penalty. Li et al. [17] addressed the urban rail trains and freight allocation plan design problem on a single urban rail line. An optimisation model for combined train service design was proposed to maximise profit resulting from the balance of revenues and costs brought by the freight service.

Multi-modal transport is a hot topic in the field of transportation research in recent years, mainly focusing on multi-modal transport route optimisation, service network design and so on. Fazayeli et al. [18] used the fuzzy programming theory to study the multi-modal transport routing problem under the uncertain demand. Liu et al. [19] considered the carbon emission factor when studying the route planning of multi-modal transport. Demir et al. [20] considered the uncertainty of transit time when studying the design of a green multi-modal transport network. Wang et al. [21] studied the transport network design of multi-modal transport under the condition of uncertain demand. Peng et al. [22] constructed a multi-objective optimisation model to minimise transportation time and cost simultaneously by considering mode switching and schedule constraints.

In the inter-modal transfer model, how to construct an optimisation model for high-speed rail express services is the key focus of the research. Constructing a time-pace network model is one of the commonly used modelling approaches. Niu et al. [23] focused on a class of multiple-depot transit vehicle scheduling problems, and a new space-time-connection graph model was proposed. Zhang et al. [24] constructed an extended time-space network model to optimise the cyclic timetable problem for a high-speed railway corridor. Gao et al. [25] proposed a time-space network model to optimise multi-type train scheduling problems under flexible operation conditions. Some three-dimensional time-space-state network models had also been proposed. Shang et al. [26] constructed a space-time-state hyper network to integrate structurally heterogeneous data sources in the study of the passenger flow state estimation problem. Liu et al. [27] designed a space-time-state path-based flow-based linear programming model for integrating the vehicle assignment and routing problem.

The decomposition algorithm framework is an effective method for solving large-scale combinatorial optimisation problems. Due to its simple framework and high solving efficiency, Lagrangian relaxation is the most commonly used decomposition algorithm [28-30]. More specifically, Brannlund et al. [31] used a Lagrangian relaxation solution approach for optimising the timetabling problem of a railway company, and the original problem was decomposed into one dynamic program for each physical train. Caprara et al. [32] proposed the Lagrangian relaxation heuristic framework to decompose the complex train timetabling problem. Hassannayebi et al. [33] decomposed the train timetabling problem into a number of sub-problems for each vehicle path. Jiang et al. [34] developed a heuristic algorithm framework based on Lagrangian relaxation for jointly designing a train scheduling and skip-stopping plan on a highly congested railway line.

Table 1 – Comparison of relevant literature

Literature	Circumstances	Optimisation content	Inter-modal transport	Modelling	Algorithm
Gao et al. [8]	High-speed railway freight transport	Demand assignment plan	Not considered	Integer programming model	CPLEX
Chao et al. [9]	High-speed railway freight transport	Special express train line planning	Not considered	Integer programming model	Simulated annealing algorithm
Li et al. [10]	High-speed rail network	Freight train line planning	Not considered	Integer programming model	Benders decomposition

Literature	Circumstances	Optimisation content	Inter-modal transport	Modelling	Algorithm
Qi et al. [13]	High-speed railway freight transport	Train timetable and stop plan	Not considered	Integer programming model	Variable neighbourhood search
Ozturk et al. [14]	Urban rail freight transport	Train timetable	Not considered	Integer programming model	Pseudo-polynomial dynamic programming algorithm
Di et al. [16]	Urban rail freight transport	Flow control and carriage arrangement	Not considered	Integer linear programming model	Benders decomposition
This paper	High-speed railway freight transport	Service plan	Considered	Based on an extended space-time network	Lagrangian relaxation

According to the above comparison with existing literature listed in *Table 1*, there is no research on the optimisation for high-speed rail service plans under transfer or inter-modal transport. In addition, most of the research on multi-modal transport is based on the optimisation of the physical route. This paper optimises the high-speed rail express service plan under transfer and inter-modal transit, and makes decisions regarding the space-time path selection of high-speed rail express demand in the combined service network composed of high-speed rail trains and flights. Compared with the existing literature, the contributions of this paper are mainly reflected in the following four aspects.

1) We concentrate on the optimisation problem of a high-speed rail express service plan under transfer and inter-modal transport, and the addressed problem is abstracted as a routing problem of high-speed rail express demand in a combined service network composed of trains and flights.
2) By constructing an extended time-space network to describe the matching relationship between high-speed rail express demand and various transportation modes, the actual constraints of express service are transformed into constraints about nodes and arcs of the designed network, and then a multi-commodity flow model is constructed.
3) The Lagrangian relaxation algorithm is designed to relax the train or flight capacity constraints, and the original problem is decomposed into the shortest path sub-problem for each batch of express shipment in the network. A heuristic algorithm framework is designed to realise the feasibility of a dual solution.
4) Taking the Shanghai-Kunming corridor as an example, the proposed model and algorithm in the study are compared with the traditional mixed integer programming model, and the efficiency of the proposed method is verified. At the same time, the direct and inter-modal transport are compared, and the advantages of inter-modal transport in expanding the express service scope are confirmed.

3. PROBLEM DESCRIPTION

3.1 Problem statement

The organisation process for high-speed rail express service is presented in *Figure 1*. Initially, the express companies are responsible for sorting the freight and loading it into railway-designated containers according to the different destinations and categories, in full compliance with the relevant transport regulations and requirements of high-speed railway. These express shipments are then collected and transported to the high-speed rail station. The railway department needs to design a unified service plan based on the parameters and requirements of the shipments waiting at a high-speed railway station. This plan is then implemented to ensure safe and timely delivery of shipments to their destinations. Currently, most service modes focus on the individual high-speed rail corridor. In order to further expand the coverage of high-speed rail express delivery, the rail-to-rail transfer mode or the rail-to-air inter-modal transport may be adopted. In this paper, these modes are collectively referred to as the inter-modal.

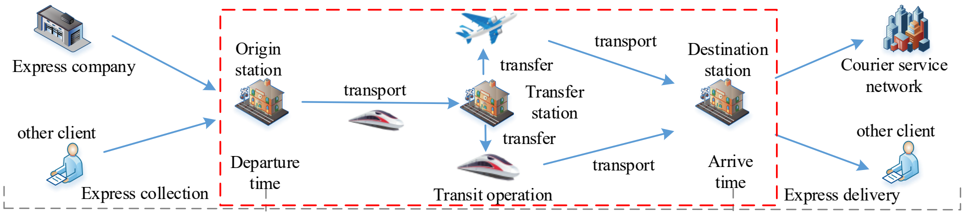


Figure 1 – Organisation process of the high-speed rail express

In the inter-modal transport, it is essential to determine the optimal service plan for each batch of express shipment from the origin station to the destination station. In the direct transport mode, each batch of express shipment needs to be assigned to the most appropriate high-speed train at its origin station, without the need for transfer during the delivery, as shown in *Figure 2(a)*. By comparison, under the inter-modal transport, express parcels are transferred to another train or flight at transfer service nodes, as shown in *Figure 2(b)*. In response to the growing demand for high-speed rail express, exploring the service plan in the transfer and inter-modal transport mode can further enrich the express service system and provide more flexible service.

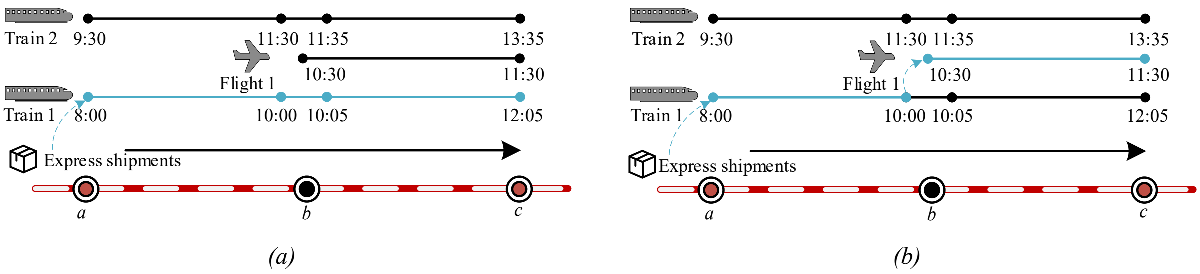


Figure 2 – Organisation plans under different modes: a) direct transport; b) inter-modal transport

In the addressed inter-modal transport, the core of the high-speed rail express service plan is to select the most suitable high-speed train and flight, based on the urgent transportation requirements of the express shipment, ensuring high-quality service delivery. It primarily entails matching the express shipment with available transportation resources and coordinating transfer connections. Specifically, within the constraints of high-speed railway and air transportation resources, decisions must be made regarding the loading location, loading time, assigned train, transfer locations and times, choice of transportation modes, transfer departure time, and arrival time for each batch of express shipment, as depicted in *Figure 3*. The express service plan incorporates not only temporal and spatial attributes but also the selection of transportation modes. Optimising this multi-attribute service plan is inherently complex and poses a significant challenge to modelling and algorithm design. This is precisely the core issue that this article addresses.

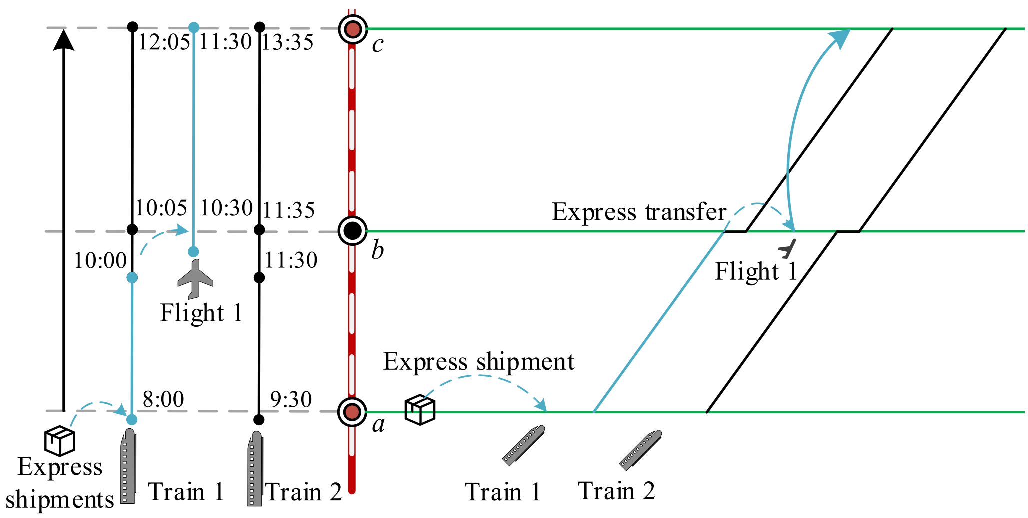


Figure 3 – High-speed rail express transfer and inter-modal service plan

To facilitate model construction, the following assumptions are introduced:

Assumption 1: High-speed rail express service uses the surplus capacity after passenger service, air transport uses freight flights, so this study assumes that express service and passenger service are separated.

Assumption 2: It is assumed that high-speed trains and freight flights strictly carry out transportation services according to their planned timetables, regardless of the newly added express trains and the adjustment of the planned timetable.

3.2 Symbol definition

Some notations used in the constructed model are defined as follows.

Table 2 – Notations used in the model

Set	Definition
P	Set of express shipments
U	Set of service points of a high-speed railway or flight
T	Set of time intervals
S	Set of transport modes
N	Set of extended space-time network nodes
A	Set of extended space-time network arcs
Index	
p, p'	Index of each batch of express shipment, $p, p' \in P$
u, u'	Index of service station point, $u, u' \in U$
t, t'	Index of time interval, $t, t' \in T$
$(u,t,m), (u',t',m')$	Index of extended space-time network node, $(u,t,m), (u',t',m') \in N$
$(u,t,m;u',t',m')$	Index of extended space-time network arc, $(u,t,m;u',t',m') \in A$
Parameter	
$c(u,t,m;u',t',m')$	The cost of arc $(u,t,m;u',t',m')$ in extended space-time network
Variable	
$x^p(u,t,m;u',t',m')$	=1 if arc $(u,t,m;u',t',m')$ is used by express parcel p; =0 otherwise

4. MATHEMATICAL MODEL FORMULATION

4.1 Network construction

The high-speed rail express service plan under the inter-modal transport is abstracted as the time-space route selection problem of express shipment to different traffic resources. It has both time-space attributes and transportation mode selection attributes, so we can construct an extended time-space network to capture the express service plan. Specifically, the time dimension describes the time attribute of each batch of express service. Without loss of generality, the time horizon is discretised according to the granularity of 1 min, and the express service time is described by integer minutes. The space dimension describes the location of each shipment, indicating where each shipment is serviced. The extended dimension describes the optional transportation modes of the express shipment. The time-space extended network can be defined as $G = (N, A)$, where N is the network nodes set, and A is the network arcs set.

1) Network nodes

The network nodes include a virtual origin node σ, intermediate nodes N' and a virtual destination node τ, i.e.,

$$N = \{\sigma, \tau\} \cup N'$$

2) Network arcs

Network arcs contain start arcs A^{start}, task arcs A^{task}, transfer arcs A^{trans} and end arcs A^{end}.

$$A = A^{start} \cup A^{task} \cup A^{trans} \cup A^{end}$$

Start arcs A^{start}: connecting the virtual origin node to the intermediate nodes.

$$A^{\text{start}} = \{(u,t,m;u',t',m') | (u,t,m) = \sigma, (u',t',m') \in N'\}$$

Task arcs A^{task}: corresponding to high-speed rail operation or flight task.

$$A^{\text{task}} = \{(u,t,m;u',t',m') | (u,t,m) \in N', (u',t',m') \in N', m = m'\}$$

Transfer arcs A^{trans}: connecting nodes of different extended dimensions, indicating the transfer of express in the network.

$$A^{\text{trans}} = \{(u,t,m;u',t',m') | (u,t,m) \in N', (u',t',m') \in N', m \neq m'\}$$

End arcs A^{end}: connecting the intermediate nodes with the virtual end node.

$$A^{\text{end}} = \{(u,t,m;u',t',m') | (u,t,m) \in N', (u',t',m') = \tau\}$$

3) Arc cost

The optimisation objective of the high-speed rail express service plan is to minimise the transportation time of all express shipments. Through network construction, the cost of various arcs in the network is related to the time attribute. For the start arcs and the end arcs, the cost can be defined as a fixed constant. The cost of the task arcs is related to the time difference between the beginning and the ending of the arc, which indicates the travel time of the high-speed train or flight corresponding to the selected arc. Similarly, the cost of the transfer arc is related to the time difference between the beginning and the ending of the arc, which indicates the transfer operation time of the express shipment.

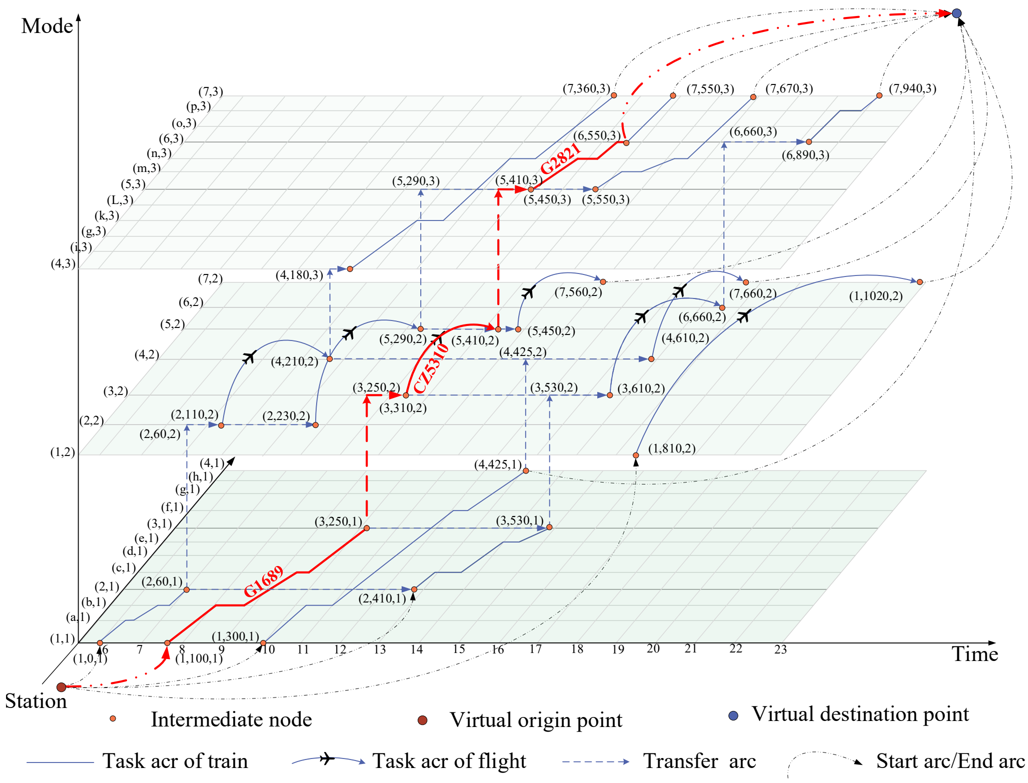


Figure 4 – Extended space-time network

In the extended space-time network, the service plan of each batch of express shipment corresponds to a space-time path, as shown by the red bold line in *Figure 4*, a batch of express shipment will depart from station 1 on high-speed rail line 1 aboard train G1689. After reaching transfer station 3, the shipment will be transferred to flight CZ5310, which will transport it by air to transfer station 5. Then, it will continue by transferring to the high-speed rail line 3, travelling on train G2821 to reach its destination station 6. Besides, the service time of express shipment corresponds to the timetable of the serving train or flight. The space-time path includes the train number, flight number and transfer information selected by the express shipment and the corresponding service time and position information.

4.2 Mathematical model

Objective function

The objective of high-speed railway express service is to minimise the total transportation time for all express shipments, comprising the travel time of serving trains or flights and the express transfer time. By establishing the network and specifying costs for different arcs, the minimal transportation time for all express shipments can be translated into the minimal cost of all selected arcs for all express shipments in the network flow model, that is;

$$\min \sum_{p\in P} \sum_{(u,t,m;u',t',m')\in A} c(u,t,m;u',t',m') \times x^p(u,t,m;u',t',m') \tag{1}$$

System constraints

1) Flow balance constraint

The flow balance constraint is the basic constraint of the network flow model, which guarantees the uniqueness of the path chosen by each batch of express shipment in the network.

$$\sum_{u,t,m:(u,t,m;u',t',m')\in A} x^p(u,t,m;u',t',m') - \sum_{u,t,m:(u',t',m';u,t,m)\in A} x^p(u',t',m';u,t,m) = \begin{cases} -1, (u',t',m') = \sigma \\ 1, (u',t',m') = \tau \\ 0, \text{otherwise} \end{cases}, \forall p \in P \tag{2}$$

2) Capacity constraint

The high-speed railway trains and flights have strict capacity constraints when providing express services, that is, the number of express shipments carried by high-speed trains or flights cannot exceed their corresponding carrying capacity. In the constructed network, each task arc is assigned a capacity attribute, corresponding to the service capacity that a train or flight can provide for high-speed railway express services. Corresponding to the network, the capacity constraint means that the number of all shipments loaded in any task arc cannot exceed the capacity of the arc, that is;

$$\sum_{p\in P} x^p(u,t,m;u',t',m') \le \mathrm{Cap}(u,t,m;u',t',m'),\ \forall (u,t,m;u',t',m') \in A^{\mathrm{task}} \tag{3}$$

3) Variable domain constraint

$$x^p(u,t,m;u',t',m') \in \{0,1\} \tag{4}$$

5. LAGRANGIAN RELAXATION ALGORITHM

5.1 Decomposition process

Considering the block diagonal structure of the mathematical model, the Lagrangian relaxation algorithm is used to solve the model. For convenience, the symbol a is introduced to simplify the network arc. The arc capacity constraints are the coupling relationship among express shipments, which compete for service resources. By introducing Lagrangian multipliers for each arc, the constraint is relaxed and the Lagrangian function is constructed as:

$$\min \sum_{p\in P}\sum_{a\in A} c(a)\times x^p(a) + \sum_{a\in A^{\text{task}}} \lambda_a \times [\sum_{p\in P} x^p(a) - \text{Cap}(a)] = \sum_{p\in P}\sum_{a\in A} c(a)\times x^p(a) + \sum_{p\in P}\sum_{a\in A^{\text{task}}} \lambda_a \times x^p(a) - \sum_{a\in A^{\text{task}}} \lambda_a \times \text{Cap}(a). \tag{5}$$

Furthermore, the Lagrangian relaxation problem is organised as:

$$\min \sum_{p\in P}\sum_{a\in A^{\text{task}}} c'(a)\times x^p(a),$$

$S.t.(2),(4),$

In which,

$$c'(a) = \begin{cases} c(a)+\lambda_a, \forall a \in A^{\text{task}}, \\ c(a), \text{otherwise}. \end{cases} \tag{6}$$

Since the coupling constraints on each shipment are relaxed into the objective function, the remaining constraints are independent for each express shipment. The original problem is decomposed into a sub-problem of a single express shipment, and the sub-problem is the shortest path problem of a single batch of express shipment in the constructed time-space extended network.

5.2 Lagrangian relaxation algorithm framework

By relaxation and decomposition, the Lagrangian algorithm framework is shown below.

Step 1: Initialise

Constructing the time-space extended network G, let the iteration number $p = 1$, initialise Lagrangian multipliers $\lambda_a > 0$, initialise the best lower bound and best upper bound $LB = -\infty, UB = +\infty$.

Step 2: Compute and update the lower bound

Use the shortest path algorithm to solve sub-problems, and compute the current lower bound LB_q by formulation (5), and update the best lower bound as $LB = \max\{LB, LB_q\}$.

Step 3: Update Lagrangian multipliers

Calculate the sub-gradient as $\nabla\lambda_a = \sum_{p\in P} x^p(a) - \text{Cap}(a)$ and update Lagrangian multipliers as $\lambda_a^{n+1} = \max\{0, \lambda_a^n + \mu^n \times \nabla\lambda_a\}$, in which μ^n is iteration step, and $\mu^n = 1/(n+1)$.

Step 4: Compute and update the upper bound

Use the heuristic algorithm in 5.3 to change the dual solutions into feasible solutions and compute the current upper bound UB_q by formulation (1), update the best upper bound as $UB = \min\{UB, UB_q\}$.

Step 5: Terminate condition

Compute the optimality gap $= (UB - LB)/UB$; if gap $\leq \varepsilon$ (the allowed minimum gap value) or $n \geq N$ (the allowed maximum iteration number), then the algorithm is terminated. Otherwise, iteration number $n = n + 1$, return to Step 2.

5.3 Heuristic algorithm

To determine the upper bound, feasible results must be achieved based on dual solutions, as dual solutions may not always be feasible. This paper presents an algorithm based on sub-gradient sorting to obtain a high-quality solution, as illustrated in *Figure 5*.

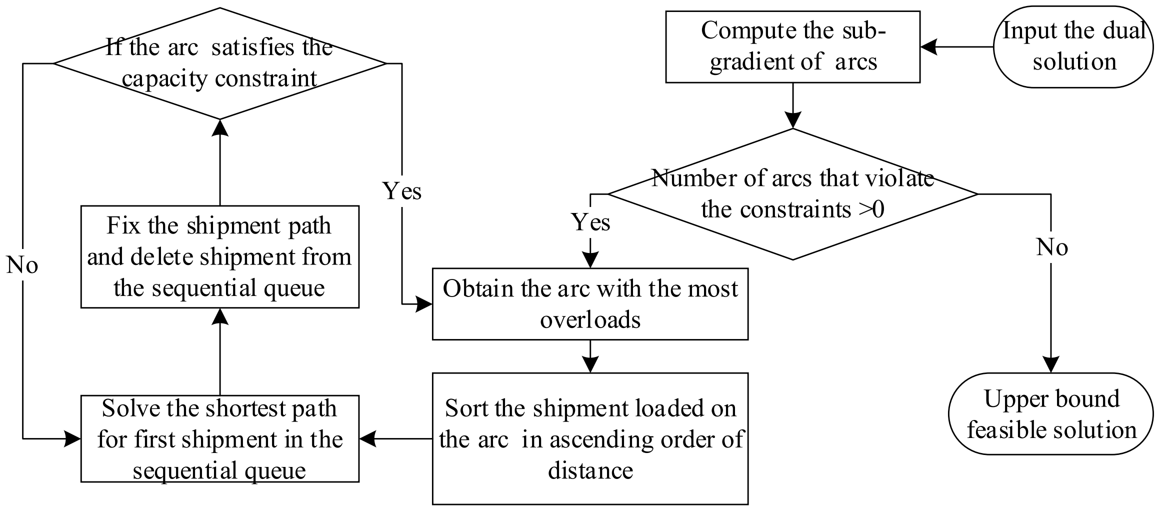


Figure 5 – Procedure of the feasible algorithm

6. NUMERICAL EXPERIMENTS

For the optimisation problem of the high-speed rail express service plan studied in this paper, we can construct a general mixed integer programming model and solve it by using a general solver. In order to verify the necessity of constructing the time-space extended network model and the efficiency of the Lagrangian relaxation algorithm, the two methods are compared, and the mixed integer programming model constructed is shown in the appendix. All experiments in this paper were implemented on a personal computer with Intel(R) Core(TM) i5-10300H CPU@2.50GHz and 16.0 GB RAM. Lagrangian relaxation algorithm is implemented by Python 3.11, and the general mixed integer programming model is solved by the DICOPT solver of commercial optimisation software GAMS. In order to make the paper more compact, the express demand data of the small-scale example and the timetable data and other various parameters in the real-world example are uploaded to the website https://github.com/ShuRuiCaolzjtu/-_-.

6.1 Small-scale example

This example is a virtual scenario, and the network consists of a high-speed corridor and 2 flights. The high-speed rail line operates 4 high-speed trains with 7 stations. The information on high-speed trains and flight schedules is shown in *Figure 6*, and the timetable information is shown in *Table 3*.

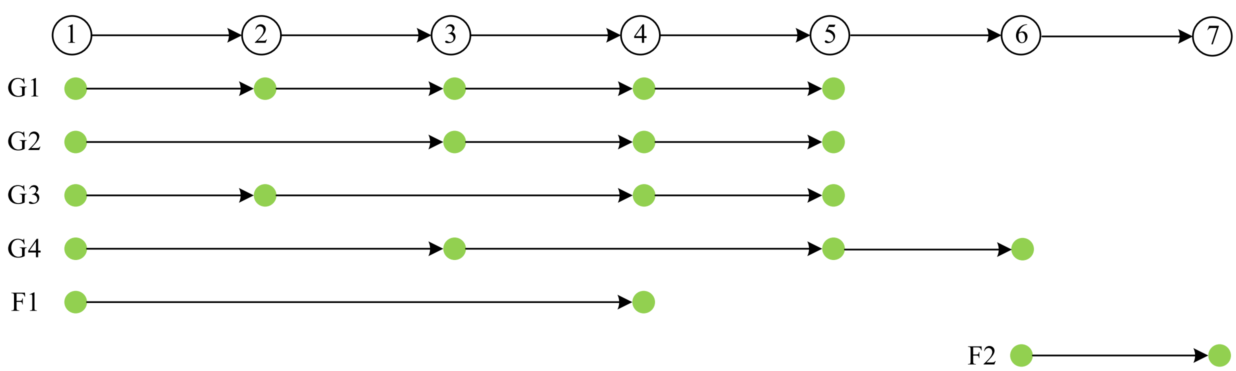


Figure 6 – Information on high-speed rail trains and flight schedules

Table 3 – Timetable information

Train/Flight	Station 1	Station 2	Station 3	Station 4	Station 5	Station 6	Station 7	Capacity
G1	8:00	8:13/8:15	8:31/8:33	8:44/8:46	9:07	--	--	10
G2	8:10	--	8:38/8:40	8:51/8:53	9:14			10
G3	8:20	8:33/8:35	--	9:01/9:03	9:24			10
G4	8:30	--	8:58/9:00	--	9:31/9:33	9:40	--	10
F1	8:05	--	--	8:15	--	--	--	10
F2	--	--	--	--	--	9:45	9:50	10

According to different high-speed rail express demands, 4 groups of small-scale examples are tested. The Lagrangian relaxation algorithm proposed in this paper and the solver method for solving mixed integer programming models are compared from computing time and objective value. The results are shown in *Table 4*.

Table 4 – Comparison of computing results

Case-demand	DICOPT		LR		Optimal solution	
	Computing time	Objective value	Computing time	Objective value	Optimal value	Gap
Case-10	0.063	228	<0.01	228	228	0.00%
Case-20	0.125	476	<0.01	476	476	0.00%
Case-30	0.187	688	0.06	688	688	0.00%
Case-40	0.25	1006	0.07	1006	1006	0.00%

The results show that both methods can obtain the optimal solution in a short time, which proves the correctness of the flow equilibrium model based on the space-time extended network and Lagrangian relaxation algorithm. Due to the small size of the example, it is difficult to reflect the advantages of the method proposed in this paper in terms of calculation time, which will be further verified in the examples in the next section.

6.2 Real-world experiments

Basic data

In order to further verify the effectiveness of the proposed method in this study, the Shanghai-Kunming corridor is taken as an example for validation analysis. The high-speed railway is one of the main corridors of China's high-speed railway line, with a total length of 2,252 km, and the number of passenger service stations on the whole line is 54. It is noted that, according to the characteristics of high-speed rail express service, this paper considers some passenger and freight hubs or important stations with large express service. The main passenger stations and the airport stations, and flight routes that undertake express services in this region are shown in *Figure 7*.

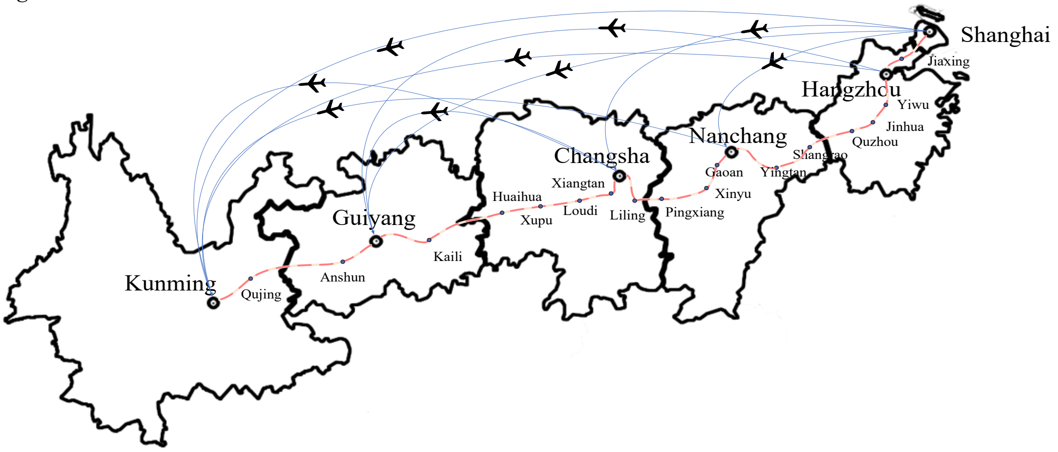


Figure 7 – Setting of the Shanghai-Kunming corridor

The operation period of the high-speed rail express is 06:00-24. The scheduled train timetable is selected from the operation data of the China railway ticket system 12306 in the fourth quarter of 2023. Not all high-speed rail trains have the capacity to provide express service; therefore, we selected a subset of high-speed rail trains as the research object. In this case study, there are 25 cities along the Shanghai-Kunming high-speed rail line that can handle high-speed rail express services. There are 19 high-speed trains and 9 flights undertaking express tasks, with their operation plans shown in *Figure 8*. Stations 1, 4, 11, 16, 22 and 25 are eligible for high-speed rail and air inter-modal transportation.

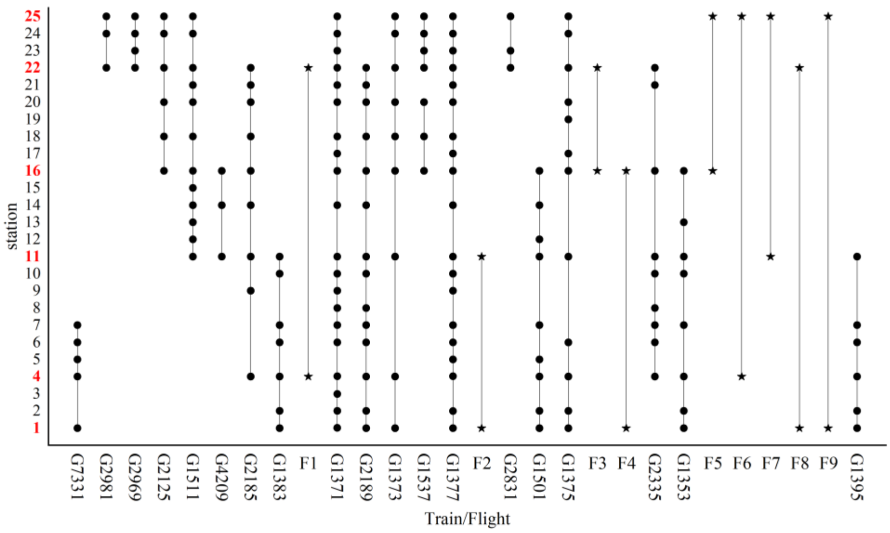


Figure 8 – Operation plans for high-speed rail trains and flights

Comparison of different solution scales

According to the different scales of express demand, a total of 7 groups of examples are set up, and the DICOPT solver and Lagrangian relaxation algorithm are used to solve them, respectively. The calculation results are shown in *Table 5*, and the variation of calculation time and objective value for different scale examples is shown in *Figure 9*. The results indicate that when the scale of express demand is small, both methods can obtain feasible solutions within a short time. However, the computing time using the DICOPT solver increases dramatically with the increase of case scale, for example, when the express demand is 50, it terminates after exceeding the allowed calculation time. When the express demand scale increases further, the DICOPT solver cannot obtain a feasible solution due to memory overflow. For the Lagrangian relaxation algorithm, although the computation time increases with the increase of the case size, the solution of the problem can be obtained in a short time, for example, when the case size is 2,000, the computation time is 163.82 s. The effectiveness of the Lagrangian relaxation algorithm proposed in this paper for large-scale numerical examples is verified by comparing the results of the two methods for 7 groups of numerical examples with different scales of express demand.

Table 5 – Comparison of different scales

Express demand	LR computing time/s	DICOPT computing time/s	LR optimised objective value/min	DICOPT optimised objective value/min
5	0.24	59.34	686	664
25	0.43	2525.72	4146	7783 (Stopped on NLP worsening)
50	0.84	10000 (overtime)	7357	11164
100	2.13	memory overflow	17401	memory overflow
500	7.57	memory overflow	91775	memory overflow
1000	31.42	memory overflow	214225	memory overflow
2000	163.82	memory overflow	413762	memory overflow

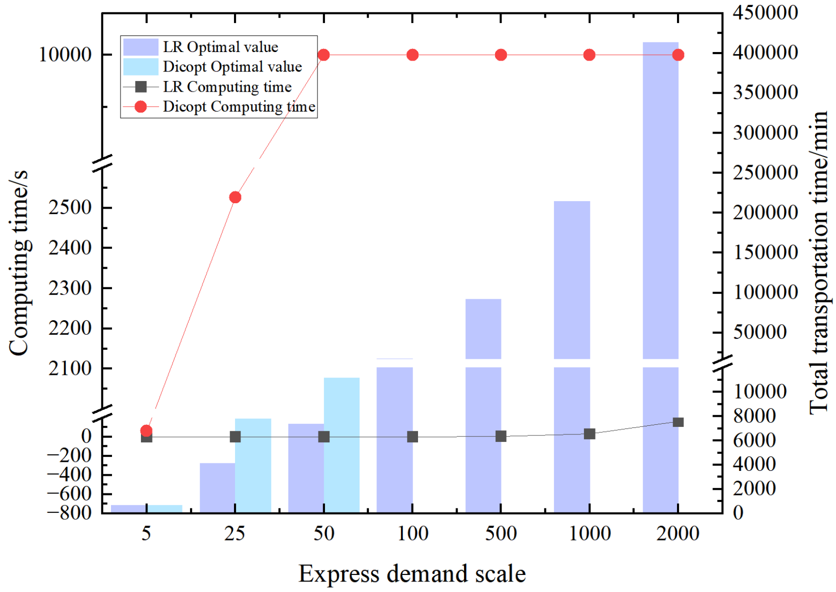


Figure 9 – Variation of calculation time and objective value for different scales

Comparison of different transportation models

Taking 2,000 batches of express demand as an example, the allowed maximum iteration number is set to 100, and the maximum gap value is allowed to be 2%. The Lagrangian relaxation algorithm proposed in this paper can solve the feasible solution of the problem in 163.82 seconds. Finally, the best upper bound is 413762, and the optimal gap value is 1.99%. The iterative process of the algorithm and the variation of gap values are shown in *Figures 10* and *11*. After the 23rd iteration, the solution results remain stable.

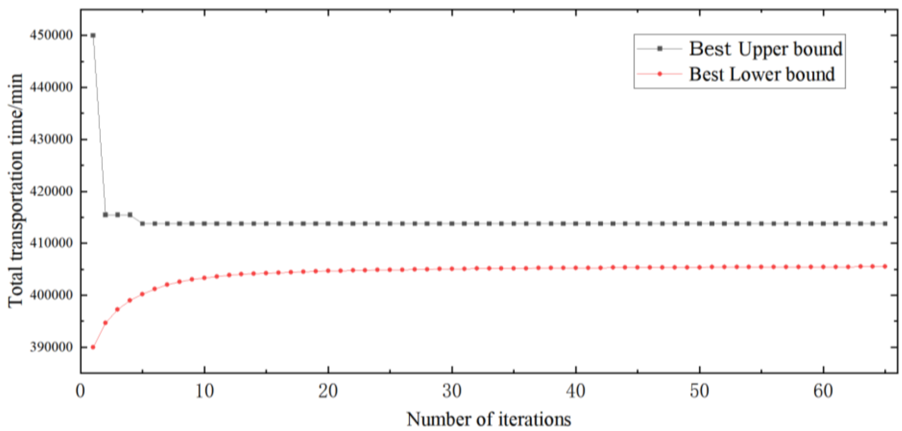


Figure 10 – Algorithm iteration process

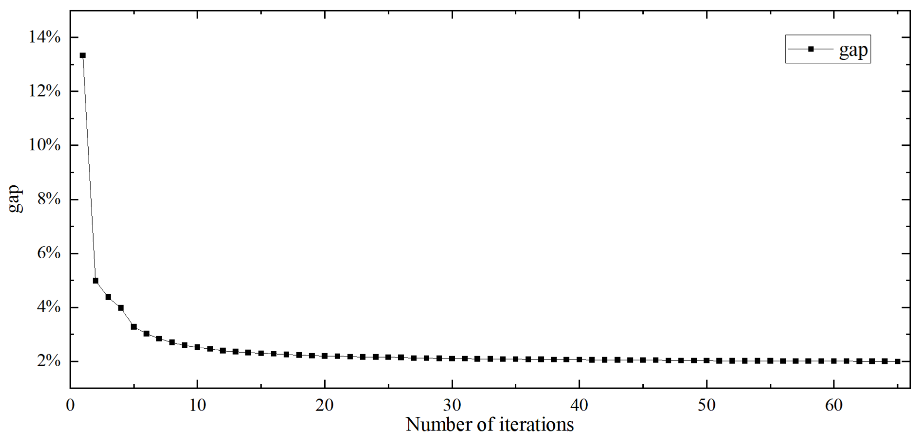
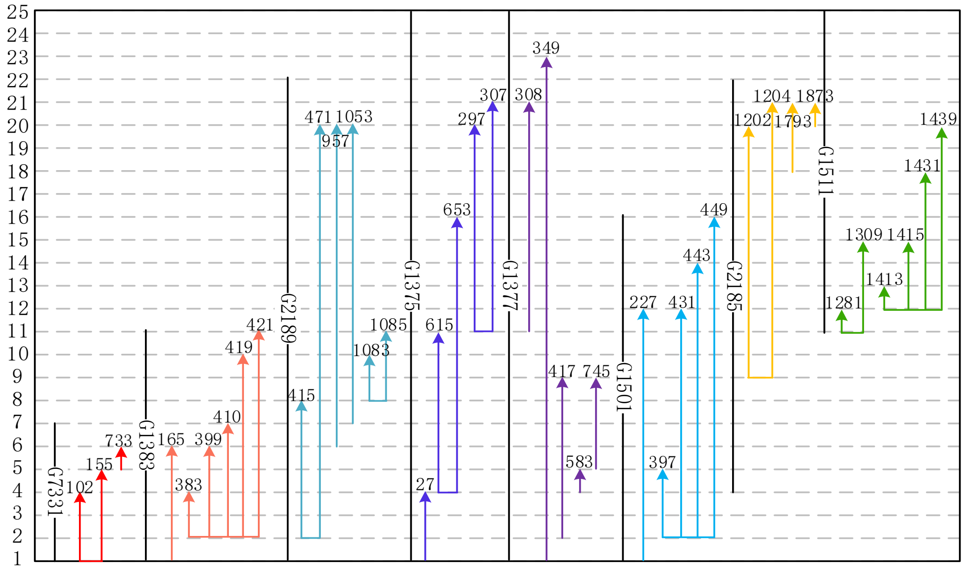
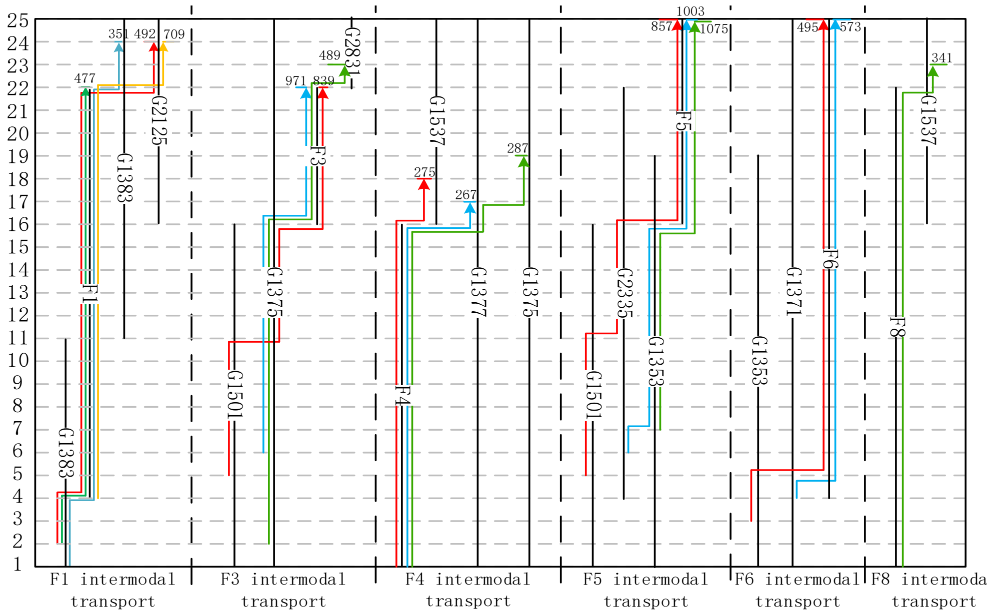


Figure 11 – Variation of gap value

The service plans of high-speed rail express under the inter-modal transport mode are sorted out. For the convenience of analysis, only a part of the express service plans is displayed, and the total express demand service plans are uploaded to http://github.com/ShuRuiCaolzjtu/ktly. According to whether the high-speed rail express demand service is transferred or not, the total express service scheme can be classified into two categories. One is the express service scheme that directly transports to the destination by high-speed rail train, as shown in *Figure 12(a)*. For example, train G1377 serves different OD express demand, such as the 17, 297, 307, 615 and 653rd express shipments. Another type is the express service with transfer operation, illustrated in *Figure 12(b)*. The 275th express shipments will be conveyed to the 16th station city via freight flight F4, and subsequently transferred to the G1537 high-speed railway train for final delivery to the destination.



(a)



(b)

Figure 12 – High-speed rail express service plan in real experiment: a) service plan with no transfer; b) transfer service plan

In order to compare the service characteristics of inter-modal and direct transportation modes on express demand, the results of the two modes are compared, as shown in *Table 6*. Under the direct transportation mode, due to the limitation of train capacity and the influence of service scope, express could not be fully served, and 66 batches of express were detained. The loading quantity and detention quantity of express at each station are shown in *Figure 13*. The express detention leads to a larger optimisation objective value. In comparison, all the

2,000 batches of express demand are served in the inter-modal transport mode, and the optimisation target is small. Compared with the two modes, the average transit time of the served express is 229.79 min in the direct mode, and 231.88 min in the inter-modal mode, and there is no significant difference in the average transit time.

Table 6 – Comparison of solution results between direct and inter-modal transportation

Transport mode	Total demand	Serviced demand	Stranded demand	Optimised objective value	Average transit time/min
Direct	2000	1934	66	1114087	229.79
Inter-modal	2000	2000	0	463762	231.88

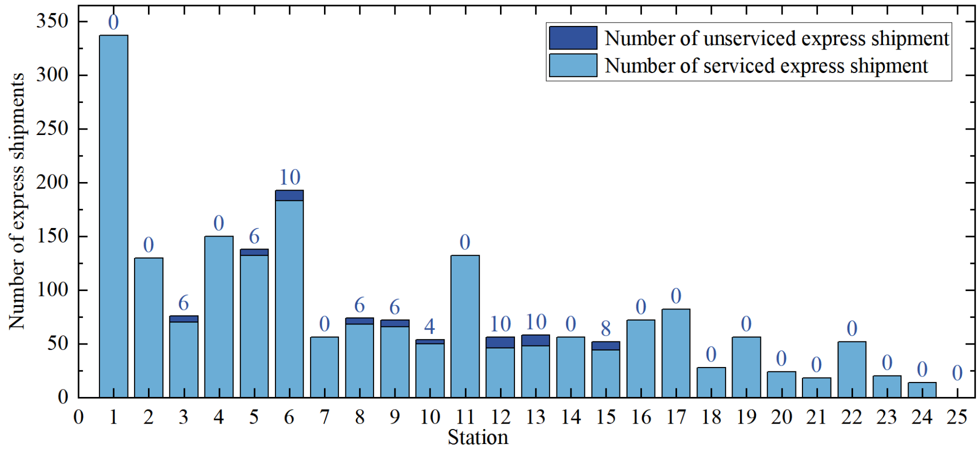


Figure 13 – Number of serviced and unserviced express shipments at stations in direct transportation mode

7. CONCLUSION

This paper optimises the service plan of high-speed rail express transportation under the inter-modal transportation, so as to improve the service scope of high-speed rail express transportation and realise the reasonable allocation of transportation resources. By constructing a three-dimensional space-time extended network to describe the route selection of express shipments, and arc costs to represent transportation and transit costs, then transforming train or flight capacity constraints into network task arc capacity constraints, a multi-commodity flow model for high-speed rail express transportation is established. Lagrangian relaxation algorithm is used to decompose the original problem into the shortest path of a single batch of express in the network. Finally, taking the Shanghai-Kunming corridor as an example, the results show that:

1) For 7 groups of examples with different demand scales, the Lagrangian relaxation algorithm and traditional solver method are used to solve them, respectively. The Lagrangian relaxation algorithm can obtain high-quality solutions within a short time. However, the general solver can only solve small-scale examples, and cannot solve medium-and large-scale examples due to memory overflow. Especially in the case with 2,000 batches of express shipments, the calculation time of the LR algorithm is 163.82 seconds, and the optimal gap value is 1.99%. This shows the applicability and effectiveness of the proposed model and algorithm.
2) Comparing the solution results of the express service plan under inter-modal and direct transportation modes, 66 batches of express could not be served completely due to the limitation of train capacity and the influence of service scope under the direct transportation mode. In the inter-modal mode, the 2,000 batches of express demand are all served, and there is no express detention. In addition, the average transit time of the serviced express has little difference between the two modes, which indicates that the inter-modal mode has advantages in expanding the scope of express service.

In practical operations, the high-speed rail express demand exhibits certain fluctuations due to factors such as the express market demand and pricing. Therefore, to enhance the robustness of service plans, studying the optimisation of high-speed rail express service plans under uncertain demand conditions is an interesting and promising direction for future research. In addition, some factors that are more suitable for the actual situation of high-speed rail express need to be further considered.

ACKNOWLEDGEMENTS

This research was supported by the National Natural Science Foundation of China (No. 72361020).

APPENDIX

In order to compare with the time-space network model and algorithm proposed in this paper, we have constructed a mixed-integer programming model for the high-speed rail express service plan under the intermodal transport mode. The sets, indices, parameters and decision variables introduced in the model are defined as shown in *Table 7*.

Table 7 – Symbol definition

Symbol	Definition	Symbol	Definition
Sets		Td_i^u	The departure time of train/flight *i* from station *u*
I	Set of train and flight	T_b	The arriving time of express shipment *b* at origin station
I_g	Set of high-speed train	M	A sufficiently large positive number
I_k	Set of freight flight	S_i^u	The stop plan for train/flight *i* at station *u*
B	Set of express shipment	T_z	Transfer time for express shipment
U	Set of station	C_i	The capacity of train/flight *i*
Indices		Variables	
i,j	Index of train or flight	x_i^b	0-1variable, indicating whether express shipment *b* is served by train/ flight *i*; if yes, the value is 1; otherwise, it is 0.
b	Index of each batch of express shipment	y_{ij}^{bu}	0-1variable, indicating whether express shipment *b* is transferred from train/flight *i* to *j* at station *u*; if yes, the value is 1; otherwise, it is 0.
u,v	Index of station	V_i^u	The number of express assigned to train/flight *i* at station *u*
O_b,D_b	The origin and destination station for express shipment *b*	A_i^u	The number of express unloaded from train/flight *i* at station *u*
Parameters		R_i^u	The number of express loaded at train/flight *i* departing from station *u*
Ta_i^u	The arrival time of train /flight *i* to station *u*		

$$min\ o\,bj = \sum_i \sum_b x_i^b \times (1 - \sum_j \sum_u y_{ij}^{bu}) \times Ta_i^{D_b} - \sum_i \sum_b x_i^b \times (1 - \sum_j \sum_u y_{ji}^{bu}) \times Ta_i^{O_b} \tag{f1}$$

$$\sum_{i \in I} x_i^b \geq 1 \quad \forall b \in B \tag{f2}$$

$$x_i^b \times (Td_i^{O_b} - T_b) > -\sum_u \sum_j y_{ji}^{bu} \times M \quad \forall i \in I, b \in B \tag{f3}$$

$$x_i^b \times (S_i^{O_b} - 1) > -\sum_u \sum_j y_{ij}^{bu} \times M \quad \forall i \in I, b \in B \tag{f4}$$

$$\sum_{i \in I} \sum_{j \in I} y_{ij}^{bu} \leq 1 \quad \forall b \in B \tag{f5}$$

$$\sum_{j \in I} \sum_{u \in U} y_{ij}^{bu} \leq 1 \quad \forall b \in B \tag{f6}$$

$$y_{ij}^{bu} \leq S_i^u \times S_j^u \quad \forall i,j \in I, b \in B, u, v \in [O_b, D_b] \tag{f7}$$

$$y_{ij}^{bu} = 0 \quad \forall i,j \in I, b \in B, u \notin (O_b, D_b) \tag{f8}$$

$$y_{ij}^{bu} \times (Ta_i^u + T_z) \leq Td_j^u \quad \forall i,j \in I, u \in U \tag{f9}$$

$$x_i^b \geq y_{ij}^{bu} \quad \forall i,j \in I, b \in B, u \in U \tag{f10}$$

$$x_j^b \geq y_{ij}^{bu} \quad \forall i,j \in I, b \in B, u \in U \tag{f11}$$

$$\sum_{i \in I} x_i^b = \sum_{u \in (O_b, D_b)} y_{ij}^{bu} + 1, \forall i,j \in I, b \in B \tag{f12}$$

$$V_i^u = \sum_{b:O_b=u} (x_i^b - \sum_{j \in I} \sum_{v>u} y_{ji}^{bv}) + \sum_{b \in B} \sum_{j \in I} y_{ji}^{bu} \; \forall i \in I, u \in U, i \neq j \tag{f13}$$

$$A_i^u = \sum_{b:D_b=u} (x_i^b - \sum_{j \in I} \sum_{v<u} y_{ij}^{bv}) + \sum_{b \in B} \sum_{j \in I} y_{ij}^{bu} \; \forall i \in I, u \in U, i \neq j \tag{f14}$$

$$R_i^u = R_i^{u-1} + V_i^u - A_i^u \quad \forall i \in I, u \in U \tag{f15}$$

$$R_i^u \leqslant C_i \quad \forall i \in I, u \in U \tag{f16}$$

$$x_i^b \in \{0,1\} \quad \forall i \in I, b \in B \tag{f17}$$

$$y_{ij}^{bu} \in \{0,1\} \quad \forall i,j \in I, b \in B, u \in \{O_b, D_b\} \tag{f18}$$

The objective function (f1) minimises the total transportation time of all express shipments. Constraint (f2) ensures that all shipments must be serviced. Constraints (f3-f4) represent the time and stop plan relationship between the express shipment and the service train or flight. Constraints (f5-f6) avoid unreasonable transfer of express shipment. Constraint (f7) creates a link between the transfer of express shipment and the train or flight stop plan. Constraint (f8) is a transfer scope constraint for express shipment. Constraint (f9) creates a relationship between the transfer time of express shipment, the departure and arrival time of the train or flight. Constraints (f11-f12) are the relationships among decision variables. Constraint (f13) is the quantity of express shipments loading on a train or flight at a station. Constraint (f14) is the quantity of express shipments unloading from a train or flight at a station. Constraint (f15) is the number of express shipments loaded onto the train or flight departing from the station. Constraint (f16) ensures that the number of express shipments loaded onto the train or flight departing from the station does not exceed the capacity of the train or flight. Constraints (f17-f18) are binary constraints for the decision variables.

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