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CELLULAR AUTOMATA MODEL FOR TRAFFIC FLOW WITH OPTIMISED STOCHASTIC NOISE PARAMETER

ABSTRACT

Based on the existing safe distance cellular automata model, an improved cellular automata model based on realistic human reactions is proposed in this paper, which aims to reproduce the characteristics of congested traffic flow. In the proposed model, the stochastic noise parameter is optimised by considering driving behavioural difference. The relative speed, gap and acceleration of the front vehicle are introduced into the optimised stochastic noise parameter oriented to describing the asymmetric acceleration behaviour of drivers in congestion. The simulation results show that an uneven distribution of acceleration trajectories of vehicles experiencing congestion exhibited on the spatial-temporal diagram of the proposed model is reproduced. Based on the analysis of the NGSIM, compared with the model with traditional stochastic noise parameter, the vehicles that move according to the proposed model can be followed more easily and more realistically. Then the actual gap of vehicles can be better reflected by the proposed model and the change of vehicle speed is more stable. Additionally, the traffic efficiency from two aspects of flow and speed shows that the proposed model can significantly improve the traffic efficiency in the medium high density region.

KEYWORDS

heterogeneous traffic flow; stochastic noise parameter; cellular automata; car-following behaviour.

1. INTRODUCTION

As traffic composition becomes increasingly complex, more and more scholars at home and abroad have paid attention to the research studies on the operation characteristics of heterogeneous traffic flow. In order to solve the problem of insufficient sample size of actual survey data, traffic simulation which can obtain a more complete traffic flow data has become a useful method for heterogeneous traffic flow research. Microscopic traffic simulation, which is one of the important simulation approaches in heterogeneous traffic flow research, can not only mimic the interaction factors and mechanisms among individual vehicles in traffic, but also accurately reflect the differences of driving characteristics of individual vehicles in heterogeneous traffic flow [1–4]. However, the effect of microscopic traffic simulation mainly depends on its accuracy [5–9]. As trucks and cars have obvious differences in vehicle performance and behaviour characteristics in heterogeneous traffic flow when compared with homogeneous traffic flow, higher accuracy of microscopic model is required in heterogeneous traffic flow. Therefore, the method of building a microscopic model that accurately reflects vehicle driving characteristics is a fundamental problem of simulation in heterogeneous traffic flow.

Cellular automata (CA) model is a commonly used microscopic model, which constructs the system discretely with time and space. The model has the advantages of simple form, high precision and flexible parameter setting, which is suitable for computer simulation. As a consequence, it has become an important model in microscopic traffic simulation [10–25]. In 1992, the most well-known CA model, i.e. the Nagel-Schreckenberg (NaSch) model was proposed by Nagel and Schreckenberg [11], which was able to present complex traffic phenomena through four simple rules including the go-and-stop traffic flow, traffic congestion (including formation, propagation and dissipation), etc. Nevertheless, some phenomena of traffic flow cannot be reflected by the NaSch model (e.g. metastability, hysteresis effect, the three-phase traffic flow theory, etc.). Based on the NaSch model, a variety of CA models which were suitable for different scenarios have been proposed, such as the velocity-dependent-randomisation (VDR) model considering slow start rule, Kemer-Klenov-Wolf (KKW) model, comfortable driving (CD) model and modified comfortable driving (MCD) model with brake light effect [12–16]. However, a common disadvantage of these models is that the acceleration and deceleration capacity of vehicles are not limited, which makes the acceleration and deceleration behaviours of the simulated vehicles inconsistent with those of the actual vehicles in heterogeneous traffic flow.

Based on the above problems, safety distance (SD) models that consider the restriction of deceleration capacity were proposed, in which the safety distance required by the rear vehicle was calculated based on the assumption that the front vehicle decelerates at the maximum deceleration [17]. Therefore, the dynamic characteristics of car-following vehicles can be well reflected in SD models, which can be widely used to simulate traffic flow [1, 18–25]. Based on the psychology of acceleration, stochastic noise parameters are often introduced into most existing SD models, which denote the probability to accelerate. An SD model in which limited acceleration and deceleration capabilities derived from safe driving principles are considered was proposed by Larraga et al. [22, 23].

In this model, rear-end collision under various conditions can be avoided while limiting the deceleration capacity of vehicles. The stochastic noise parameter is modelled as a linear function based on the velocity of the vehicles in this model, which means that the low-speed vehicles have to wait longer before continuing their journey. By considering the characteristics of heterogeneous traffic flow, Li et al. [24] proposed a new SD model, where the computing method for the safety distances has been appropriately extended so as to avoid the collision between cars and trucks. The vehicle impulsive accelerated motion in position update rule was further modified by Guzman et al. [25] to uniform accelerated motion. This proposed model overcomes the limitation that low-speed or stationary vehicles in the previous SD models cannot smoothly approach other vehicles. However, the linear stochastic noise parameter based on the speed of the vehicles is still retained in the above proposed SD models, which results in a significant problem that only speed is considered, whereas the influence of other factors on the driver's acceleration behaviour is ignored. This setting makes it that the low-speed vehicles in congestion are prone to abnormal acceleration behaviour. Therefore, this will result in an inconsistence of vehicle gap between the simulated traffic flow and the actual situation (shown in Section 3.1). In addition, car-following velocity and car-following stability in the platoon as well as the fundamental diagram are also adversely affected. In short, it can be seen that the existing SD models are still insufficient in describing the following behaviour of low-speed or stationary vehicles. The reason is that the existing stochastic noise parameter is not the most suitable for traffic flow and needs to be optimised so as to improve the accuracy of SD models.

This paper aims to propose an improved safety acceleration (I-SA) model with optimised stochastic noise parameter. Firstly, the acceleration factor is defined by analysing the running state of the front vehicle in the model. Besides, by introducing the acceleration factor into the stochastic noise parameter, the single decision variable of velocity in this parameter is extended to multiple decision variables of acceleration, maximum velocity, relative speed and gap. Based on the model, when the front vehicle is running fast and the distance between the target vehicle and the front vehicle is gradually increasing, the target vehicle will increase the acceleration probability so as to ensure that the gap and the difference of velocity remain within a reasonable range. The main contributions of this new model are as follows: (1) an improved cellular automata model based on realistic human reactions is proposed in this paper. In the proposed I-SA model, the vehicle can learn traffic information from environment and adjust acceleration probability in real time. Compared with CTCA model without the stochastic noise parameter, the I-SA model can effectively show the asymmetric acceleration behaviour of different drivers; (2) compared with SD model using linear stochastic noise parameters, the width of congestion can be reduced and the efficiency of congestion elimination is improved in the proposed I-SA model. Under the same density and truck ratio, the I-SA model can effectively improve traffic flow and speed; (3) and the platoon move according to the I-SA model can return to a stable state after experiencing congestion waves, making the improved road traffic system more stable.

The remainder of this paper is structured as follows: the I-SA model is introduced in detail in Section 2. The comparisons on the simulation results of the I-SA model, CTCA model and SD model are conducted from both the microscopic and macroscopic levels in Section 3. Finally, the conclusions from the above sections are made in Section 4.

2. I-SA MODEL

Consistent with the SD model, the vehicle update rules in the I-SA model are divided into four steps, namely safety distance, stochastic noise parameter, speed update and position update from Step 1 to Step 4, respectively. Compared with the SD model, Step 1, Step 3 and Step 4 in the I-SA model do not change, and the detailed explanation and formula derivation of these steps can be found in Refs. [23] and [24]. However, the stochastic noise parameter in Step 2 in the I-SA model has been optimised, with multiple decision variables constructed based on the vehicle dynamic information perception.

Step 1: Safety distance. The criterion of car-following behaviour selection in the model is whether the target vehicle will collide with the front vehicle when the target vehicle and front vehicle decelerate at the maximum deceleration. Based on this criterion, the minimum safety distances for the target vehicle to accelerate, slow down or maintain its velocity are calculated respectively as the criteria for subsequent vehicle speed updating.

$$d_n^{acc}(t) = \max\{0, d_n[v_n(t) + a_n^{acc}] - d_{n+1}[v_{n+1}(t) - a_{n+1}^{max}]\} \quad (1)$$

$$d_n^{keep}(t) = \max\{0, d_n[v_n(t)] - d_{n+1}[v_{n+1}(t) - a_{n+1}^{max}]\} \quad (2)$$

$$d_n^{dec}(t) = \max\{0, d_n[v_n(t) - a_n^{dec}] - d_{n+1}[v_{n+1}(t) - a_{n+1}^{max}]\} \quad (3)$$

where $d_n^{acc}(t)$, $d_n^{keep}(t)$, $d_n^{dec}(t)$ denote the safety distances required by the target vehicle to accelerate, maintain and decrease its velocity at the current time step; $v_n(t)$ means the velocity of the target vehicle at the current time step; a_n^{acc}, a_n^{dec}, a_{n+1}^{max} refer to the acceleration of the target vehicle, the deceleration of the target vehicle and the maximum deceleration of the front vehicle respectively. $d_n[v_n(t)]$ means the distance of the target vehicle when it drives with $v_n(t)$ for one time step and decelerates to the moment when the gap between the target vehicle and the front vehicle is minimum.

Step 2: Stochastic noise parameter. In the SD model, the stochastic noise parameter is a single variable function of speed [22–25]. However, the stochastic noise parameter is optimised in the I-SA model, which reflects the driver's consideration of other factors when accelerating. The optimised stochastic noise parameter is shown in *Equations 4 and 5*.

$$p_n^{acc}(t) = \begin{cases} p_d & a_{n+1}(t) \geq 0 \\ p_c & a_{n+1}(t) < 0, v_{n+1}(t) - v_n(t) \leq 0 \\ p_c + s(t)\cdot(p_d - p_c) & a_{n+1}(t) < 0, v_{n+1}(t) - v_n(t) > 0 \end{cases} \quad (4)$$

$$s(t) = \exp\left[a_{n+1}(t)\cdot\frac{v_n^{\max}}{v_{n+1}(t) - v_n(t)}\cdot\frac{d_n^{acc}(t)}{d_n(t)}\right] \quad (5)$$

where $p_n^{acc}(t)$ means the acceleration probability of the target vehicle at the current time step; p_d and p_c respectively denote the maximum and minimum of acceleration probability; $s(t)$ refers to the acceleration factor; $a_{n+1}(t)$ represents the acceleration of the front vehicle at the current time step; v_n^{max} indicates the maximum velocity of the target vehicle; $d_n(t)$ shows the gap between the target vehicle and the front vehicle at the current time step.

As shown in *Equation 4*, when the acceleration of the front vehicle is greater than or equal to zero (the front vehicle is accelerating or maintaining its velocity), the driver usually considers that the driving condition is getting better. Under the circumstances, the driver's acceleration expectation is the highest in order to keep following closely. Therefore, the acceleration probability reaches the maximum in the model. There are two cases when the acceleration of the front vehicle is less than zero (the front vehicle is decelerating). In the first case, when the velocity of the target vehicle is greater than or equal to that of the front vehicle (the two vehicles are in the approaching state), the driver usually considers that the driving condition is getting worse. At that moment, the acceleration pressure of the driver increases gradually, causing that the driver's acceleration expectation is minimised in order to maintain the desired safe gap. Therefore, the acceleration probability reaches the minimum. In the second case, as the velocity of the target vehicle is less than that of the front vehicle, the two vehicles are gradually away from each other even if the front vehicle is decelerating. Thus, it can be seen that the acceleration probability varies with the above variables, rather than being a fixed value. *Equation 5* is used to describe the acceleration probability in this case. As shown in *Equation 5*, the acceleration factor is constructed to describe the acceleration selection decisions of the driver. In the acceleration factor, the acceleration of the front vehicle and gap will exert different effects on the acceleration psychology of the driver.

The braking state of the front vehicle will cause the driver to accelerate cautiously, while the increasing gap will encourage the driver to accelerate. In addition, since the driver can judge the change of speed difference by relative motion of vehicles, the driver's acceleration decisions are also affected by the speed difference. Similarly to the impact of gap on drivers' acceleration expectation, drivers' acceleration expectation augments with the increase of the speed difference. Unlike the existing stochastic noise parameters, *Equation 5* adopts the form of exponential function. The reason is that the slope of the exponential function increases gradually, i.e. as the value of the independent variable increases, the influence on the dependent variable increases. Drivers are more sensitive to changes in large values than small ones. The basic characteristics of exponential function are more in line with the driver's psychology. Therefore, compared with linear function, exponential function is more suitable for *Equation 5*.

Step 3: Speed update. The speed is updated by comparing the gap of actual vehicles with the three safety distances calculated in Step 1 (*Equation 6*). where p_{rand} is the randomisation probability.

Step 4: Position update. The position at the next time step of the vehicle refers to the sum of the position at the current time step and the updated speed.

$$x_n(t+1) = x_n(t) + v_n(t+1) \tag{7}$$

where $x_n(t+1)$ indicates the position of the target vehicle at the next time step.

3. NUMERICAL SIMULATIONS AND RESULTS ANALYSIS

The simulation scenario is a basic single lane section of motorway with the length of 10,000 cells (5 km), in which the periodic boundary conditions are used. The simulation step is 1 s and the total simulation time is 3,600 s. The data of the first 3,100 s are excluded, with only the last 500 s being collected. In Section 3.4, the final results come from the average of 5 samples. The definitions and values of parameters are shown in *Table 1*, which are consistent with those in Ref. [20].

3.1 Analysis of gap

The spatial-temporal diagram analysis is an important method in traffic microscopic simulation. The coordinates of spatial-temporal diagram include

$$v_n(t+1) = \begin{cases} min\{v_n(t) + a_n^{acc}, v_n^{max}\} & d_n(t) \geq d_n^{acc}(t) \& rand() \leq p_n^{acc} \\ v_n(t) & d_n^{keep}(t) \leq d_n(t) < d_n^{acc} \| v_n(t) = v_n^{max} \\ max\{v_n(t) - a_n^{dec}, 0\} & d_n^{dec}(t) \leq d_n(t) < d_n^{keep}(t) \\ max\{v_n(t) - a_n^{max}, 0\} & v_n(t) > 0 \& d_n(t) < d_n^{dec}(t) \end{cases}$$
$$\text{if } d_n^{keep}(t) \leq d_n(t) < d_n^{acc}(t) \& (rand() \leq p_{rand}), v_n(t+1) = max\{v_n(t+1) - a_n^{dec}, 0\} \tag{6}$$

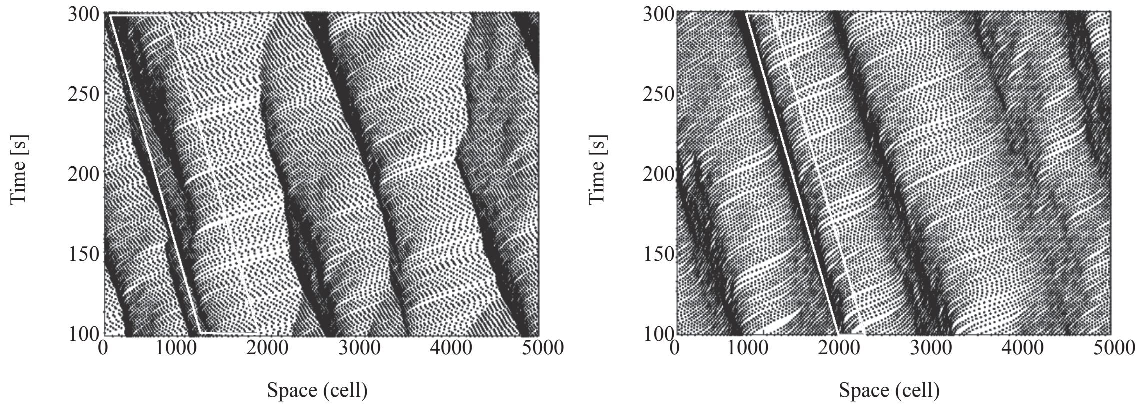
Table 1 – The definitions and values of parameters and variables

Parameter	Definition	Value in specific units	Value in SI units
l_{car}	The length of cars	10 cells	5 m
l_{truck}	The length of trucks	30 cells	15 m
v_{car}^{max}	The maximum velocity of cars	75 cells/s	135 km/h
v_{truck}^{max}	The maximum velocity of trucks	45 cells/s	81 km/h
a_{car}^{max}	The maximum deceleration rate of cars	7 cells/s^2	3.5 m/s
a_{truck}^{max}	The maximum deceleration rate of trucks	5 cells/s^2	2 m/s
a_{car}^{acc}	The acceleration rate of cars	6 cells/s^2	3 m/s
a_{truck}^{acc}	The acceleration rate of trucks	4 cells/s^2	2 m/s
a_{car}^{dec}	The deceleration rate of cars	5 cells/s^2	2.5 m/s
a_{truck}^{dec}	The deceleration rate of trucks	4 cells/s^2	2 m/s
p_{rand}	The randomization probability	0.3	—
p_c	The minimum acceleration probability	0.8	—
p_d	The maximum acceleration probability	1	—
r	The truck ratio	—	—

time and the position of a vehicle, by which the positions of the vehicle at each time step can be described and the change of the gap can be observed more intuitively. It is obvious that the gap is an important microscopic index reflecting the car-following characteristics. Therefore, a spatial-temporal diagram analysis is adopted in this paper so as to study the impacts of the stochastic noise parameter on the car-following characteristics of vehicles.

The benefit of the SD model with the stochastic noise parameter over other CA models is that the SD model can reflect the heterogeneous acceleration behaviour of drivers. In order to verify that the optimisation of the stochastic noise parameter in this paper does not change above basic characteristics *Figure 1* presents the spatial-temporal diagrams with the space range of 0 to 5,000 cells of the I-SA model and the CA based car–truck traffic flow (CTCA) model. The CTCA model is an SD model without considering stochastic noise parameters and other rules that are consistent with I-SA model. In *Figure 1*, the horizontal and vertical coordinates refer to space and time respectively, and each scattered point represents the position of vehicle at each time step. In the initial state, it can be assumed that cars are on the road with uniform distribution, where both the speed and acceleration are zero. By observing the framed part in *Figure 1a*, the trajectory curve distribution of vehicles in the downstream section of congestion wave is extremely close and uniform, which indicates that the gap between the front vehicle and rear vehicle in each car-following combination is almost the same. There is a reasoning to the phenomena that each vehicle follows the same acceleration rules when leaving the congested section, and the operation of the vehicle is not disturbed by human reactions. Nevertheless, combined with the previous study (see Ref. [22]), it is evident that the driver's behaviours are affected by traffic conditions (relative velocities, acceleration and the gaps between vehicles), which interferes with the stability of the trajectory. The human reactions to the surrounding traffic conditions are considered in the I-SA model, reflected by the stochastic noise parameter, which makes each vehicle have a specific acceleration rule. As shown in *Figure 1b*, compared with the CTCA model, the gaps of curves are not completely consistent. Within the time and space shown in *Figure 1b*, according to the statistics of the gap between the rear vehicle and the front vehicle within 30 s after the rear vehicle accelerates to leave the congested section, it is found that the maximum gap is 12 m, while the minimum distance is only 2 m. Hence, the I-SA model preserves its basic features, even when the stochastic noise parameter calculus has changed to use exponential function.

To explore the benefits of the I-SA model over the SD model at the micro level, the spatial-temporal diagrams of the SD model and the I-SA model are compared in *Figure 2*. In *Figure 2*, the two columns are the spatial-temporal diagrams of the SD model and the I-SA model, respectively. Each row has the same density, namely 15 veh/km, 40 veh/km and 60 veh/km, respectively. When the density reaches 15 veh/km, the traffic is in free flow. The vehicles in free flow are evenly distributed and the slopes of the vehicle trajectories are constant. By comparing *Figures 2a and 2b*, it can be found that there is no difference between the spatial-temporal diagrams obtained from the SD model and the I-SA model. The reason for this phenomenon is that the number of vehicles is low in this transportation system and large



a) CTCA model

b) I-SA model

Figure 1 – Spatial-temporal diagram of the CTCA model and the I-SA model with density of 40 veh/km

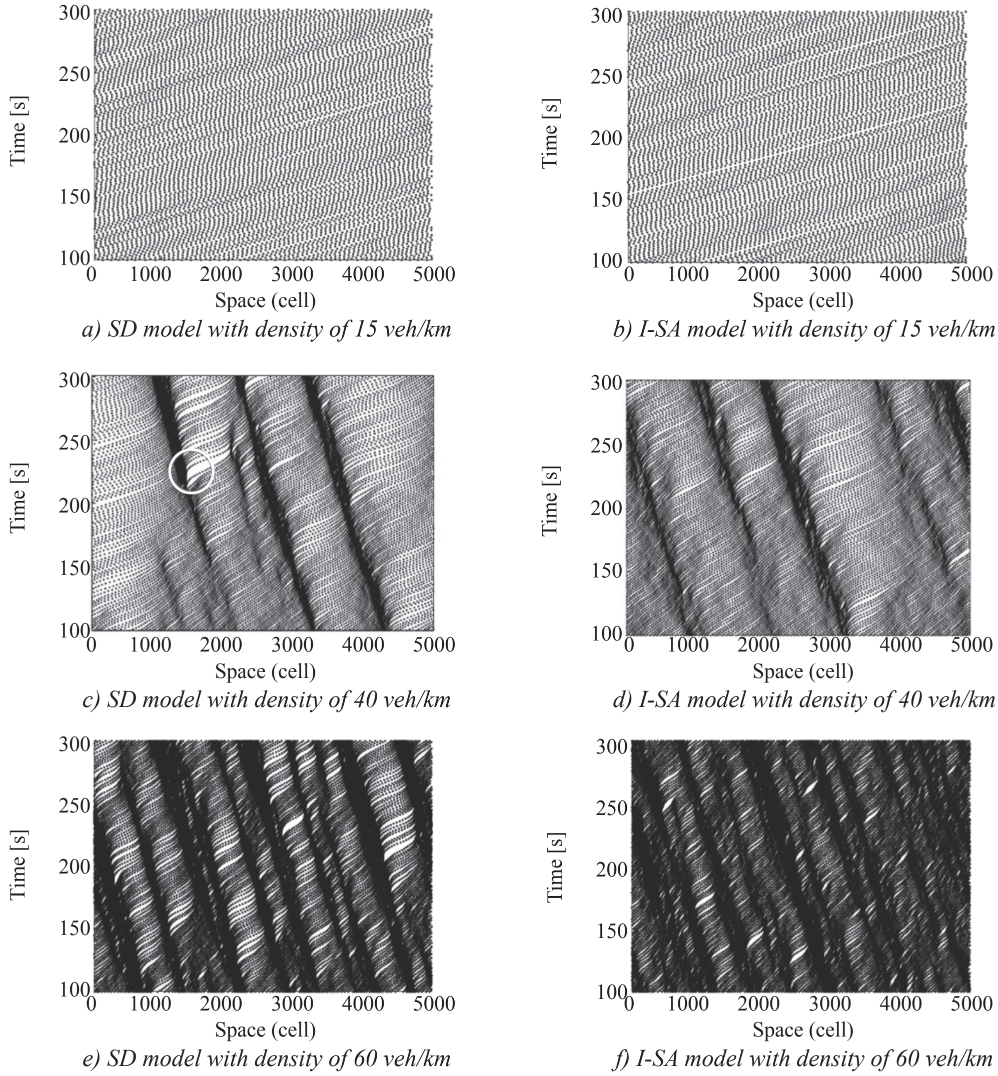


Figure 2 – Spatial-temporal diagram of the SD model and the I-SA model

gaps are maintained between vehicles. Besides, each vehicle runs at free flow speed without deceleration, which makes the acceleration probability of the two models consistent in most time steps.

As shown in *Figures 2c and 2d*, the traffic is in synchronised flow. There is a phenomenon that the slopes of vehicle trajectories become larger, rather than fixed values. The speed of vehicles in synchronised flow decreases and stationary vehicles begin to appear in the traffic flow. There is a significant difference in the gap between the stationary vehicle and the front vehicle in both the SD model and the I-SA model. It can be seen that there is an excessive gap in the traffic flow in the SD model. The maximum gap between the vehicle with zero speed and the front vehicle reaches 400 cells (as indicated by the circle), indicating that the stationary vehicle starts to accelerate when the gap reaches 200 m. In the SD model, this phenomenon occurs 26 times in the whole simulation process. In addition, the phenomenon where the stationary vehicle begins to accelerate when the gap reaches 100 m to 150 m occurs 62 times. However, in the I-SA model, there is no excessive gap between the stationary vehicle and the front vehicle. As shown in *Figure 2d*, the maximum gap between the stationary vehicle and the front vehicle is 25 m. NGSIM is a research program initiated by the Federal Highway Administration (FHWA) in 2004, in which vehicle trajectory data of two groups of motorways [17] were collected including vehicle position, headway, vehicle type, vehicle length, etc. and from which it is easy to obtain the following gap of stationary vehicles. In order to compare the real following gap with that in simulations, the gaps of car-following-car combinations without lane-changing behaviour in the NGSIM are used for statistical analysis. The statistical gaps are from the five innermost lanes of Section I-80 whose acquisition period ranges from 5:00 p.m. to 5:15 p.m. Based on the analysis, the maximum gap between the stationary vehicle and the front vehicle is only 17 m when the density reaches 40 veh/km. It can be found that compared with the I-SA model, the maximum gap between the stationary vehicle and the front vehicle in the SD model is seriously inconsistent with the statistical analysis results of the NGSIM data. In addition, the propagation of congestion waves is promoted by the stationary vehicle and low-speed vehicle in the traffic flow. In *Figure 2c*, the stationary vehicle marked in the circle remains stationary under the action of the existing stochastic noise parameters and fails to leave the congested area, which aggravates the severity and scope of congestion. Therefore, the congestion wave expands in the process of propagating upstream. However, as the stationary vehicle can follow the front vehicle in a timely manner to leave the congested sections in the I-SA model, the shape of the congestion wave is more stable in propagation. In *Figure 2e*, when the density reaches 60 veh/km, the maximum gap between the stationary vehicle and the front vehicle decreases. The main reason for this result is that the number of vehicles in the traffic flow gradually increases and the available road blank space is compressed. By comparing *Figures 2e and 2f*, it can be seen that the maximum gap in the spatial-temporal diagram in the I-SA model is still smaller than that in the SD model, and the vehicles in the I-SA model follow more closely, which makes the distribution of congestion waves in the spatial-temporal diagram in the I-SA model more uniform. The above phenomenon shows that the optimised stochastic noise parameters can make the road space utilisation more balanced under high density.

3.2 Analysis of speed change

Based on the spatial-temporal diagram, the accuracy of the I-SA model reflecting the real vehicle gap is proved. To further understand the influences of the optimised stochastic noise parameter on the acceleration process of vehicles from a microscopic perspective, a car-following experiment will be designed in this section in order to summarise and analyse the time sequence of velocity.

Car-following experiment design

In the initial state, it is assumed that a platoon of 5 cars is on the road. The vehicle names from the leading to the trailing vehicle are Vehicle 1 to Vehicle 5, respectively. At the same time, the speed and gap of the vehicles are 50 cells/s (90 km/h) and 30 cells (15 m), respectively. In the simulation, the speed variation process of Vehicle 1 is designed as follows: Vehicle 1 drives at a constant speed of 50 cells/s (90 km/h) in the range of 0 to 60 s, then gradually decreases to 0 during the 61 s to 70 s. It stays stationary in the range of 71 s to 80 s, and gradually returns to 50 cells/s (90 km/h) during the 81 s to 88 s, and maintains 50 cells/s (90 km/h) after 89 s. Other vehicles in the platoon update the speeds and positions based on the SD model or the I-SA model.

Comparative analysis of car-following speed

The time sequences of speed in the SD model and the I-SA model are shown in *Figures 3a and 3b*, respectively. By comparing the vehicle speed changes in the SD model and the I-SA model, it can be found that the speed difference between two adjacent vehicles in the I-SA model is significantly different from that in the SD model. The first difference occurs during the 81 s to 92 s. In the SD model, the speed

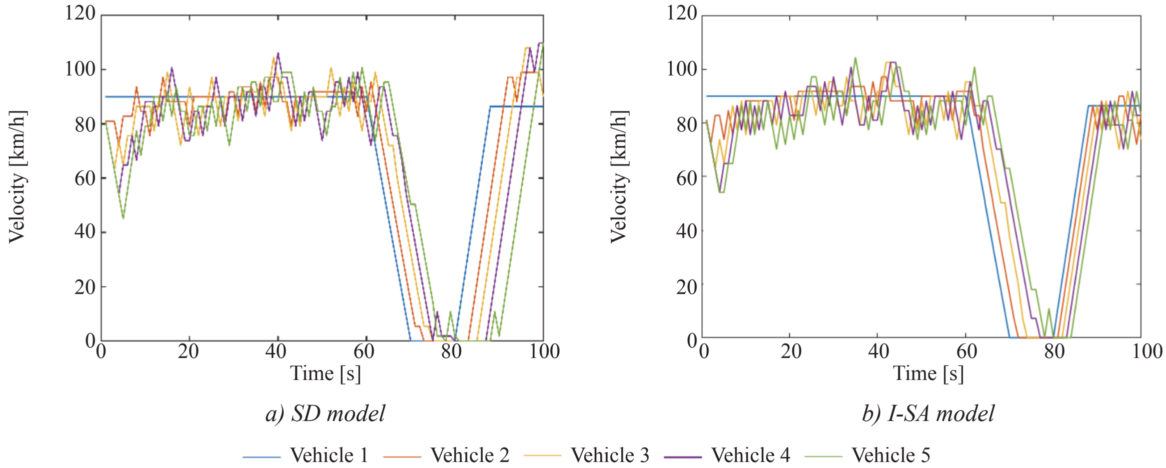


Figure 3 – Velocity time series

of Vehicle 1 increases from 0 at 80 s to 32.4 km/h at 83 s. However, Vehicle 2 is still stationary, resulting in a maximum speed difference between Vehicle 1 and Vehicle 2 of 32.4 km/h. The acceleration behaviour of Vehicle 2 causes the speed dispersion of vehicles of the platoon to be extremely strong during acceleration, which is not existent in the I-SA model. As shown in *Figure 3b*, the time sequence curves of speed in the I-SA model are relatively dense. During the 81 s to 92 s, a small speed difference between the stationary vehicle and the front vehicle can be maintained in the I-SA model, which is 10 km/h.

The basic reason behind the above phenomenon is that the acceleration probability performed by vehicles in SD model is simple, and the driving state of the front vehicle is not considered when the vehicle is accelerating, while velocity becomes the unique factor in the acceleration behaviour. When the vehicle is stationary and the front vehicle is accelerating, the vehicle accelerates with a low probability, which causes the vehicle to remain stationary for 4 s to 5 s. However, in the I-SA model, the calculation of acceleration probability is updated, and the acceleration, maximum velocity, relative speed and gap are considered, whereby an accurate judgment of the state of the front vehicle can be quickly made by the vehicle. When the front vehicle is accelerating, the vehicle can accelerate to follow closely even if it is in a low speed. Therefore, in the I-SA model, the time sequence curves of speed are closely arranged during the acceleration.

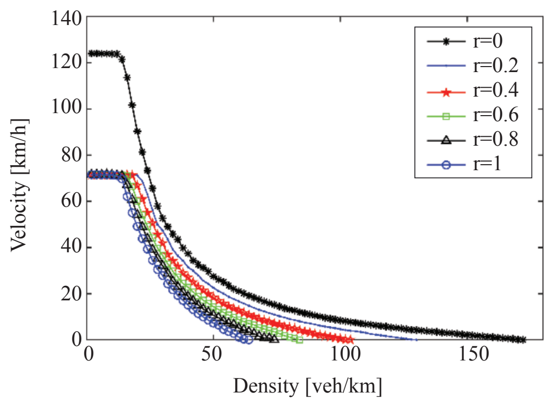
The second difference can be found in the period after 92 s. In the SD model, it can be found that when Vehicle 1 maintains 50 cells/s (90 km/h), the speed of Vehicle 2 to Vehicle 5 is much higher than that of Vehicle 1, particularly the speed of Vehicle 4 reaches 110 km/h at 99 s. However, in the time after 89 s, the maximum speed of the vehicles in the I-SA model reaches only 92 km/h. This difference indicates that the platoon in the I-SA model can quickly return to a stable state after experiencing congestion waves. The main reason for this result is that the excessive speed difference between Vehicle 3 and Vehicle 2 increases the gap between them. At that moment, the safe acceleration distance is fully met. Vehicle 3 needs to accelerate continuously to reduce the gap from the front vehicle. However, in the I-SA model, a vehicle can follow the front vehicle more closely when accelerating. Hence, the speed discreteness of the platoon in the I-SA model is weaker than that in the SD model.

3.3 Comparative analysis of fundamental diagrams

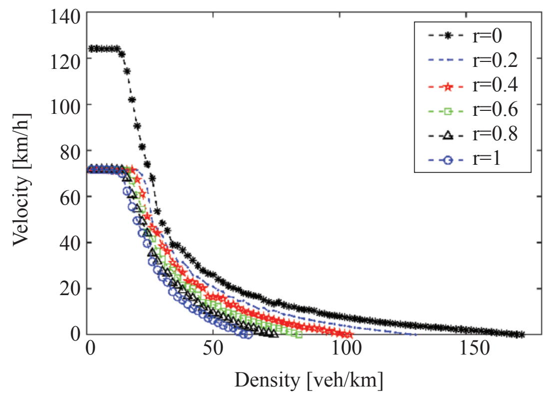
Based on the above analysis, the car-following behaviours of vehicles in homogeneous traffic flow can be optimised by the I-SA model. In order to understand the effects of changes in microscopic car-following behaviours in heterogeneous traffic flow at a macroscopic perspective, the heterogeneous traffic flow including cars and trucks is simulated in this section. Speed-density fundamental diagram and flow-density fundamental diagram under different truck ratios are represented in *Figures 4a and 4b* respectively. As shown in *Figure 4*, each traffic flow curve can be divided into three phases, namely free flow, synchronised flow and jam [26-28].

When the density is less than 14 veh/km, the traffic flow is in free flow. Consistent with the speed-density diagram, the free flow branch of the flow-density diagram under different truck ratios in the SD model and the I-SA model are overlapped, which means that the optimised stochastic noise parameters have no effect on the flow and velocity in free flow conditions. Obviously, the reason for this phenomenon has been explained in the analysis of the spatial-temporal diagram in Section 3.1, i.e. when the vehicle is running at a high speed, the gap is much larger than the acceleration safety distance, which leads to the similar acceleration probability in both the SD model and the I-SA model. Therefore, the significant difference in flow and speed between the SD model and the I-SA model does not exist under the same conditions.

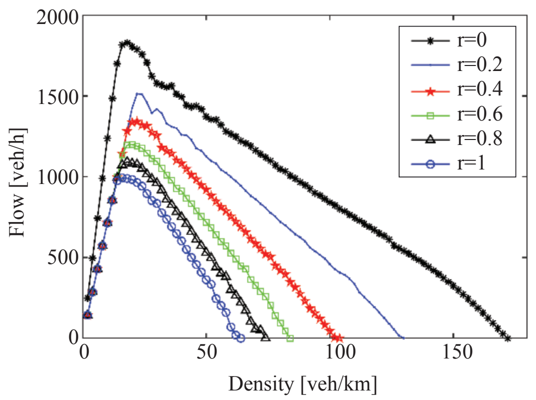
With densities range from 20 veh/km to 30 veh/km, the traffic is in synchronised flow. A sudden drop in flow near the critical density can be observed in *Figure 4b*. By comparing the SD model with the I-SA model, it can be seen that the sudden drop phenomenon of the SD model is always more obvious than that of the I-SA model regardless of the truck ratio. The reason for this phenomenon is that the stability of the traffic system with density near the critical density is quite weak, the traffic system is vulnerable to external disturbance and breakdown and its flow is difficult to maintain for a long time [3, 28-30]. However, the optimised stochastic noise parameter changes the acceleration characteristics of stationary and low-speed vehicles, which reduces the speed and gap dispersion of vehicles and improves the stability of the transportation system.



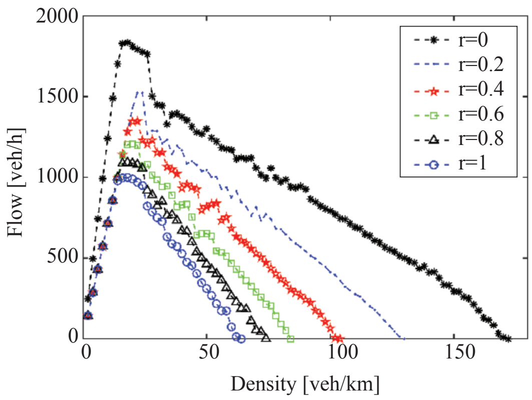
a) The velocity-density diagram of I-SA model



b) The velocity-density diagram of SD model



c) The flow-density diagram of I-SA model



d) The flow-density diagram of SD model

Figure 4 – The fundamental diagrams

When the density is greater than 24 veh/km, the congestion waves begin to appear frequently, resulting in a sharp decline in the speed of traffic flow and an increase in the number of low-speed and stationary vehicles in traffic flow. Hence, the flow and velocity in the SD model are significantly lower than those in the I-SA model at the same density and truck ratio. However, it can be seen in *Figure 4* that the branch in congestion of the fundamental diagrams under each truck percentage obtained from the SD model and the I-SA model overlap, which means that the optimised stochastic noise parameter exerts no influence on the flow and speed in crowded conditions. The main reason for this result is that the congestion is heavy, causing the possibility of meeting the acceleration conditions by the gap between vehicles in rare cases. Therefore, in most cases, the gap is less than the acceleration safety distance, and the vehicle is in decelerating or stationary state. The difference of the acceleration probability between the I-SA model and the SD model cannot have a significant impact on the flow and speed.

Statistical significance test is an effective method used to detect whether there is a difference between the experimental group and the control group and whether the difference is significant in experiments. To further quantify the improvement of the I-SA model on the flow and speed under the medium high density, the variance test of the statistical significance test is used to investigate the difference of speed and flow obtained by the two models under different densities. Because the variance test requires the sample to meet the normal distribution and the variance is homogeneous, before the variance test, the normality hypothesis and variance homogeneity hypothesis of speed and flow are detected based on the lillietest normal test function and vartestn square difference homogeneity test function of the MATLAB. The verification results of the normality hypothesis and variance homogeneity hypothesis are shown in *Table 2* and *Table 3* respectively.

In the lillietest normal test function, if h=0, it can be considered that the data obey the normal distribution, and if h=1, it can be considered that they do not obey the normal distribution. It can be seen

Table 2 – The normality hypothesis of speed and flow from SD model and I-SA model

Truck ratios	Speed from SD model		Speed from I-SA model		Flow from SD model		Flow from I-SA model	
	h-value	p-value	h-value	p-value	h-value	p-value	h-value	p-value
0	1	0.0010	1	0.0010	0	0.5000	0	0.5000
0.2	1	0.0010	1	0.0010	0	0.4208	0	0.5000
0.4	1	0.0010	1	0.0010	0	0.5000	0	0.5000
0.6	1	0.0010	1	0.0093	0	0.5000	0	0.5000
0.8	1	0.0098	1	0.0267	0	0.5000	0	0.5000
1.0	1	0.0167	0	0.0789	0	0.5000	0	0.3788

Table 3 – The variance homogeneity hypothesis of speed and flow from SD model and I-SA model

Truck ratios	Standard deviation				p-value		Sum of squares of deviations	
	Speed from SD model	Speed from I-SA model	Flow from SD model	Flow from I-SA model	Speed	Flow	Speed	Flow
0	37.0148	36.9089	486.302	504.924	0.9790	0.7298	0.00070	0.1193
0.2	25.2639	25.3480	411.035	436.492	0.9789	0.6321	0.00070	0.2292
0.4	26.1012	26.0374	388.488	409.863	0.9861	0.7035	0.00030	0.1448
0.6	26.5475	26.2761	352.215	365.764	0.9479	0.8102	0.00428	0.0577
0.8	26.9169	26.5299	338.400	345.795	0.9312	0.8975	0.00745	0.0166
1.0	27.1518	26.6411	317.097	318.629	0.9165	0.9788	0.01100	0.0007

from *Table 2* that at the significance level α=0.05, the p value of flow under each truck ratio is greater than 0.05. Hence, the assumption that the flow follows a normal distribution (h=0) is accepted. Conversely, it can be concluded that the assumption that the velocity obeys the normal distribution is not acceptable. As shown in *Table 3*, the p-value of speed and flow under each truck ratio is greater than 0.05. Therefore, in the vartestn square difference homogeneity test function, it can be considered that the normal population variances of speed and flow are homogeneous. To sum up, the speed distribution under each truck ratio meets the assumption of variance homogeneity, but does not meet the assumption of normality, while the flow distribution meets the assumption of variance homogeneity and normality. Therefore, the variance test is used to test the significance of flow difference between the SD model and the I-SA model, respectively.

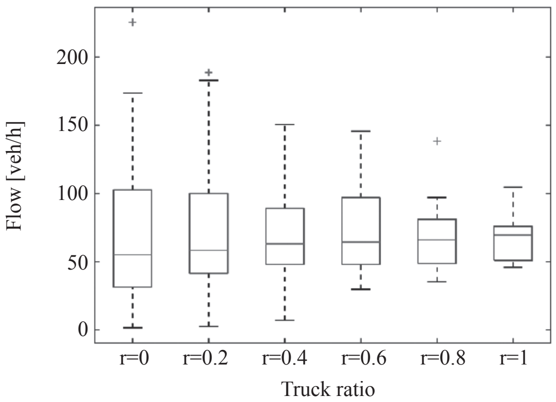
Based on the above results, specific density intervals are divided for each truck ratio to test the significance of the flow differences between the SD model and the I-SA model in each density interval. *Table 4* represents the one-way ANOVA of flow from the SD model and the I-SA model. In *Table 4*, the F-value in Interval 1 and Interval 3 under each truck ratio is less than the F-test threshold, and the p-value is much greater than 0.05, which means that there is no significant difference between the flows of the SD model and the I-SA model in the interval above. The F-values in Interval 2 are 5.14 and 4.56 when the truck ratio is 0 and 0.2, respectively, which is greater than the respective F-test threshold of 4.004 and 4.057, indicating the conclusion that significant differences exist. Based on the statistical analysis of the flow difference between the SD model and the I-SA model under the above truck ratio, the flow difference reaches the maximum value (i.e. 225 veh/h) when the density ranges from 24 veh/km to 34 veh/km and the truck ratio is 0, which is 17.9% of the flow from the SD model under the same density.

It should be emphasised that when the truck ratio is 0.4, 0.6, 0.8 and 1.0, the F-value in Interval 2 is lower than the corresponding F-test value, and the p-value is greater than 0.05. A problem will exist when only F-value and p-value are used to judge the significant difference, i.e. the conclusion that the data have significant differences does not seem to be recognised. Furthermore, the speed distribution under each truck ratio does not meet the assumption of normality, resulting in the parameter

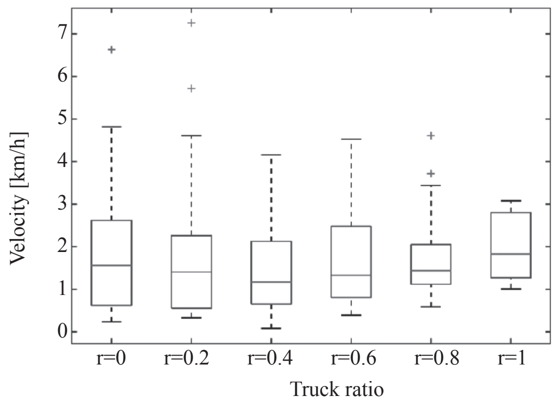
Table 4 – The one-way ANOVA of flow from SD model and I-SA model

Truck ratios	Density interval		Sum of squares (Groups)	Degree of freedom (Groups)	Degree of freedom (Total)	F-test threshold	F-value	p-value
0	Interval 1	2 veh/km – 26 veh/km	532.276	1	25	4.279	0	0.9675
	Interval 2	28 veh/km – 88 veh/km	176184.2	1	61	4.004	5.14	0.027
	Interval 3	90 veh/km – 70 veh/km	26347.1	1	81	3.962	0.4	0.5306
0.2	Interval 1	2 veh/km – 24 veh/km	67.0673	1	23	4.325	0	0.9866
	Interval 2	26 veh/km – 72 veh/km	150644.5	1	47	4.057	4.56	0.0380
	Interval 3	74 veh/km – 128 veh/km	21528.6	1	55	4.023	0.41	0.5238
0.4	Interval 1	2 veh/km – 22 veh/km	29.5568	1	21	4.381	0	0.9901
	Interval 2	24 veh/km – 50 veh/km	56513	1	27	4.242	2.89	0.1010
	Interval 3	52 veh/km – 104 veh/km	25360.4	1	37	4.121	0.75	0.3935
0.6	Interval 1	2 veh/km – 22 veh/km	58.7456	1	21	4.381	0	0.9845
	Interval 2	24 veh/km – 54 veh/km	61508.4	1	31	4.183	2.11	0.1567
	Interval 3	56 veh/km – 84 veh/km	8255.8	1	29	4.210	0.25	0.6198
0.8	Interval 1	2 veh/km – 24 veh/km	69.4961	1	23	4.325	0	0.9810
	Interval 2	26 veh/km – 60 veh/km	42973.3	1	35	4.139	0.92	0.3433
	Interval 3	62 veh/km – 74 veh/km	262.1	1	13	4.844	0.03	0.8636
1.0	Interval 1	2 veh/km – 24 veh/km	84.2625	1	23	4.325	0	0.9765
	Interval 2	26 veh/km – 52 veh/km	34321	1	27	4.242	1.01	0.324
	Interval 3	54 veh/km – 64 veh/km	158.1	1	11	5.117	0.02	0.8991

test method not being suitable for testing the significance of speed difference. To further quantify the improvement of the I-SA model on the flow under the remaining truck ratios (0.4, 0.6, 0.8 and 1), and test the significance of speed difference, the value obtained with the speed (flow) of the I-SA model subtracting the speed (flow) of the SD model is summarised and analysed under the same density and truck ratio. *Figures 5a and 5b* represent the boxplot of speed difference and flow difference, respectively. It can be directly observed from *Figure 5a* that when the truck ratio is 0.4, 0.6, 0.8 and 1, the maximum flow difference reaches 150 veh/h, 146 veh/h and 138 veh/h, respectively. Additionally, the average value of the flow difference when the truck ratio is 0.4, 0.6, 0.8 and 1 is even greater than that of the flow difference when the truck ratio is 0 and 0.2. The significant difference in the flow obtained from the SD model and the I-SA model also exists when the truck ratio is 0.4, 0.6, 0.8 and 1, although the p-value obtained from the variance test is small. In *Figure 5b*, the speed difference reaches the maximum value (i.e. 7.26 km/h) when the density ranges from 26 veh/km to 32 veh/km and the truck ratio reaches 0.2, which is equivalent to 17% of the speed from the SD model under the same conditions.



a) Flow difference



b) Speed difference

Figure 5 – Speed difference and flow difference in Interval 2

4. CONCLUSION

It should be noted that there is a problem of insufficient accuracy in describing the car-following behaviours in the existing SD model. The excessive speed difference and gap between the low-speed vehicle and the front vehicle always appear in the simulations. Therefore, the I-SA model considering the multi-decision variable stochastic noise parameter is proposed in this paper. Acceleration of the front vehicle, speed difference and gap are all considered in the I-SA model, where the vehicle acceleration behaviour is accurately modelled.

The accuracy of the model is verified in the paper on both microscopic and macroscopic levels. The microscopic analysis results show that the basic advantage of the stochastic noise parameter is reproduced in the I-SA model, i.e. compared with the CA model without the stochastic noise parameter, the asymmetric acceleration behaviour of vehicles can be reflected by the I-SA model, which is consistent with previous studies [21, 22]. In the modelling of jams, the impact of the I-SA model on jams from two aspects of jam velocity and gap is studied in this paper. The main conclusions are as follows: (1) based on the spatial-temporal diagrams, the real gap between the stationary vehicle and the front vehicle can be reproduced by the I-SA model with the optimised stochastic noise parameter, which reduces the width of congestion waves and alleviates congestion. In addition, the distribution of vehicles in the simulation section is more uniform, and the utilisation of road space is more balanced; (2) in previous studies, the spatial-temporal diagrams of traffic under fixed density was used in the analysis of jam velocity [22]. The previous results show that the jam velocity simulated by the SD model is 14.3 km/h, close to the field observation. However, the jam velocity obtained by this method is easily disturbed by random factors, such as the initial speed and deceleration duration of the leading vehicle, resulting in strong randomness of the jam velocity obtained from multiple spatial-temporal diagrams. Hence, the car-following experiments in which the above factors are controllable are carried out to analyse the jam velocity. Based on the car-following experiments, the problem that the excessive speed difference between the stationary vehicle and the front vehicle exists in the SD model can be solved by the I-SA model. The speed curves of each vehicle are arranged more closely during the acceleration, which makes the platoon quickly return to a stable state after experiencing congestion waves. At the macroscopic level, the main conclusions are as follows: (1) the results from the fundamental diagram analysis show that the I-SA model can reduce the discreteness of the speed and the gap and improve the stability of the transportation system with density near the critical density. Additionally, when the density is higher than 24 veh/km and lower than the congestion density, the flow and velocity at the same density and truck ratio can be improved by the I-SA model; (2) based on the results of the statistical significance test and the box plot of flow difference, a significant difference in flow between the SD model and the I-SA model under the medium high density is observed; (3) according to the box plot of flow and speed difference, the flow difference between the SD model and the I-SA model reaches the maximum value (i.e. 225 veh/h) when the density ranges from 24 veh/km to 34 veh/km and the truck ratio is 0, which is 17.9% of the flow from the SD model under the same density. The speed difference reaches the maximum value (i.e. 7.26 km/h) when the density ranges from 26 veh/km to 32 veh/km and the truck ratio reaches 0.2, which is equivalent to 17% of the speed from the SD model under the same condition.

Although the I-SA model corrects some shortcomings of the existing cellular automata models, there are still many deficiencies in the proposed model that can be made up for in the future work:

1) The model needs to be further improved. Lane-changing behaviour does not exist in the I-SA model. Therefore, the impact of lane-changing behaviour should be taken into account in the acceleration rules of the car-following model in the future works.
2) Parameters need to be calibrated specifically. Some parameters in the model are taken from the existing research results. In the extended model, the parameters need to be further calibrated according to more detailed and accurate data.

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一种具有优化随机噪声参数的交通流元胞自动机模型

摘要

在现有安全距离元胞自动机模型的基础上，本文提出了一种基于真实人类反应的改进元胞自动机模型，旨在再现拥挤交通流的特征。在该模型中，通过考虑驾驶行为差异对随机噪声参数进行了优化。在优化的随机噪声参数中引入了相对速度、间隙和前车的加速度，以描述拥挤状态下驾驶员的非对称性加速行为。仿真结果表明，在该模型的时空图上，拥堵车辆的加速度轨迹分布不均匀。在分析*NGSIM*数据集的基础上，与传统随机噪声参数模型相比，该模型能够使车辆更容易、更真实地跟驰行驶。因此，该模型能够更好地反映车辆的实际间隙，并且车速的变化更加稳定。此外，从流量和速度两个方面对交通效率进行了分析，结果表明，该模型能显著提高中高交通流密度范围内的交通效率。

关键词：
异质交通流；随机噪声参数；元胞自动机；跟驰行为

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