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Multi-Objective Optimisation of Timetable for Urban Rail Transit during the End-of-Operation Period for Large-Scale Events

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ABSTRACT
This study proposes a timetable multi-objective optimisation model for urban rail transit during the end-of-operation period, addressing dynamic passenger demand under limited train capacity. The model simultaneously minimises the unreachable rate of last-train origin-destination (OD) points and passenger flow-induced congestion at the station. Key decisions involve optimising dwell times and headways while incorporating dynamic transfer spatiotemporal constraints and train capacity limitations. To resolve the computational complexity from non-convex and non-linear terms, a multi-objective whale optimisation algorithm is employed. Case studies under large-scale events reveal that moderately extending dwell times (1.5-2 minutes) alleviates congestion, while excessive extensions prove ineffective due to line length limitations. For headways, 8-10 minutes generally optimises departure frequency, though specific Pareto-optimal solutions may flexibly adopt 4-8 minute intervals. The findings enable operators to balance accessibility and congestion mitigation based on event characteristics, enhancing operational efficiency and passenger satisfaction. The research provides actionable strategies for improving last-train transfer coordination and station flow management during special operational periods.

KEYWORDS
timetable multi-objective optimisation; end-of-operation period; large-scale events; unreachable rate of last train origin-destination (OD) points; passenger flow-induced congestion at the station; whale optimisation algorithm.

1. INTRODUCTION

Motivation: During the post-event dispersal period of large-scale events, concentrated passenger flows often lead to significant short-term surges in ridership. Combined with the limited operating time of nighttime rail transit during the end-of-operation period, this situation poses considerable challenges to both evacuation efficiency and operational safety. Therefore, optimising the last-train timetable under such conditions has substantial theoretical and practical significance for enhancing the resilience of urban transportation systems and supporting major events.

Problem statement: Existing research on last-train timetable optimisation has mainly focused on improving transfer accessibility and OD accessibility. However, dedicated studies on timetable coordination for abnormal scenarios, such as large-scale events, remain scarce. Addressing this research gap is crucial for balancing evacuation efficiency with operational constraints and ensuring safety during the end-of-operation period.

Objectives: The optimisation objectives of this study are twofold: 1) Minimise the unreachable passenger rate: Reduce the proportion of passengers who cannot complete their journeys due to last-train timetable limitations (*Equation (3)*). 2) Minimise station congestion: Reduce the total passenger congestion across all stations during the end-of-operation period (*Equation (4)*). A set of Pareto-optimal solutions is generated to provide flexible decision-making options for operators based on practical requirements.

Methodological approach: A multi-objective mixed-integer nonlinear programming (MINLP) model is developed, considering both transfer and non-transfer passengers, train capacity limitations and station congestion effects. The model is solved using the multi-objective whale optimisation algorithm (NSWOA) to obtain a set of Pareto-optimal solutions.

Expected outcomes: The research aims to provide optimised last-train timetable strategies that enhance both operational efficiency and passenger service quality during post-event dispersal. These solutions are intended to offer data-driven decision support for operators managing large-scale events under constrained operating conditions.

Paper organisation: The remainder of this paper is organised as follows: Section 2 reviews the related research on last-train timetable optimisation. Section 3 describes the problem definition and key assumptions. Section 4 formulates the mathematical model for last-train timetable optimisation under large-scale event scenarios. Section 5 introduces the solution algorithm for solving the proposed model. Section 6 provides a case study analysis to validate the effectiveness of the proposed approach. Finally, Section 7 concludes the study and outlines future research directions.

2. LITERATURE REVIEW

2.1 Existing research on last-train timetable optimisation under normal conditions

During the post-event dispersal period of large-scale events, the large-scale passenger flow impact in a short period is particularly significant. Combined with the constraints of the end-of-operation period for nighttime rail transit services, this creates unique spatiotemporal aggregation characteristics. In this period, transfer connection efficiency not only directly affects station congestion management but also determines the degree of OD accessibility within the limited time window. Particularly under the rigid constraints of the last train, how to enhance networked transportation efficiency through collaborative optimisation of the train timetable becomes a key issue in balancing evacuation efficiency and operational safety. Exploring timetable optimisation methods for this unique scenario has significant theoretical value and practical guidance for improving the resilience of urban transport systems and enhancing the capacity to support major events.

Chen et al. [1] introduced the concepts of transfer accessibility and OD accessibility, and pointed out that the scheduled time of the last train in the timetable defines the arrival and departure times of the last train at each station, determining whether a transfer is possible. Therefore, the accessibility of last train services largely depends on the timetable of the last train.

Existing research on the coordinated optimisation of urban rail transit last train timetables can be mainly divided into two categories. The first category focuses on optimising the service accessibility between origin-destination (OD) pairs in the network, improving the success rate of passengers using urban rail transit services to reach their destinations; the second category emphasises optimising the success of transfer connections at transfer stations, improving the success rate of transferring from the last train service on one line to the last train service on another line during the end-of-operation period. However, research on the optimisation of last train timetables under abnormal conditions, such as large-scale events, is scarce both domestically and internationally. Most studies focus on peak periods rather than the end-of-operation period.

OD accessibility optimisation

Category one: Chen et al. [1] developed a mixed-integer programming model to determine the scheduled departure time of the last train. The model aims to maximise the weighted sum of the origin-destination (OD) pairs that can be covered by the last train service in the network. Furthermore, they proposed a method combining genetic algorithms with the Dijkstra algorithm for timetables to address related issues in large-scale networks. Wang et al. [2] optimised the latest time for passengers to reach their destinations using urban rail transit services, based on improving last train accessibility. They proposed an improved genetic algorithm based on Q-learning (QGA) to solve the mixed-integer programming model for last train timetable optimisation in large-scale urban rail transit networks. Zhang et al. [3] further considered transfers between the last train and non-last train services. Building on the improvement of network accessibility, they aimed to reduce passenger travel time. They decomposed the problem into two levels and proposed an iterative algorithm. At the upper level, an adaptive large neighbourhood search (ALNS) method was developed to generate new train timetables in the urban rail transit (URT) network. These timetables were then evaluated through the lower-level passenger flow problem, undergoing an optimisation-evaluation iteration until the termination condition was met. Ning et al. [4] developed a bi-objective mixed-integer programming model. The model seeks to achieve two primary objectives: the first is to maximise the number of reachable passengers, while the second is to minimise the total remaining travel distance for all passengers. Yang et al. [5] applied a spatiotemporal network framework to develop two '0-1' linear programming models, incorporating considerations of passenger path selection behaviour. Furthermore, the Lagrangian relaxation algorithm was employed to transform the hard constraints in the model into the objective function, resulting in a relaxed model. Wen et al. [6] aimed to enhance the accessibility of all passenger flow demands during the end-of-operation period. The objective function sought to maximise the number of reachable passengers at the OD points during the end-of-operation period. They developed a mixed-integer nonlinear programming model for coordinating the optimisation of the last train timetable in urban rail transit and used this model to perform timetable optimisation.

Transfer connection optimisation

Category two: Kang et al. [7] explored the optimisation of last train timetables and bus bridging services within the context of urban rail transit networks. They developed a mixed-integer programming model for the coordinated operation of last trains and bus bridges. The model focuses on two main objectives: on the one hand, maximising the frequency of feasible train-to-train and train-to-bus transfer connections, and on the other hand, minimising the overall waiting time for passengers transferring from trains to buses. Wang et al. [8] aimed to minimise the average transfer waiting time, transfer connection failures and the negative impact of random passenger flow, without imposing excessive costs on the operator. They also sought to enhance the robustness of the optimised train rescheduling scheme, ensuring that the overall optimisation performance remains stable despite disturbances from specific passenger flow scenarios, while considering all possible passenger flow states. Yang et al. [9] proposed a distributionally robust last train coordination planning problem with a dwell time adjustment strategy. They developed a new distributionally robust chance-constrained programming model to enhance the robustness of the last train under uncertain transfer passenger flow. Chen et al. [10] developed three last train timetable optimisation models, aiming to improve transfer accessibility with heterogeneous walking times by synchronising the arrival and departure times at transfer stations. They also proposed a discrete approximation method to reformulate these nonlinear models. Jiang et al. [11] developed a last train transfer coordination optimisation model for normal operational scenarios. The objective function is oriented towards transferring passenger flow demand intensity, and the constraints include the flexibility range of last train departure times in the network and the rigid locking mechanism for key line timetables. The model significantly enhances the coverage capacity of last train services by improving the transfer connection match. Engineering validation shows that the model can effectively expand the coverage of transfer services. Notably, existing research on abnormal scenarios such as large-scale events primarily focuses on traffic management technologies. Zhang et al. [12] innovatively proposed a multi-dimensional collaborative management system for large-scale events. By implementing zonal management (core area, buffer zone, evacuation zone), multi-modal coordination (shuttle vehicles - pedestrian systems - dedicated bus lines) and time-period segmentation strategies, they developed a three-dimensional guidance framework for managing spectator passenger flow. Niu et al. [13] overcame the limitations of traditional forecasting methods by integrating multi-source data to construct a spatiotemporal coupling feature matrix. They employed the random forest algorithm to perform dynamic OD passenger flow forecasting at a 5-minute time granularity following the conclusion of large-scale events.

2.2 Timetable optimisation under abnormal conditions

Research on urban rail transit timetables under abnormal conditions, such as large-scale events, has also been conducted. For example, Guo et al. [14] developed a mixed-integer nonlinear optimisation model to address issues such as severe platform overcrowding and passenger congestion at urban rail transit stations during holidays and large-scale events. The model significantly alleviates congestion at key stations. Yuan et al. [15] explored the optimisation of train timetables and skip-stop plans, aiming to minimise the total passenger waiting time and station congestion. Lu et al. [16] discussed the systematic study of robust passenger flow control strategies and the joint optimisation of train timetables on congested urban rail transit lines. Xu et al. [17] investigated the impact of unexpected events on rail transit timetables and studied a train rescheduling algorithm for high-density urban rail systems, aiming to mitigate the impact of delays on overall railway operations. They developed a mixed-integer linear programming model to generate an optimal rescheduled timetable and assist dispatchers in making intelligent decisions for dynamically adjusting train schedules.

2.3 Strengths and weaknesses of existing studies

1) Existing studies on enhancing transfer accessibility and OD accessibility for optimising last train timetables primarily concentrate on timetable coordination under standard operational conditions. However, they often overlook exceptional scenarios, such as holidays and large-scale events. In contrast, research on abnormal scenarios related to large-scale events mainly concentrates on traffic organisation schemes, traffic support measures and passenger flow forecasting. Currently, there is a lack of research on the last train timetable optimisation under abnormal conditions.
2) Existing research on urban rail transit timetable optimisation tends to adopt idealised assumptions, such as fixed or ignored transfer passenger flow and infinite train capacity, which limit their practical application. In contrast, this study investigates and formulates a more realistic and relevant problem, namely the optimisation of subway network train timetables and skip-stop plans, considering both non-transfer and transfer passengers, strict train capacity and passenger waiting times.
3) Most existing research on last train timetables only considers the second-to-last and last trains, whereas this study focuses on the entire end-of-operation period for last train timetable optimisation.

Based on this, this study focuses on the practical demands of the end-of-operation period in the context of large-scale events and constructs a last train timetable coordination optimisation model for this period. The model aims to maximise the OD accessibility of the last train and minimise passenger flow-induced congestion at stations during the end-of-operation period, with a focus on improving operational efficiency and passenger experience. Considering the characteristics of high passenger flow and significant travel demand during the end-of-operation period under large-scale event scenarios, a mixed-integer non-convex optimisation model is developed. The model is solved by using the multi-objective whale optimisation algorithm, generating a series of Pareto-optimal solutions. Sensitivity analysis is then conducted to provide more parameter adjustment options for operators to select from, in order to enhance the operational efficiency and passenger service quality during the end-of-operation period in subway systems.

3. PROBLEM DESCRIPTION

In the context of large-scale events, the number of passengers during the end-of-operation period exceeds that of regular passengers. In passengers' transfer behaviour, if passengers arrive at a transfer station just after missing a train, they are required to wait for several minutes or even longer for the next train. If passengers miss the last train, they must resort to alternative transportation methods, significantly reducing network accessibility and passenger satisfaction. Therefore, the formulation of timetables for the end-of-operation period under abnormal conditions, such as large-scale events, becomes particularly critical.

This study proposes a dynamic collaborative scheduling method for a bidirectional operating network, deconstructing the up and down lines into independent service units, and selecting the end-of-operation period from 21:30 to 24:00. Based on the spatiotemporal distribution characteristics of transfer passenger flow (Table 1), a phased, differentiated optimisation objective system is established. Under the context of large-scale events, the end-of-operation period includes both peak and last train services. Therefore, this study comprehensively considers both types of services: for non-last train services, the objective is to minimise passenger flow-induced congestion at stations; for last train services, the objective is to improve the OD accessibility within the network. The optimisation focuses on the train timetables for each line during the end-of-operation period to enhance nighttime operational efficiency and passenger satisfaction.

Table 1 – Comparative analysis of transfer demand characteristics across different train services

Comparison factors	Last train service number	Non-last train peak service number
transfer passenger flow	relatively lower passenger flow	high passenger flow
transfer waiting time	longer	shorter
transfer frequency	sensitive	relatively sensitive
transfer accessibility	partially accessible	fully accessible
station dwell situation	generally non-existent	common phenomenon
focus points	transfer efficiency, path accessibility	efficiency and comfort

3.1 Transfer connection characterisation

Unidirectional transfer accessibility

Unidirectional transfer connection refers to a situation at a transfer station where passengers from only one train can transfer to another train on a different line, and reverse-direction transfers are unavailable. The feasibility of a successful transfer is determined by the transfer connection time, denoted as Z. If $Z \geq 0$, the transfer is accessible in that direction, otherwise, it is not accessible. The connection time is given by the formula (1).

$$Z = D_{sk}^{i} - A_{sl}^{\mathrm{j}} - W_{\mathrm{slk}} \tag{1}$$

In the equation:

D_{sk}^{i} : the departure time of the i-th train on line k from station s;

A_{sl}^{j} : the arrival time of the j-th train on line l at station s;

W_{slk} : the transfer walking time for passengers between line k and line l at station s.

In the transfer direction, if a passenger can successfully board a train i on the transfer line after alighting from a train j on the originating line, this connection process is defined as transfer accessibility. Conversely, if this connection is not possible, it is referred to as transfer inaccessibility. In this case, the transfer situation for the last train in this direction may result in one of the three scenarios shown in *Figure 1*.

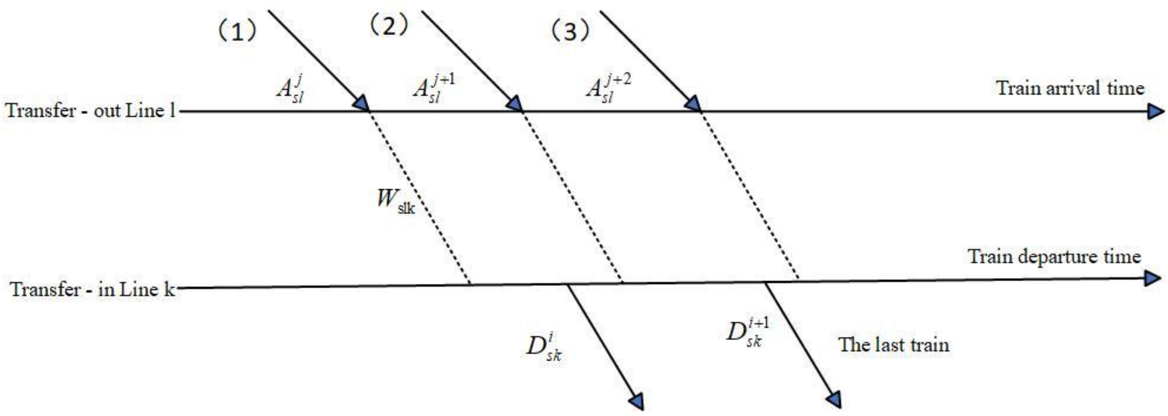

Figure 1 – Schematic diagram of unidirectional transfer connectivity at transfer stations

When passengers travelling on train j on line l transfer to line k, if train i has not yet departed from the station, these passengers can choose either train i or train i+1 to complete their transfer.

For passengers travelling on train j+1 on line l, when transferring to line k, if the last train i+1 has not yet departed from the station, they can only transfer using the last train i+1. However, in this case, some passengers may “miss” train i just in time, thus having to wait longer for the last train, leading to a significant increase in waiting time.

If passengers travelling on train j+2 on line l attempt to transfer to line k but the last train i+1 has already departed, these passengers will be unable to complete the transfer and will be compelled to resort to alternative modes of transportation. This scenario undoubtedly diminishes network accessibility and adversely affects the overall passenger travel experience.

Bidirectional transfer accessibility

In transfer stations where line connections occur, the transfer directions exhibit symmetry. Specifically, at transfer station s, the direction from line l to line k is symmetric to the direction from line k to line l. We define the situation where two trains operating in opposite directions are able to achieve mutual transfer as bidirectional transfer accessibility.

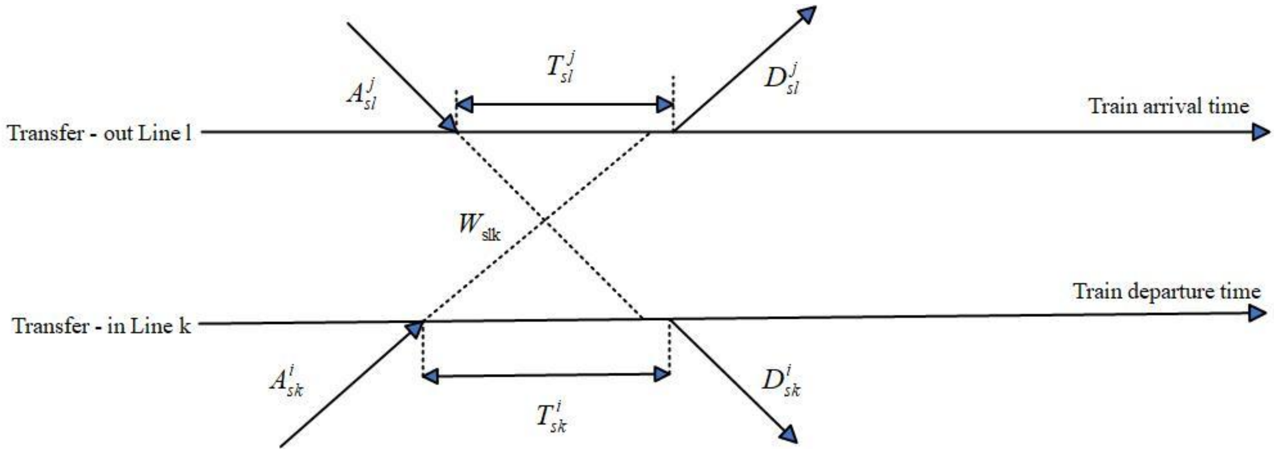

Figure 2 – Illustration of bidirectional transfer connection relationships at transfer stations

3.2 OD accessibility analysis

For the OD pair from p to q, by imposing transfer count restrictions and no-circuit constraints, a finite number of possible paths can be identified. If, at any transfer station along a path, the transfer connection fails, that path is considered unreachable. However, if any path within the finite set of paths connecting OD pair p-q is reachable, then the OD pair p-q is considered accessible. When multiple paths are reachable, the OD pair p-q is accessible. As shown in *Figure 3*, where S1 to S6 are transfer stations and the arrows represent different lines, there are three possible paths for the OD pair MN: {(M-S1-S2-N), (M-S3-S4-N), (M-S5-S6-N)}. Suppose that the transfer at S3 in the path (M-S3-S4-N) fails, then this path is unreachable. At the same time, if all the transfer connections at stations S5 and S6 in the path (M-S5-S6-N) are successful, then the OD pair MN is accessible (from M to N).

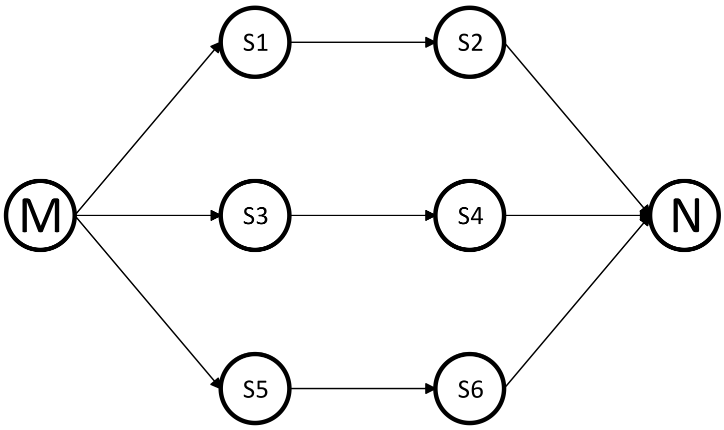

Figure 3 – Valid paths between the OD pair MN

4. MODEL FORMULATION

4.1 Model assumptions and notation explanation

It is assumed that during the end-of-operation period, passenger heterogeneity is relatively weak, and the variation in transfer walking time is minimal. Specifically, for the same transfer direction at a transfer station, the transfer walking time is consistent for all passengers.

The valid paths between OD pairs adopt a physical acyclic topology structure.

All passengers, during the transfer process, will choose the train with the shortest arrival time on the transfer line within the end-of-operation period to complete their transfer.

The train's car body turnover is not considered, and the two operational directions of each line are treated as two independent lines.

Assumption 2 specifies that the transfer walking time is known as a parameter related to operating lines that passengers transfer between, which can reflect the variability of transfer walking times between different lines. This is reasonable since the transfer walking time depends on the transfer channel layout and can be estimated from the average walking speed of passengers and the transfer channel length. This assumption is widely used in research on network-level train timetabling (Wong et al. [18], Wang et al. [19], Yin et al. [20]).

4.2 Notation explanation

Table 2 – Notation explanation

Notation	Definition
Set index	
L	set of rail lines in the network, l $\in$ L
S^L	set of all stations on line l, $\{1,2,3,...,s_l\}$
Q^L	set of trains on line L, $\{1,2,3,...,j_l\}$
X_l	set of all transfer stations on line l, $X_l \in S^L$
J	set of all last trains
System parameters	
$T_{sl}^{\max}$	the maximum dwell time at station s on line l
$T_{sl}^{\min}$	the minimum dwell time at station s on line l
$W_{sll'}$	the walking time for transferring from line l to line l' at station s
$\underline{R}_l^{s(s-1)}$	the lower limit of the running time for a train on line l between adjacent stations (s-1) and s
$\bar{R}_l^{s(s-1)}$	the upper limit of the running time for a train on line l between adjacent stations (s-1) and s
$\bar{H}_l$	the upper limit of the headway between two adjacent trains on line l
$\underline{H}_l$	the lower limit of the headway between two adjacent trains on line l
$C_{\max}$	the maximum capacity of a train
$T_l^{\max}$	the latest train closure time
$\beta_s^{l',l}$	the transfer ratio between line l and line l' at station s

Notation	Definition
$\alpha_{S}^{l,j}$	the alighting rate of train j on line l at station s
λ_{s}^{l}	the average passenger arrival rate for those travelling to station s on line l
K_{pq}	the ratio of the passenger flow between OD pairs to the total capacity
$h_{l,j,s}$	penalty coefficient, a piecewise function, designed to prevent the accumulation of large numbers of passengers at the station
T_{AD}	minimum departure time
System variables	
A_{sl}^{j}	the arrival time of the j-th train of line l at station s
D_{sl}^{j}	the departure time of the j-th train of line l from station s
$X_{ll^{\prime}jj^{\prime}}^{s}$	a 0-1 variable, where a value of 1 indicates a successful transfer between line l and line l' at station s, and a value of 0 indicates an unsuccessful transfer
$C_{l,j,s}^{remaining}$	the remaining capacity on train j of line l after passengers disembark at station s
$P_{l,j,s}^{on}$	the actual number of passengers boarding train j of line l at station s
$P_{l,j,s}^{al}$	the actual number of passengers alighting from train j of line l at station s
$P_{l,j,s}^{waita}$	the actual number of passengers waiting for train j of line l at station s
$P_{l,j,s}^{st}$	the actual number of passengers waiting and stranded at station s for train j of line l
$g_{l,j,s}^{l^{\prime},j^{\prime}}$	the number of passengers transferring between train j of line l and train j' of line l' at station s
Y_{OD}	a 0-1 variable that measures the accessibility between OD pairs, where 1 indicates that the OD pair is accessible, and 0 indicates that it is not accessible
Z_{OD}^{m}	a 0-1 variable that measures the accessibility of the m-th path between OD pairs, where 1 indicates that the path is accessible, and 0 indicates that it is not accessible
$R_{pq,sll^{\prime}jj^{\prime}}^{m.n}$	a 0-1 variable ($R_{pq,sll^{\prime}jj^{\prime}}^{m.n}=X_{ll^{\prime}jj^{\prime}}^{s}$) that measures the transfer accessibility of the m-th path between OD pair pq at station s (the n-th station)
$h_{l,j,s}$	the penalty coefficient, a piecewise function, is designed to prevent the accumulation of large numbers of passengers at the station

4.3 Summary of objective functions

To clearly define the multi-objective optimisation framework used in this study, the following objective functions are considered:

1) Minimise the unreachable passenger rate: Reduce the proportion of passengers who cannot complete their journeys due to last-train timetable limitations (*Equation (3)*).
2) Minimise station congestion: Reduce the total passenger congestion across all stations during the end-of-operation period (*Equation (4)*).

The objective of *Equation (2)* is to maximise the OD accessibility rate of the last train services within the network. To facilitate the solution process, *Equation (2)* is transformed into *Equation (3)*, with the objective function Z_1 being the minimisation of the proportion of unreachable passenger flow at the regional OD points: minimising the unreachable rate of passengers on the last train services.

$$Z_1 = \max \sum_{j \in J} \sum_{pq \in OD} (1 - Y_{pq}) K_{pq} \tag{2}$$

$$Z_1 = \min \sum_{j \in J} \sum_{pq \in OD} (Y_{pq} - 1) K_{pq} \tag{3}$$

Objective function Z_2 : Minimise the total passenger flow-induced congestion across all stations for all train services.

$$Z_2 = \min \sum_{l \in L, j \in Q^L, s \in S} P_{l,j,s}^{waita} \tag{4}$$

$P_{l,j,s}^{waita}$: the number of passengers waiting for the j-th train of line l at station s within the time period between two consecutive train services.

4.4 Constraint conditions

Timetable transfer constraint

1) Dwell time constraint

To ensure safe operation between sections and maintain passenger service levels, the train dwell time should be greater than the minimum dwell time and less than the maximum dwell time.

$$T_{sl}^{\min} \le D_{sl}^{j} - A_{sl}^{j} \le T_{sl}^{\max}, \ \forall l \in L, s \in S^L, j \in Q^l \tag{5}$$

2) The headway constraint between two adjacent trains on line L

To ensure the safety of train operations and maintain passenger service levels, the train headway should be greater than the minimum headway and less than the maximum headway.

$$H_l^{\min} \le D_{sl}^{j} - D_{sl}^{j-1} \le H_l^{\max}, \quad \forall l \in L, s \in S^L, j \in Q^l \setminus \{1\} \tag{6}$$

3) Train running time constraint between two adjacent stations on line L

The running time between stations should meet the actual operational requirements. It should be greater than the minimum running time and less than the maximum running time.

$$R_{s(s-1),l}^{\min} \le A_{sl}^{j} - D_{(s-1)l}^{j} \le R_{s(s-1),l}^{\min}, \quad \forall l \in L, s \in S^L \setminus \{1\}, j \in Q^l \tag{7}$$

To ensure station operational safety (train separation), the interval between the arrival time of a train at the same station and the departure time of the previous train should not be less than the minimum arrival-departure interval T_{AD} 。

$$T_{AD} \le A_{sl}^{j} - D_{sl}^{j-1} \ , \ \forall l \in L, s \in S^L \setminus \{1, s_l\}, j \in Q^l \setminus \{1\} \tag{8}$$

4) Constraint for measuring the success of transfer connections

The success of the transfer between train j of line l and train j' of line l' at station s is represented by introducing a 0-1 variable $X_{ll'jj'}^{s}$. M is a very large positive number.

$$M_1(X_{ll'jj'}^{s} - 1) \le D_{sl'}^{j'} - A_{\text{sl}}^{\text{j}} - W_{\text{sll}'} < M_1 X_{ll'jj'}^{s}, \forall l \cap l' \in L, s \in X_l, j \in Q^l \tag{9}$$

5) The constraint on the latest arrival time

To avoid interference with nighttime maintenance, the closing time at the terminal stations of each line should not be later than the latest train departure time $T_l^{\max}$.

$$A_{sl}^{j} \le T_l^{\max} \quad , \quad \forall l \in L, s = s_l, j = J \tag{10}$$

6) Accessibility calculation

Y_{pq}, Z_{pq}^{m} are both 0-1 variables, Y_{pq} is used to measure the accessibility between OD pairs， and Z_{pq}^{m} is used to measure the connectivity of the m-th path between OD pairs.

Assuming that there are M accessible paths between the OD pair pq, the OD pair pq is considered reachable as long as at least one path in the network remains connected. In the OD selection, only the terminal stations of each line are considered. This is because if the origin station of one line is reachable to either terminal station of another line, then all stations along that line are also considered reachable.

$$Y_{pq} \le \sum_{0}^{M} Z_{pq}^{\mathrm{m}}, \qquad \mathrm{pq} \in \mathrm{OD}, \mathrm{m} \in \mathrm{M} \tag{11}$$

7) Connectivity calculation

$R_{pq,sll^{/}}^{m.n}$ is a 0-1 variable ($R_{pq,s}^{m,n} = X_{ll^{/}jj^{/}}^{s}$ jj/ are the last train services of each line), evaluating the transfer accessibility of the m-th path between OD pair pq at station s (the n-th station). Within the m-th path of the OD pair pq, if a transfer fails at any station, the path is considered unreachable. Assuming that this path contains n transfer stations, its connectivity is expressed by the following equation.

$$M_2(Z_{pq}^{m} - 1) \le \sum_{1}^{N} (R_{pq,s}^{m,n} - 1) < M_2 Z_{pq}^{m}, \quad \mathrm{pq} \in OD, n \in N, R_{pq,s}^{m,n} = X_{ll^{/}jj^{/}}^{s} \tag{12}$$

Train passenger capacity constraint

1) Station waiting passenger constraint

t_0 : Start time of the study period. For non-transfer stations, $P_{l,j,s}^{waita}$ includes passengers arriving at the station from external sources during the headway between two consecutive train departures, as well as passengers who remained at the station after the departure of the previous train. For transfer stations, $P_{l,j,s}^{waita}$ includes passengers transferring from other lines. In this equation, λ_s^l represents the average passenger arrival rate for line l at station s. Its value can be obtained from historical AFC data estimation or forecasting.

$$P_{l,j,s}^{waita} = \begin{cases} (D_{sl}^{j} - t_0)\lambda_s^l + \sum_{l^{/} \in X_{l,u}, j^{/} \in Q^{L^{/}}} g_{l,j,s}^{l^{/},j^{/}}, \forall l \in L, s \in S^L, j = 1 \\ (D_{sl}^{j} - D_{sl}^{j-1})\lambda_s^l + \sum_{l^{/} \in X_{l,u}, j^{/} \in Q^{L^{/}}} g_{l,j,s}^{l^{/},j^{/}} + P_{l,j-1,s}^{st}, \forall l \in L, s \in S^L, j \in Q^l \setminus \{1\} \end{cases} \tag{13}$$

2) Station passenger stranding constraint

The number $P_{l,j,s}^{st}$ of stranded passengers after the departure of train j on line L is equal to the number $P_{l,j,s}^{waita}$ of waiting passengers at station s minus the number $P_{l,j,s}^{on}$ of boarding passengers.

$$P_{l,j,s}^{st} = P_{l,j,s}^{waita} - P_{l,j,s}^{on}, \qquad \forall l \in L, s \in S^L, j \in Q^l \tag{14}$$

3) Actual boarding passenger constraint

The actual boarding passenger count is the minimum value between the actual waiting passengers $P_{l,j,s}^{waita}$ and the remaining train capacity $C_{l,j,s}^{remaining}$.

$$P_{l,j,s}^{on} = \min\{P_{l,j,s}^{waita}, C_{l,j,s}^{remaining}\}, \quad \forall l \in L, s \in S^L, j \in Q^l \tag{15}$$

4) Train remaining capacity

The remaining capacity $C_{l,j,s}^{remaining}$ of the train at the starting station is equal to the train capacity $C_{\max}$, and the remaining capacity at the rest s stations is calculated as the remaining capacity $C_{l,j,s-1}^{remaining}$ at station s−1, minus the number $P_{l,j,s-1}^{on}$ of passengers boarding at station s−1, plus the number $P_{l,j,s}^{al}$ of passengers alighting at station s.

$$C_{l,j,s}^{remaining} = \begin{cases} C_{\max}, & \forall l \in L, s=1, j \in Q^l \\ C_{l,j,s-1}^{remaining} + P_{l,j,s}^{al} - P_{l,j,s-1}^{on}, & \forall l \in L, s \in S^L \setminus \{1\}, j \in Q^l \end{cases} \tag{16}$$

5) Alighting passenger constraint

The alighting passenger number $P_{l,j,s}^{al}$ of train j on line L at station s equals the alighting rate $\alpha_S^{l,j}$ multiplied by the number of passengers on board when reaching station s.

$$P_{l,j,s}^{al} = \begin{cases} 0, & \forall l \in L, s=1, j \in Q^l \\ \alpha_S^{l,j} * (C_{\max} - C_{l,j,s-1}^{remaining} + P_{l,j,s-1}^{on}), & \forall l \in L, s \in S^L \setminus \{1\}, j \in Q^l \end{cases} \tag{17}$$

6) Transfer passenger constraint

The number $g_{l,j,s}^{l',j'}$ of passengers transferring from train j of line L to train j/ of line L/ at station s, is equal to the number $P_{l,j,s}^{al}$ of passengers alighting from train j of line L at station s, multiplied by the transfer ratio $\beta_s^{l',l}$ between the two lines.

$$g_{l,j,s}^{l',j'} = P_{l,j,s}^{al} * \beta_s^{l',l} * X_{ll'jj'}^{s}, \quad \forall l \in L, s \in S^L, j \in Q^l \tag{18}$$

5. MODEL SOLUTION ALGORITHM

Given the NP-hard nature of the formulated mixed-integer nonlinear programming (MINLP) model, particularly due to the high-dimensional decision space and complex constraints from multi-line and multi-station coordination, conventional exact solvers (e.g. CPLEX, Gurobi) are often computationally infeasible for large-scale instances. Therefore, this study adopts a metaheuristic approach.

Specifically, we employ a multi-objective version of the whale optimisation algorithm (WOA), enhanced with non-dominated sorting and tailored variation strategies (NSWOA), to solve the optimisation model. This improved algorithm maintains WOA's core advantages, including global search capability, computational efficiency, and its ability to handle both continuous and discrete decision variables (Jangir et al. [21]). Furthermore, the integration of non-dominated sorting allows for effective exploration of the Pareto front in multi-objective settings.

Compared to other metaheuristic methods such as genetic algorithms (GA) and particle swarm optimisation (PSO), the proposed NSWOA offers simpler parameter tuning, fewer control parameters, and improved performance on scheduling and transportation problems, as supported by recent studies (Sharawi M et al. [22]).

While NSWOA is adopted in this study based on its algorithmic strengths and practical applicability, future research will include comparative evaluations with other multi-objective swarm intelligence algorithms to further validate its effectiveness.

The flowchart of the multi-objective whale optimisation algorithm is shown in *Figure 4*. Its detailed design process is as follows.

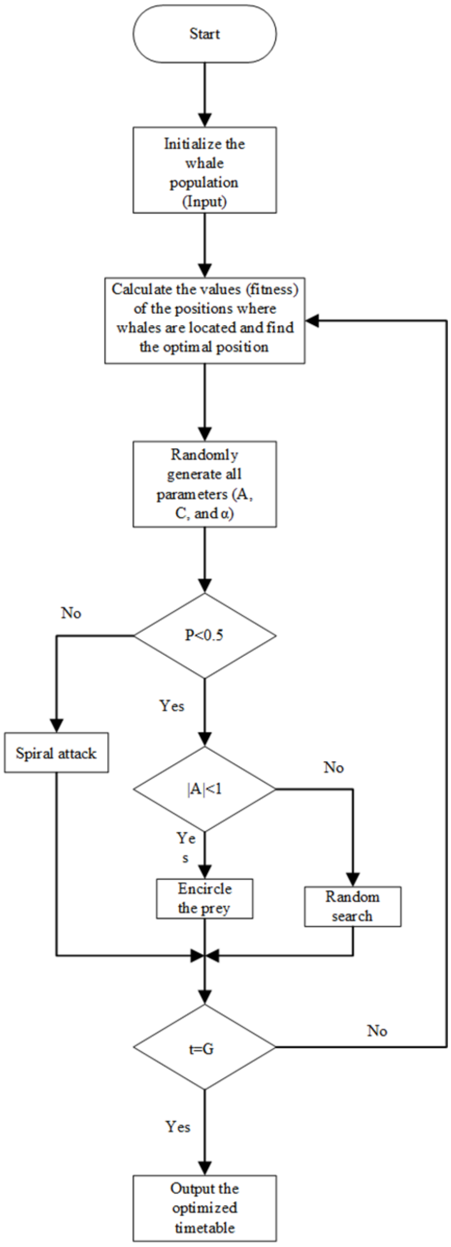

Figure 4 – Flowchart of the multi-objective whale optimisation algorithm

1) Positional vector encoding strategy

Each whale's position vector represents a solution to the problem, and the encoding of the position vector consists of dwell times between stations, running times between stations, and running times between successive train departures on the same line. Other decision variables can be derived using *Equation (19)*.

$$
\begin{aligned}
&A_{sl}^{j} = D_{1l}^{j} + \sum_{k=1}^{s} R_{kl}^{j} + \sum_{k=1}^{s-1} T_{kl}^{j}\ , && \forall l \in L, s \in S^{L} \setminus \{1\}, j \in Q^{l} \\
&D_{sl}^{j} = A_{sl}^{j} + T_{sl}^{j}, && \forall l \in L, s \in S^{L}, j \in Q^{l} \\
&D_{1l}^{j} = D_{1l}^{j} + H_{l}^{j}, && \forall l \in L, s = 1, j \in Q^{l}
\end{aligned}
\tag{19}
$$

The row vector structure of train j on line L is shown in the figure below. The first element represents the headway, followed by alternating dwell times and running times between stations. Since there is no running time at the terminal station, the last element is the dwell time at the terminal station. As this study focuses on scheduling within an end-of-operation period, the number of train services is not fixed, resulting in a variable row vector structure. In this study, a maximum of 30 train services is preset. Row vectors corresponding to trains the arrival times of which at the terminal station exceed the latest operation time constraint are removed. Finally, all remaining row vectors are sequentially concatenated into a single large row vector.

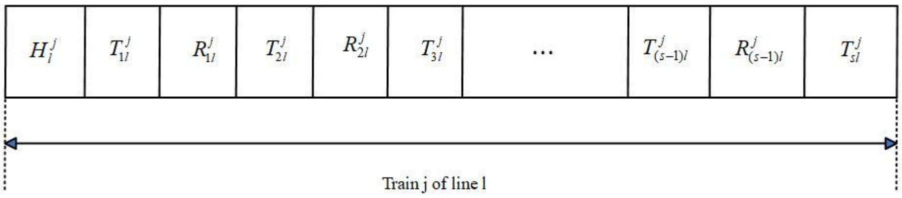

Figure 5 – Row vector schematic diagram

2) Generate the initial population

Set the population size to G and randomly generate G initial solutions. Verify these solutions against the constraints (3-9) to (3-12) in the model. If a solution satisfies all constraints, it is considered feasible; otherwise, regenerate the initial solution until an initial whale population consisting of G feasible solutions is obtained.

3) Iterative evolution

There are three iterative update strategies: the search and foraging mechanism, the shrinking encircling mechanism, and the spiral updating mechanism. In each iteration, the position update strategy for each whale is determined based on the value of the random number p and the magnitude of the coefficient vector $\vec{A}$. As the iterations progress, the whale population gradually converges toward the optimal solution.

4) Algorithm termination criterion

The algorithm's evolution termination conditions consist of two criteria. If either criterion is met, the algorithm stops and outputs the Pareto front.

1) The maximum number of iterations: when the population evolution reaches the maximum number of generations, the algorithm terminates and outputs the current Pareto front.
2) The termination is determined based on both the variation of the Pareto set and the stability of diversity indicators.
   — Pareto front stability: The algorithm determines stability by comparing the changes between the current and previous generations of the Pareto front. Specifically, the Euclidean distance is used to measure the variation between the two fronts. If the maximum distance between them is smaller than the predefined threshold epsilon, the current Pareto front is considered relatively stable.
   — Diversity stability: Diversity stability is evaluated by calculating the diversity of the current Pareto front and comparing it with that of the previous generation. The minimum value is used as a reference point for diversity calculation to assess the uniformity of individual distribution in the objective space.

6. CASE STUDY ANALYSIS

This study takes the Shanghai Metro as the research background and selects metro data from 30 April 2015. The metro network is modelled by considering the up and down directions as two separate lines (as shown in *Figure 6*). A total of 13 metro lines, from line 1 to line 13, are selected. For each OD pair, up to three valid paths are considered, with the number of transfers on each path limited to a maximum of three. Parameter settings are as follows. The end-of-operation period is set from 21:30 to 24:00. The dwell time at each station in the timetable ranges from 0.5 to 1 minute, while the running time between stations ranges from 2 to 3 minutes. The minimum headway and minimum arrival-departure interval at stations are both set to 4 minutes. The maximum headway at the first station of each line is set to 6 minutes. The transfer time is set to 2 minutes, and the train capacity is set to 1,500 passengers. The maximum number of train services during the end-of-operation period is preset to 30. The remaining arrival rate, alighting rate and transfer rate are estimated based on time-segmented statistics of metro data from 30 April 2015. The OD passenger flow proportion heatmap for the end-of-operation period is shown in *Figure 7*.

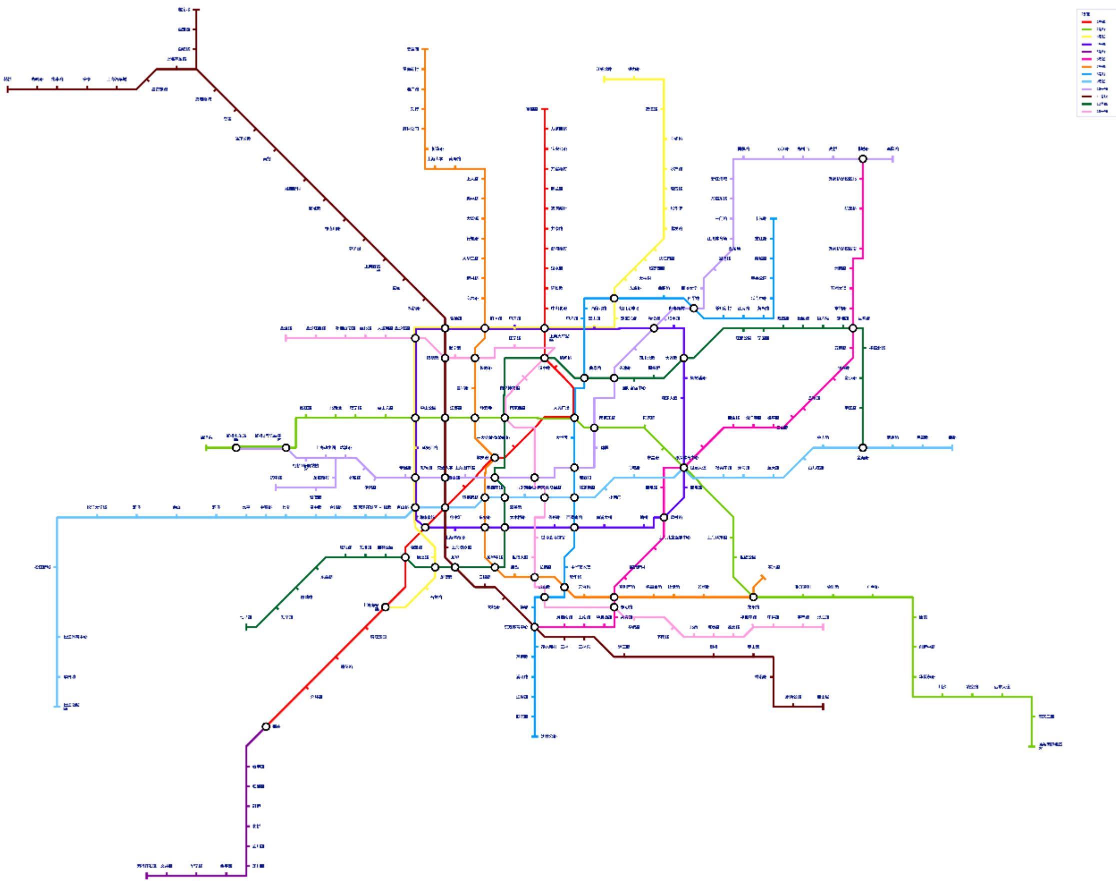

Figure 6 – Partial urban rail transit map of Shanghai in 2015

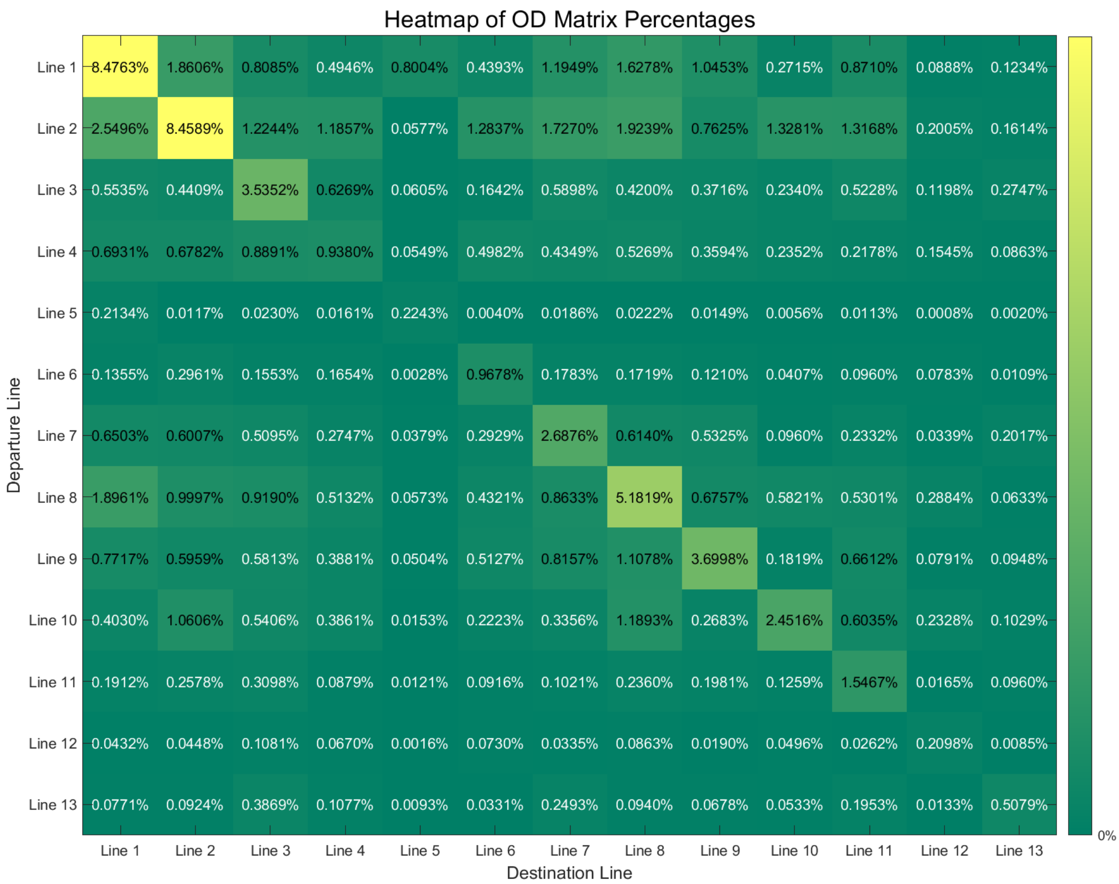

Figure 7 – Heatmap of OD passenger flow ratio during the end-of-operation period

6.1 Results analysis

Due to the heterogeneity of the metro network topology, the total travel time of different lines exhibits significant variation (with the longest line reaching up to 60 minutes). Under the fixed end-of-operation period constraint, dwell time is positively correlated with the number of stations, resulting in a more than 60% reduction in train service generation capacity for high-density station lines (e.g. line 13) compared to low-density station lines (e.g. line 5).

As shown in *Figure 8*, a total of nine Pareto-optimal solutions are obtained upon the termination of the algorithm. Since both objectives in this study are minimisation problems, the red * symbols represent the Pareto-optimal set, while the blue ▲symbols denote non-Pareto-optimal solutions.

Due to the varying weights of passenger flow-induced congestion at stations and the unreachable rate of last train OD points under different large-scale event scenarios, urban rail transit operators can balance these two factors and select the most suitable timetable scheme accordingly.

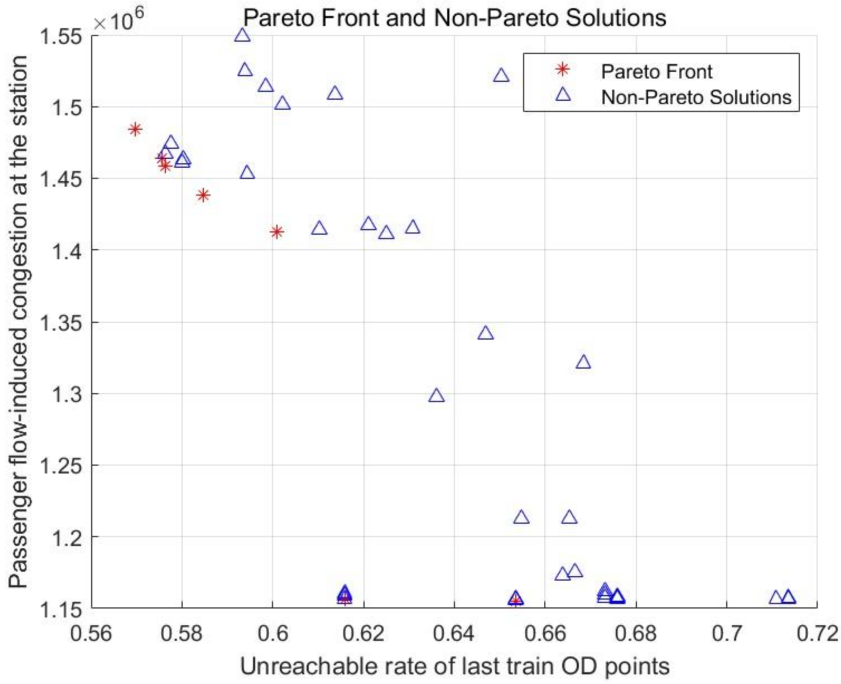

Figure 8 – Pareto and non-Pareto solution sets

6.2 Before-and-after timetable comparison

The original timetable corresponds to the current operational plan implemented by Shanghai Metro, which adopts fixed departure intervals and uniform station dwell times. This static scheduling strategy does not account for the substantial variations in passenger demand during the late-night period. As a result, the timetable often causes a mismatch between supply and demand, leading to a significant proportion of passengers being unable to complete their trips and causing localised overcrowding at certain stations.

In contrast, the optimised timetable generated by the proposed model dynamically adjusts both departure intervals and station dwell times based on predicted passenger flow characteristics during the end-of-operation period. This demand-responsive approach aims to simultaneously enhance trip accessibility for passengers and alleviate station congestion.

Since the proposed model generates a Pareto optimal set representing trade-offs between the two conflicting objectives, it is necessary to select a representative solution for comparative analysis with the real-world timetable. In this study, the compromise solution located near the "knee point" of the Pareto front is selected as the representative optimised timetable, as it reflects a balanced trade-off between minimising the unreachable passenger rate and reducing station congestion. This solution is considered to offer a practical balance between operational feasibility and passenger service quality.

To further assess the proposed model's performance, we compare it with two other widely used optimisation algorithms: NSGA-II and NSWOA. Both algorithms are employed to optimise the same set of objectives (unreachable passenger rate and station congestion) under the same conditions.

The following comparative analysis specifically focuses on two key performance metrics:
1) Unreachable rate of last train OD points – representing the proportion of passengers unable to complete their journeys.
2) Passenger flow-induced congestion at the station – reflecting the cumulative congestion intensity across all stations during the end-of-operation period.

The comparison illustrates the performance improvements achieved by the representative optimised timetable over the existing operational plan, and highlights how the proposed approach outperforms other algorithms in addressing the dynamic nature of passenger demand and station congestion.

Table 3 – Objective function value comparison (before vs after optimisation)

Performance metric	**Unoptimised**	**NSGA-II optimised**	**NSWOA optimised**
Unreachable rate of the last train OD points	65.53%	63.21%	60.08%
Passenger flow-induced congestion at the station	1.52×10^6	1.49×10^6	1.41×10^6

Table 3 presents the comparative results of the two key performance metrics under different timetable schemes, including the current operational plan (unoptimised), the NSGA-II optimised timetable and the NSWOA optimised timetable.

It is observed that the unreachable rate of last train OD points decreases from 65.53% to 60.08% with the proposed NSWOA-based optimisation, representing a 5.45% reduction relative to the unoptimised scenario. Similarly, the passenger flow-induced congestion at the station decreases from 1.52×10^6 to 1.41×10^6, corresponding to an overall reduction of 7.24%.

The unreachable rate of last train OD points decreases from 63.21% to 60.08% with the proposed NSWOA-based optimisation, representing a 4.95% reduction relative to the NSGA-II optimised scenario. Similarly, passenger flow-induced congestion at the station decreases from 1.49×10^6 to 1.41×10^6, corresponding to an overall reduction of 5.37%.

6.3 Sensitivity analysis

To further evaluate the performance of the optimisation model and solution method, a sensitivity analysis is conducted on two key parameters: station dwell time and the end-of-operation period.

Station dwell time

In this study, the dwell time is extended in increments of 0.5 minutes while keeping the total time span unchanged. The dwell time windows are set to 1–1.5 minutes and 1.5–2 minutes, respectively. Two Pareto result diagrams are obtained, as shown in *Figure 9*, and the Pareto-optimal solution sets are plotted as a line chart in *Figure 10*. From the figures, it can be observed that:

Shorter dwell times, such as 0.5–1 minute, can improve train turnover rates. However, they may result in more passengers missing their transfers due to insufficient time to pass through the transfer corridors within the limited transfer window.

Further extending the dwell time, such as to 1–1.5 minutes or 1.5–2 minutes, increases the stopping duration of the last train at transfer stations. This, in turn, enhances the number of successfully connected transfer directions, thereby improving the reachable rate. Additionally, more passengers may have the opportunity to board the train.

However, excessively extending dwell time, such as to 2–2.5 minutes, may prevent trains on certain lines with a large number of stations (e.g. line 7 with 33 stations) from reaching the terminal station within the end-of-operation period. Therefore, congestion reduction cannot rely solely on increasing dwell time.

Since both objectives in this study are minimisation problems, the optimal dwell time is recommended to be within the range of 1.5–2 minutes.

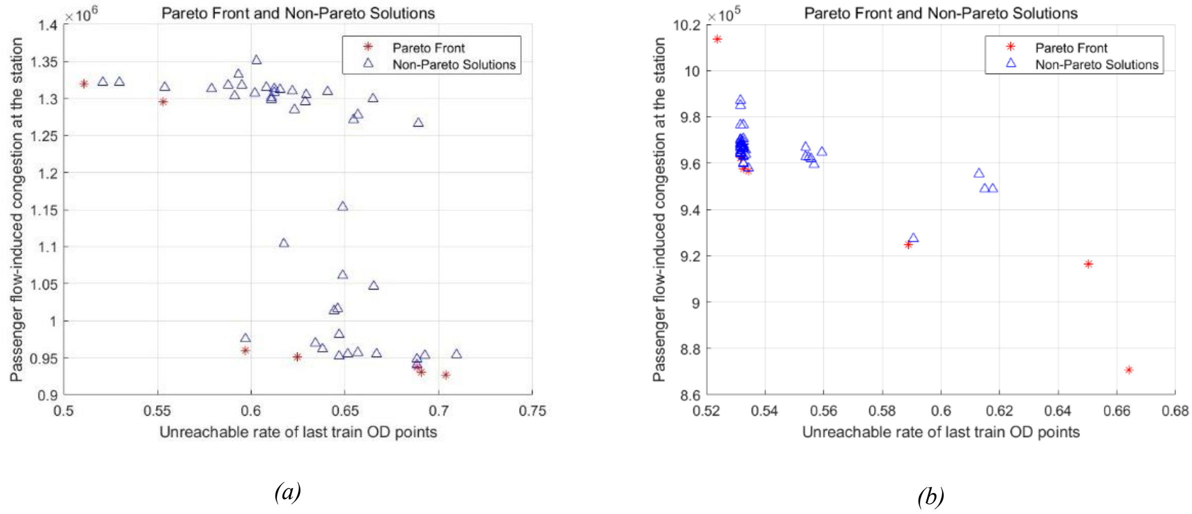

Figure 9 – Pareto optimal solution sets for different dwell times: a) Pareto optimal solution set for dwell time of 1-1.5 minutes; b) Pareto optimal solution set for dwell time of 1.5-2 minutes

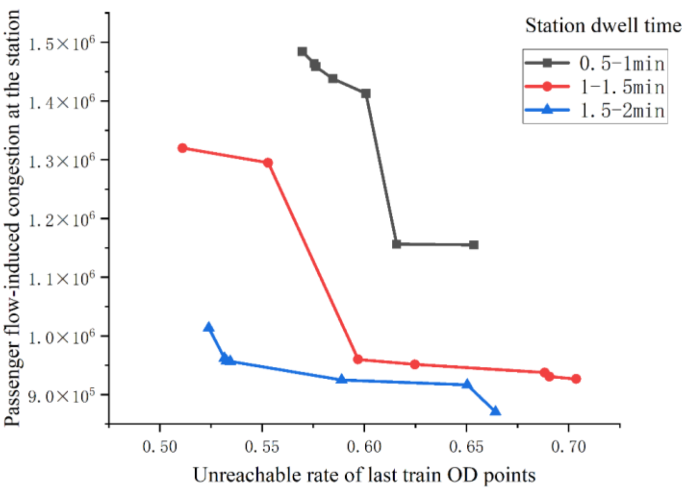

Figure 10 – Sensitivity analysis of dwell time

Headway

Different headways correspond to varying numbers of departures within the same operation period, which in turn affects station congestion levels. A sensitivity analysis is conducted on headway by extending it from the original 4–6 minutes to 6–8 minutes and 8–10 minutes, respectively. Two Pareto-optimal solution sets are obtained, as shown in *Figure 11*. A comparison of the Pareto-optimal solution sets for different headways is presented in *Figure 12*.

As shown in *Figure 11*, there are a total of four Pareto-optimal solutions when the headway is set to 8–10 minutes. However, *Figure 12* reveals that under the same level of passenger flow-induced congestion or the same network reachability, the other objective (network reachability or congestion level) generally outperforms the Pareto-optimal solutions of the other two headway settings. Therefore, the optimal headway is recommended to be 8–10 minutes.

It is important to note that certain Pareto-optimal solutions do not fall within the optimal 8–10 minute headway range, as indicated by the green diamonds in *Figure 10*. For instance, at one optimal solution (with an unreachable rate of 0.57 and passenger flow-induced congestion of 1.35×10^6), a headway of 6–8 minutes is observed. Similarly, at another optimal solution (with an unreachable rate of 0.62 and passenger flow-induced congestion of 1.15×10^6), a headway of 4–6 minutes is more favourable. Therefore, rail transit operators can select the appropriate headway based on operational requirements for different large-scale events.

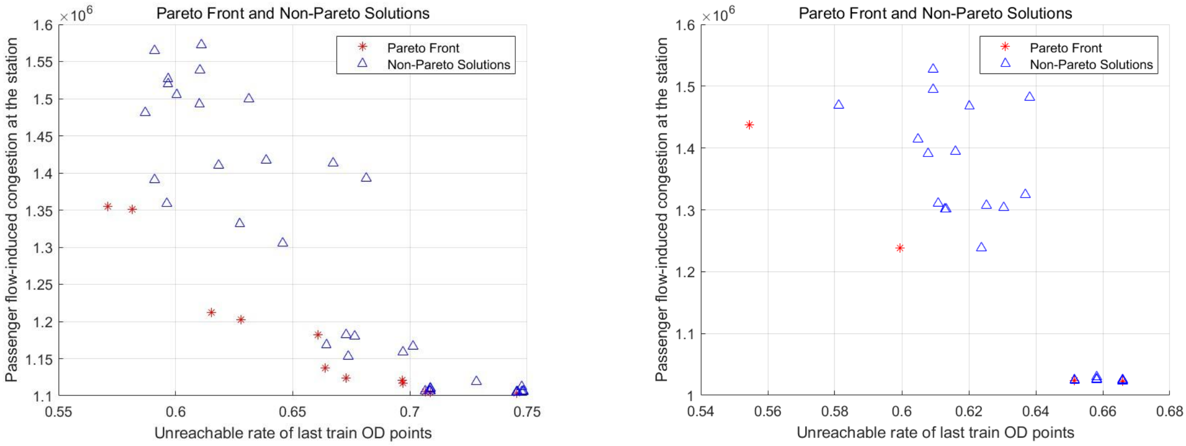

Figure 11 – Pareto optimal solution set for different headways: a) Pareto optimal solution set for headway of 6-8 minutes; b) Pareto optimal solution set for headway of 8-10 minutes

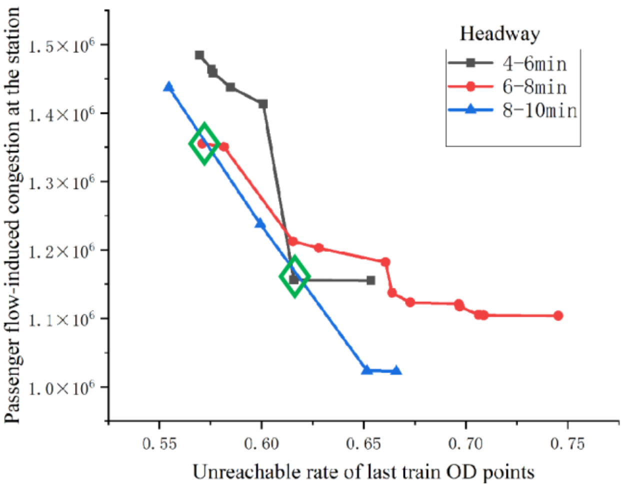

Figure 12 – Headway sensitivity analysis

7. CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

To enhance transfer coordination in the metro network during the end-of-operation period and balance dynamic passenger demand with limited train capacity, this study proposes a mathematical model for optimising the last-train timetable in the metro network. The objective is to improve passenger service quality and station operational safety. Given that the proposed model involves a certain number of non-convex and nonlinear terms, leading to high computational complexity, a multi-objective whale optimisation algorithm is employed to solve the problem and generate a set of Pareto-optimal solutions.

Finally, a sensitivity analysis is conducted on dwell time and headway. The results indicate that an appropriate dwell time yields better optimisation outcomes. However, excessively prolonging the dwell time may prevent certain lines from reaching their terminal stations within the end-of-operation period. The optimal dwell time is recommended to be within the range of 1.5–2 minutes.

For headway, when set to 8–10 minutes, its Pareto-optimal solutions exhibit superior performance in the other objective compared to those of the other two headway settings, given the same level of passenger flow-induced congestion or the same network unreachable rate. However, exceptions still exist. Therefore, rail transit operators should make selections based on the specific characteristics of different large-scale events.

This study enhances the overall service quality and operational safety of the metro network by providing operators with a broader range of parameter adjustment options.

Although the model simplifies certain aspects (e.g. fixed station congestion thresholds and deterministic demand), the conducted sensitivity analysis on key parameters such as dwell time and headway provides strong evidence that the optimisation results are robust in capturing the key trade-offs between operational efficiency and service quality. Additionally, the critical trends identified, such as the relationship between dwell time and terminal reachability, are consistent with general operational principles observed in metro systems. Therefore, despite the limitations, the current results can serve as a valuable reference for practical scheduling decisions, especially in event-based or peak-period operational scenarios.

7.2 Scope of applicability

The present model is mainly applicable to single-mode metro systems during large-scale events and special operational periods (e.g. end-of-service coordination). In these cases, the optimisation framework proposed in this study can effectively support metro operators in making informed timetable adjustments.

It should be noted that the number of train services (30) is not a fixed limitation of the model, but rather a parameter determined by the characteristics of the specific case study. Since metro lines in the case study are relatively long and turnaround times are significant, the number of last-train services required during the analysed period is realistically around this scale. For larger networks or extended operational periods, the model can be flexibly adjusted by changing this parameter to suit specific operational needs.

For broader applications involving multi-modal coordination or long-term planning under uncertainty, further model refinement is necessary. Additionally, further validation through simulation-based methods and integration with stochastic models will be conducted in future studies to enhance the robustness and applicability of the proposed approach.

7.3 Limitations and future work

Some limitations in the current study deserve further exploration in future research:

1) Station size inconsistencies: This study assumes a fixed critical congestion threshold for all stations, without considering potential variations in station size and passenger flow characteristics. Future work could address this by developing more detailed models that incorporate station-specific capacities to better reflect real-world conditions.
2) System stochasticity and uncertainty: Public transportation systems inherently involve random factors, such as variations in vehicle running times, fluctuations in passenger demand and changes in driver behaviour. These factors were simplified or assumed to be constant in the current study. Future research could relax these assumptions and incorporate stochastic elements into the model to enhance its robustness and adaptability to real-world uncertainties.
3) Lack of multi-modal transfer optimisation: This study focused on optimising the transfer coordination within a single mode of transportation – metro systems. However, real-world transportation often involves multiple modes, such as metro-bus, metro-ferry, metro-aeroplane or metro-train transfers. Future research should explore multi-modal transfer coordination to improve the integration of various transportation modes, ultimately achieving seamless, door-to-door travel and improving overall system efficiency.

These future directions can help address the limitations of the current study and further enhance the optimisation of metro networks in large-scale, real-world applications.

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