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CONTRIBUTION TO THE OPTIMISATION OF THE CARGO TRANSPORTATION PROBLEM

ABSTRACT

The paper deals with modelling of the problem concerning the transportation of various kinds of cargo with one transport means from one source to one or more destinations. The mathematical model of such a problem can assume different forms, often involving a non-linear criterion function and either a linear or non-linear constraints, with an optimal solution being achievable by the dynamic programming method. The solution to the problem concerning transportation of different cargoes can be approached as a problem of either a simple or a complex distribution of a single source. The example presented in the paper illustrates how an optimum structure of container carriage by sea can be determined.

KEY WORDS

knapsack problem, dynamic programming, container carriage

1. INTRODUCTION

Problems concerning transportation of cargoes can vary, depending on the number of kinds of cargo, on the number of kinds and types of transport means, as well as on the number of sources and destinations, and modes of transportation.

There are four different types of problems concerning transportation of cargo, such as the following:

1. those concerning the transportation of one single kind of cargo from more than one source to more than one destination (the two-index transportation problem);
2. those concerning more than one kind of cargo being distributed among more than one destination either through different modes of handling or by different means of transport (the multi-index transportation problem);
3. those concerning the sequence of cargo distribution between the source and the destination, with either the minimum distance/cost or the maximum income/profit being achieved (the travelling salesman problem);
4. those concerning the transportation of different kinds of cargoes by one means of transport from one single source to one or more than one destinations (knapsack problem).

In the first case, the *two-index transportation problem* or simply the transportation problem as most frequently denominated in literature, concerns the supply of a certain number of destinations with the same kind of cargo from a certain number of sources with fixed unit transportation costs on particular routes, taking into account the cargo quantities available at sources (offer/production) and cargo quantities required at destinations (demand/requirements). The searching for the solution to this problem is aimed at the one resulting in the lowest total transportation cost possible [3, p. 121].

This type of problem can be solved with linear programming (the simplex method), which is a time-consuming method because great number of sources and destinations are involved. Owing to this reason, specific algorithms have been developed in approaching the linear programming transportation problem, i.e. methods for installation of the basic programme to be followed by one of the basic programme-improving methods leading to the optimum solution. Dynamic programming, as a possible solution-finding method, is only efficient where the number of sources and destinations is small (2, 3, or 4 at the maximum) or where transport costs are non-linear functions [8, p. 29].

The multi-index transportation problem is a linear programming problem requiring the quantities to be determined, which shipped from a particular source to a particular destination by a certain means of transport or a transportation mode result in the minimum transport costs. Where the problem is defined in this way, we are dealing with the three-index transportation problem which can be solved by means of the gradual cargo distribution method or by the multiplicator method, that is to say just like the two-index transportation problem, by means of methods for installation and improvement of the basic programme, an optimum solution being also achievable by means of the simplex method [13, p. 230].

The travelling salesman problem belongs to the narrower class of non-linear programming called the integer one, particularly the 0-1 programming. In the performance of cargo distribution, care should be taken that starting from the 0-point of departure visiting includes a number of scheduled points of destination with known distances between them as well as their visiting order, in dependence on the criterion applied, result either in the minimum distance/time consumption or in the maximum income/profit earned.

The two-index transportation problem having been described in detail in literature and already widely applied, the multi-index transportation problem having been presented in Z. Zenzerović's paper [13] before, and the commercial traveller problem being the one dealt with by H. Pašagić, at the Faculty of Transport and Traffic Engineering [6], this paper deals with the fourth type of cargo transportation problems and with solutions based on the dynamic programming.

2. DEFINITION OF THE TRANSPORTATION PROBLEM CONCERNING CARGOES OF DIFFERENT KINDS

The fourth type of cargo transportation problem arises when a certain number of consignments of different weights and cargo kinds has to be transported by particular means of transport, e.g.: a truck, a ship, a plane, a railway, etc. Their total weight is not to exceed the maximum weight allowed for the respective means of transport or its carrying capacity. Transportation of different consignments means different profits earned or different transport costs. Profit amounts also depend on the specific features of cargo kinds carried and on their share in the total carrying capacity of the respective means of transport. The assumption is that profits obtained from different cargoes can be measured by a common measurement unit, that the profit obtained from one kind of cargo is independent on the distribution of the remaining carrying capacity among other kinds of cargo, and finally that the profit total represents the sum of particular profit amounts. The same conclusion is valid for transportation costs.

The purpose is to have the available capacity distributed among different kinds of cargo and/or to decide upon the number of single-sort cargo consignments for transportation which will result either in the maximum profit or the minimum costs.

The criterion function represents the maximum transportation profit/minimum transportation costs, whereas the constraints refer to the occupancy of the means of transport involved with consignments of particular cargo kinds of different weights.

The mathematical model for this type of the problem can take the following forms:

a) the criterion function and constraints expressed by a linear function,

b) the criterion function is linear and constraints are non-linear functions,

c) the criterion function is non-linear and constraints are either linear or non-linear functions.

The problem corresponding to the mathematical model described under a) above is approached by linear programming methods, and/or by the integer linear programming or the dynamic programming method as either a simple or complex distribution problem. In this case linear programming is given priority because practical problems are mostly solved by computers provided with software and also because linear programming enables the postoptimum analysis.

Where the problem described under b) above is dealt with, its constraints being expressed in non-linear form, linear programming cannot be applied, but the problem is recommended to be approached by the dynamic programming method. This is seldom the case in practice because of the linear connection existing between the number of consignments and the weight of particular consignments.

In case of c) the problem may involve a non-linear criterion function and linear constraints. In case of problems for which an appropriate mathematical model can be formulated with at least one or more non-linear links either in the criterion function or within the group of restrictions, the non-linear programming is applied. Quite a number of methods have been developed with the aim of finding solutions to these problems through solutions to particular subclasses of non-linear programming, such as: non-linear programming with linear constraints, the square, integer, separable, geometrical, and a series of other types of programming with certain forms of non-linear links in their mathematical models.

In practice, for example, calculations are frequently required regarding optimum profitability, economy, and productivity which are considered the most significant indicators of commercially successful business operation. These indicators being expressed in the form of fractions, for the calculations searching their maximums, the fractioned linear programming method is applied, provided constraints are expressed in the linear form.

In case of the mathematical model of transportation problems which comes down to linear programming, the assumption is that the transport cost (or the profit from transport) on any route is proportional with the quantity of goods being transported on the respective route. By this assumption the corresponding criterion function is conditioned to assume linear interdependence. However, it often happens in practice that the problems concerning transportation of cargo refer to the cost or to the profit of non-linear form in relation to the cargo quantity being transported (e.g. rebates on larger quantities). In such cases, the cost/profit related to certain cargo quantities can be given in the form of discrete values and this is where the dynamic programming method should apply.

With regard to the mentioned forms of cargo transportation problems, the dynamic programming method is given preference over other possible methods listed before, the main reason arising from the fact that the objective and the limitation function are not necessarily linear. This brings us to the essential feature of the dynamic programming, which is its limited strictness in terms of the linearity of the model structural equation (or non-equation) [5, p. 41].

The problem concerning transportation of different kinds of cargo is a typical example for the dynamic programming problem not including the time component, yet owing to its artificial decomposition into a series of cargo kinds corresponding to a series of stages the problem is reduced to a multi-stage process gradually becoming optimised by means of the dynamic programming method.

3. FORMULATION OF MATHEMATICAL MODEL FOR TRANSPORTATION OF DIFFERENT KINDS OF CARGO

Dynamic programming represents one of the operational research methods for determining the decision-making optimal strategy concerning multi-stage processes in cases where correlated decisions are made for particular years within a particular period or for particular activities concerning the formulated problem. By this method the problem is approached through stages in such a manner that in each stage the estimates of the optimum values obtained in the preceding stage are used. This process leading to the sequence of optimal decisions is known as the Bellman principle.

For the mathematical formulation of the general problem, concerning the optimum planning process, the basic terms should be introduced and defined, as the following: *system, process, multi-stage process, criterion function, policy.*

System denotes a physical system (technical, economic, etc.) analytically defined as the vector of states [7, p. 214]:

$$r(t)=[r_1(t), r_2(t), \dots, r_N(t)] \tag{1}$$

The state vector components $r(t)$ determine the system features, whereas the number N is called the system dimension. Vector $r(t)$ may be marked as p, the p denoting the initial state of the system.

Process can be described as the system behaviour in the course of time. If there are relations which determine their initial and each subsequent state [7, p. 214]:

$$p_0 = p,\ p_{n+1} = W(p_n);\ n = 0,1,2,\dots;\ W\text{-operator,} \tag{2}$$

then the set of vectors $(p_0, p_1, \dots)$ represents the system behaviour in discrete moments of time $n=1,2,\dots$ and is defined as a process, i.e. a special type of process called the *multi-stage process.*

The relation (2) can be also presented in the following way:

$$p_n = W^n(p) \tag{3}$$

meaning that operator W has been applied n – times. Accordingly, the multi-stage process is defined by the initial state system p by means of transformation $W(p)$, which can be symbolically presented as: $[p, W(p)]$.

Classification of multi-stage processes is the following [9, p. 3-10]:

- Finite and infinite processes (processes with finite/infinite number of states);
- Discrete and continuous processes (states change in discrete/continuous moments of time);
- Stationary and non-stationary processes (invariability/variability of the system working process over time);
- Deterministic and stochastic processes (the process behaviour is completely known/unpredictable, changeable).

The cargo transportation problem being a process consisting of a finite number of steps with state changes taking place within discrete periods of time, the form of transformation (change) does not depend on time and the transformation is completely known (determined), the next paragraphs are going to deal with the following multi-stage processes: finite, discrete, stationary and deterministic processes.

Recursive relations play an important role within the dynamic programming. For the function type $\sum_{i=0}^{N} G(p_i)$, provided the transformation W is known in expressions (2) and (3), recursive relations are [7, p. 216]:

$$f_N(p) = \sum_{i=0}^{N} G(p_i), \quad N = 0, 1, 2, \dots$$

i.e.

$$f_N(p) = G(p) + f_{N-1}(p_1) = G(p) + f_{N-1}[W(p)], \quad N = 1, 2, \dots \tag{4}$$

The term denoting a multi-stage process, previously defined by the set of vectors $[p, p_1, p_2, \dots]$ and by relation $p_n = W^n(p)$, can be extended if another parameter q is introduced in the transformation W, this parameter also depending on the time and its selection influencing the process. In that case the process is determined by the set of vectors:

$p_{n+1} = W(p_n, q_n); \quad n = 0, 1, 2, ..., N$ (5)

The selection of this value (q_i) is called *determining of the permitted solution.* This value is selected with the aim of reaching the optimum solution to function $F(p, p_1, p_2,...; q, q_1, q_2,...)$, called *the criterion function or the optimisation criterion function.*

The multi-stage process (N-stage) providing the solution is described by the set of vectors:

$[p, p_1, \dots, p_{n+1}; \; q_0, q_1, \dots, q_N]$,

$p_{n+1} = W(p_n, q_n) \quad za\ 0 \leq n \leq N$ (6)

The group of permitted solutions $[q_0, q_1, \dots, q_n]$, where $q_n = q_n(p, p_1,..., p_n; q_0, q_1,...,q_{n-1})$, is called the *policy*. The optimal policy is the one providing the best solution (the maximum or the minimum of a function F).

The dynamic programming method is mostly applied to planning and management problems containing the plan structure selection, the realisation dynamics and the distribution of resources expressed in natural or value units, provided that a multi-stage process has been dealt with.

According to the *stage contents*, there are two types of problems to be distinguished [9, p.3-2]:

a) *dynamic problems with the time component as a stage* (investment optimisation, production planning, distribution of resources, management of reserves, maintenance, replacement of equipment),
b) *non-dynamic problems or problems without the time component,* being subject to decomposition to stages according to logical criteria (distribution of resources, transport duties, system reliability, quantity of spare parts, shortest route within the network).

According to the *mode of resources distribution,* the dynamic programming problems can be divided into the following two groups:

a) *simple distribution problems,*
b) *complex distribution problems.*

The problem-solving process with the dynamic programming method is the following:

1) defining the optimum income or cost function;
2) derivation of functional equation for the optimum income or cost function;
3) application of the functional equation in determining decisions representing the optimum policy concerning the problem observed.

In other words, it is necessary to define the problem and formulate the mathematical model, i.e. the criterion function and constraints.

The mathematical model for simple distribution problem is:

$$f(N)[Q] = \max_{0 \leq x(N) \leq Q}(\min) Z\{A\}, \quad N = 1, 2, \dots,$$

where

$$A = g(N)[x(N)] + f(N-1)[Q - x(N)]$$

subject to:

$x(1) + x(2) + \dots + x(N) = Q$ (7)

with: all variables nonnegative and integral,

where:

- Q – the total quantity of resources distributed among N stages (activities),
- $x(i)$ – the amount or quantity of resources expressed in natural or value units allocated to the i-th stage (activity); $i=1,...,N$,
- $g(i)[x(i)]$ – income or costs of the i-th stage (activity) being dependent on the quantity invested in the i-th stage (activity),
- $f(N)[Q]$ – maximum income or minimum cost obtained through the investment of the total quantity Q in N stages (activities).

The complex distribution problem arises when there is a certain quantity of resources expressed in natural or value units to be distributed among a certain number of stages, provided that:

$Q = a(1) \cdot x(1) + a(2) \cdot x(2) + \dots + a(N) \cdot x(N),$ (8)

where $a(i)$ represents the coefficient or the resource expenditure of quantity Q for the i-th stage per unit $x(i)$.

In practice $a(i) > 1$, and if $a(i)=1$ the complex distribution problem is reduced to the simple distribution problem.

Therefore, the mathematical model for complex distribution problem is:

$$f(N)[Q] = \max_{0 \leq x(N) \leq Q/a(N)}(\min) Z\ \{A\}, \quad N = 1, 2, \dots,$$

where

$$A = g(N)[x(N)] + f(N-1)[Q - a(N) \cdot x(N)]$$

subject to:

$a(1)x(1) + a(2)x(2) + \dots + a(N)x(N) = Q$ (9)

with: all variables nonnegative and integral.

The problem concerning transportation of cargo can be approached either as a complex distribution problem or as a simple distribution one. In case it is approached as a complex distribution problem, the complex distribution model (9) shall apply with the following symbols taken into consideration:

- Q – carrying capacity of the means of transport,
- i – kind of cargo, $i=1,...,N$,
- $a(i)$ – i-th cargo consignment size,
- $x(i)$ – number of consignments of i-th cargo kind,
- $g(i)[x(i)]$ – income or cost of the i-th cargo kind transport being dependent on the quantity of the transported i-th cargo kind,
- $f(N)[Q]$ – maximum income (minimum costs) obtained in the transport of N cargo kinds in total with a transport means of carrying capacity Q.

4. OPTIMISATION OF CONTAINER CARRIAGE BY SEA

The cargo transportation problem occurs in every mode of transport, such as: road, railway, sea, air and inland waterway transport. An optimal solution to the problem is seen in the selection of the most favourable mode of transport with respect to transport costs or income, that is to say, for the minimum transport costs or the maximum income.

Where the container sea carriage technology is concerned, the problem of transportation occurs when transportation of containers requires organisation of the carriage by sea from several ports of departure to several ports of destination with the minimum distance to be crossed (time at sea), and maximum profit or minimum transport costs to be achieved. The container ship structure consists exclusively of containers, with the possibility of RO/RO cargo. Containers intended for carriage by container ships are subject to the uniform international standardisation (length, width and height), thus enabling conditions to be met in any case of the formulated model.

In determining the optimal structure of container carriage by sea, the corresponding number of various containers should be selected from the total container quantity available at the port of departure in terms of container types, weights, and possibly the RO/RO cargo in order to enable the ship's maximum profit or minimum transport costs and to make her deadweight and earning capacity exploited to the fullest extent.

One way to the solution of this problem is the dynamic programming method. The problem being formulated requires a mathematical model consisting of the criterion function and constraints.

The criterion function in case of container carriage can be the maximum income (profit) obtained by a container ship or the minimum costs. Considering that in the service of expensive container ships an exceptional significance is attached to making as much profit as possible on account of the fact that containerisation itself as a technological and transport process is a very expensive and highly profitable mode of the maritime container technology, the ship's profit represents the most frequently selected criterion among all.

Where the use of the formulated mathematical model in this way in respect of container carriage by sea is concerned, the assumption concerning the problem of container carriage by sea is that there is a sufficient number of different containers in terms of type, weight and size available on the seaborne trade market and moreover that they exceed the ship's earning capacity.

For illustration purposes, an example has been taken concerning the determination of a container ship's optimum carrying structure, where the existing literature on container transport [4, p. 67] has been consulted for the data input.

A 29 434-ton container ship of the capacity of 1762 TEU is supposed to carry a certain number of four different types of containers. The different kinds of cargo have been stowed in containers at the port of departure, their unit weights and profit per container unit being presented in Table 1.

Table 1: Unit weight and profit by type of container

Type of container	Weight per unit (in tons)	Profit per container (in US$)
20' OT	15	63
20' TC	16	69
40' RF	25	72
40' DB	21	67

Note: 20' OT (open top, 20 feet length), 20' TC (tank container, 20 feet length), 40' RF (refrigerated container, 40 feet length), 40' DB (dry box container,40 feet length)
Source: S. Kos [4, p. 67]

It is necessary to determine the number of particular types of containers which are going to realise the maximum profit from the carriage and to supplement the ship's capacity. To make the calculation simpler, just one portion of container ship was taken into account, with capacity of 200 tons, the portion having been booked by a single forwarder.

For each container type separate profit tables are created with their respective deadweight following intervals dependent on the unit weight of each particular container. Where the transport up to 10 TEU is involved, the profit behaves linearly with the quantity (full tariff per container unit), and in case of the quantity of 10 TEU or above a 5% rebate is granted.

The mathematical model is:

Criterion function

$$\max F(x) = 63x(1) + 69x(2) + 72x(3) + 67x(4)$$

Constraints

$$15x(1) + 16x(2) + 25x(3) + 21x(4) \leq 200$$
$$x(1) \leq 13, \quad x(2) \leq 12, \quad x(3) \leq 8, \quad x(4) \leq 9$$
$$x(1) + x(2) + x(3) + x(4) \leq 42$$

x(1), x(2), x(3), x(4) - integer positive values.

For the purpose of defining recursive relations, the following symbols are introduced:

Q – ship's deadweight (in tons),

$x(i)$ – number of the i-th type of container,

$a(i)$ – weight per unit of the i-th type of container,

$g(i)$ – profit per unit from the carriage of the i-th type of container,

$f(i)[Q]$ – function of the profit obtained by the container ship from the carriage of $x(1), x(2), x(3), x(4)$ containers.

From the condition of integer variables $x(i)$ there follows:

$$x(i) \leq Q / a(i), \text{ i. e.}$$

$$0 \leq x(1) \leq 13; \quad 0 \leq x(2) \leq 12; \quad 0 \leq x(3) \leq 8;$$
$$0 \leq x(4) \leq 9.$$

Recursive relation is defined by:

$$f(i)[Q] = \max_{0 \leq x(i) \leq Q/a(i)} \{A\}, \tag{10}$$

where

$$A = g(i)x(i) + f(i-1)[Q - a(i)x(i)].$$

Profit functions for particular container types are:

$$f(1)[Q] = \max_{0 \leq x(1) \leq 13} \{g(1)x(1)\}, \tag{11}$$

$$f(2)[Q] = \max_{0 \leq x(2) \leq 12} \{g(2)x(2) + f(1)[Q - 16x(2)]\}, \tag{12}$$

$$f(3)[Q] = \max_{0 \leq x(3) \leq 8} \{g(3)x(3) + f(2)[Q - 25x(3)]\}, \tag{13}$$

$$f(4)[Q] = \max_{0 \leq x(4) \leq 9} \{g(4)x(4) + f(3)[Q - 21x(4)]\}. \tag{14}$$

On the basis of data presented in Table 1, the unit profits for particular container types have been calculated and entered in Tables 2-5.

Table 2: Profit for container type I

Q	$x(1)$	$g(1)[Q]$
0 – 14	0	0
15 – 29	1	63
30 – 44	2	126
45 – 59	3	189
60 – 74	4	252
75 – 89	5	315
90 – 104	6	378
105 – 119	7	441
120 – 134	8	504
135 – 149	9	567
150 – 164	10	598
165 – 179	11	658
180 – 194	12	718
195 – 200	13	778

Using the complex distribution mathematical model (9) and recursive relations (11)-(14), resulted in Table 6 representing the maximum profit amounts obtainable from the carriage of four types of containers by the 200-ton earning capacity portion of the ship.

On the basis of results presented in Table 6, the optimum solution reads as follows:

f(4)[Q] = 849 US$, x(1) = 8 (20' OT), x(2) =
= 5 (20' TC), x(3) = 0 (40' RF) i x(4) = 0 (40' DB),

meaning that the carriage of the available four types of containers can result in the maximum profit of 849 US$, including 8x20' OT containers and 5x20' TC containers. The optimum programme does not include carriage of 40' DB and RF (the third and fourth type containers).

Table 3: Profit for container type II

Q	$x(2)$	$g(2)[Q]$
0 – 15	0	0
16 – 31	1	69
32 – 47	2	138
48 – 63	3	207
64 – 79	4	276
80 – 95	5	345
96 – 111	6	414
112 – 127	7	483
128 – 143	8	552
144 – 159	9	621
160 – 175	10	655
176 – 191	11	721
192 – 200	12	786

Table 4: Profit for container type III

Q	$x(3)$	$g(3)[Q]$
0 – 24	0	0
25 – 49	1	72
50 – 74	2	144
75 – 99	3	216
100 – 124	4	388
125 – 149	5	342
150 - 174	6	410
175 – 199	7	478
200	8	547

Table 5: Profit for container type IV

Q	$x(4)$	$g(4)[Q]$
0 – 20	0	0
21 – 41	1	67
42 – 62	2	134
63 – 83	3	201
84 – 104	4	268
105 – 125	5	318
126 – 146	6	381
147 – 167	7	445
168 – 188	8	509
189 – 200	9	572

Table 6: Table of maximum profit

Q	$f(4)[Q]$	$x(1)$	$x(2)$	$x(3)$	$x(4)$
0 - 20	849	8	5	0	0
21 - 41	788	1	11	0	1
42 - 62	758	4	8	1	2
63 - 83	756	5	7	1	3
84 - 104	751	5	7	0	4
105 - 125	666	9	4	1	5
126 - 146	660	10	3	1	6
147 - 167	652	10	3	0	7
168 - 188	647	11	2	0	8
189 - 200	572	13	0	0	9

5. CONCLUSION

The cargo transportation problem occurs in all modes of transport, such as: road, railway, sea, air and inland waterway transport. An optimum solution to the problem is seen in the selection of the most favourable variant of transport with respect to transport costs or revenue, that is to say for the minimum transport costs or the maximum revenue.

Problems concerning the transportation of cargo can vary. Depending on the mode of cargo transport and/or on the number of cargo kinds, on the number of sorts and types of transport means, as well as on the number of sources and destinations, the problems concerning transportation of cargo have been classified in four type groups (the two-index transportation problem, multi-index transportation problem, commercial traveller problem and knapsack problem)

The fourth type of the cargo transportation problem characterised by inadequate attention in literature, and frequent presence in practice, as well as its solution by means of the dynamic programming method represents the topic this paper has been dedicated to.

The problem concerns the transportation of consignments of different weights and different kinds of cargo by a means of transport, such as: truck, ship, plane, railway, etc. The available carrying capacity of the respective means of transport should be distributed in the manner enabling either the maximum profit or the minimum cost to be obtained from the carriage of a certain number of consignments of particular cargo kinds.

Since in cases presented in the paper particular profits from the transportation of particular kinds of cargo have been mostly non-linear with respect to cargo quantities (due to various rebates, benefits, stimulations etc.), cargo distribution cannot be solved by linear programming but recommendably by the dynamic programming method. The mathematical model for this problem has a non-linear criterion function and constraints in the form of either a linear or non-linear function.

By way of illustration, the paper has presented how an optimal carriage structure for a container ship is determined, the presented methodology being also applicable in determining the optimal transportation structure within any other mode of transport.

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SAŽETAK

PRILOG OPTIMIZACIJI PROBLEMA PRIJEVOZA TERETA

U radu je prikazano modeliranje problema prijevoza raznovrsnog tereta jednim prijevoznim sredstvom iz jednog ishodišta do jednog ili više odredišta. Matematički model takvog problema može imati različite oblike, često s nelinearnom funkcijom cilja i ograničenjima u obliku linearne ili nelinearne funkcije za koji se optimalno rješenje dobiva metodom dinamičkog programiranja. Problem prijevoza raznovrsnog tereta rješava se kao problem jednostavne ili složene raspodjele jednog izvora. Na konkretnom primjeru ilustrirano je određivanje optimalne strukture prijevoza kontejnera brodom.

KLJUČNE RIJEČI

problem ranca, dinamičko programiranje, prijevoz kontejnera

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