Qiushi ZHANG, Master candidate[1]
E-mail: zqstoan3@163.com
Jing QI, Ph.D.[2]
E-mail: qijing400@vip.sina.com
Yongtian MA, Master candidate[1]
E-mail: mytian_1996@163.com
Jiaxiang ZHAO, Master candidate[1]
E-mail: 1343306391@qq.com
Jianjun FANG, Ph.D.[1]
(Corresponding author)
E-mail: jianjun@buu.edu.cn
[1] School of Urban Rail Transit and Logistics
Beijing Union University
No.97, North 4th Ring East Road, Chaoyang District,
Beijing, China
[2] School of Tourism, Beijing Union University
No.97, North 4th Ring East Road, Chaoyang District,
Beijing, China

Transport Economics
Original Scientific Paper
Submitted: 11 Oct. 2021
Accepted: 29 Mar. 2022

TIME DIFFERENTIAL PRICING MODEL OF URBAN RAIL TRANSIT CONSIDERING PASSENGER EXCHANGE COEFFICIENT

ABSTRACT

Passenger exchange coefficient is a significant factor which has great impact on the pricing model of urban rail transit. This paper introduces passenger exchange coefficient into a bi-level programming model with time differential pricing for urban rail transit by analysing variation regularity of passenger flow characteristics. Meanwhile, exchange cost coefficient is also considered as a restrictive factor in the pricing model. The improved particle swarm optimisation algorithm (IPSO) was applied to solve the model, and simulation results show that the proposed improved pricing model can effectively realise stratification of fares for different time periods with different routes. Taking Line 2 and Line 8 of the Beijing rail transit network as an example, the simulation result shows that passenger flows of Line 2 and Line 8 in peak hours decreased by 9.94% and 19.48% and therefore increased by 32.23% and 44.96% in off-peak hours, respectively. The case study reveals that dispersing passenger flows by means of fare adjustment can effectively drop peak load and increase off-peak load. The time differential pricing model of urban rail transit proposed in this paper has great influences on dispersing passenger flow and ensures safety operation of urban rail transit. It is also a valuable reference for other metropolitan rail transit operating companies.

KEYWORDS

urban rail transit; time differential pricing; bi-level programming model; passenger exchange coefficient.

1. INTRODUCTION

In recent years, the rate of using public transport has become higher and higher. Under the circumstances, optimisation of game between urban rail transit and conventional bus which account for over 80% transport volume together is regarded as a key to distribute passenger flows and make effective use of public transport. Additionally, balancing passenger demands and profits of public transport operating companies remains a hot research topic. This paper establishes a new time differential pricing model for urban rail transit from the perspective of regulatory role of economic leverage and urban rail transit operational efficiency. The purpose is to guide passengers to arrange their travels rationally.

The fare optimisation issue of public transport has been extensively studied by many scholars. Currie [1] studied Melbourne's 'early bird' fare system (free train travel before 7 a.m.) and found that, while reducing fare revenue to some extent, it significantly reduced the level of full trains in the first peak hour (7 a.m. to 8 a.m.). Following the introduction of the five-zone fare system with no transfer fees in Haifa, Israel, Sharaby [2] found an 18.6% increase in the number of passengers switching from the private car system to the public transport system, demonstrating that fare optimisation provides better route choices for passengers. Kamel [3] proposed a time-based system fare strategy based on factors such as passenger travel choice patterns and departure times, and tested in Toronto, Canada. Results showed that time-based fare strategy spreads traffic demand to the shoulder of peak period and alleviates congestion during peak period. Yook [4] developed a distance-based bi-level programming fare model and used the Utah transit system as an example for experiments, proving that the model can effectively increase the demand for public transport and make the travel structure more rational. Borndörfer [5] argued that fares for public transport should not be based solely on the welfare maximisation perspective, developed a cost-constrained cooperative game model and applied it to the Dutch transport network. The reliability of the model is demonstrated. Zhang [6] compared and analysed a generalised cost minimum model and a bi-level programming model based on a generalised cost function in terms of multiple generalised utility values, and the study results showed the latter was superior in terms of both iterative results and efficiency. Liu [7] constructed a bi-level programming model based on the minimum travel time of an urban passenger transport system and confirmed that a reduction in urban rail transit fares would result in a reduction in the total generalised travel cost for passengers.

From the above references, it can be seen that all the public transport systems in different countries have the same characteristics, are time-varying and restricted by multiple factors. Neither maximisation of passenger welfare nor minimisation of travel distance alone can reasonably alleviate congestion during peak periods. Therefore, pricing strategies and models need to be systematically optimised. By combining time differential pricing strategies and models, a bi-level programming model can be built to better achieve multi-objective optimal decision making. The upper level model carries out profit maximisation of an operating company as the main objective, whereas the lower level model does minimisation of generalised cost of passenger travel as the main objective. The optimal value of fare can be solved by iteratively solving fare through mutual constraints relationship between the upper level and low level models. It is worth noting that for urban rail transport, the same line would present different passenger utilisation rates as city function changes over time. Generally, passenger utilisation rates of different lines represent a dynamic change. Unfortunately, the current pricing model is not flexible enough to cope with this problem.

From the viewpoint of algorithmic solution, Cheraghalipour [8] utilises and compares two kinds of traditional heuristic algorithms–genetic algorithm (GA) and particle swarm optimisation algorithm (PSO), as well as two hybrid algorithms (PSO-GA and GA-PSO) and a modified algorithm (GPA) to solve bi-level programming model, respectively. Results showed that the GPA has a good performance among them. Mostafa [9] proposes a fast heuristic algorithm and two hybrid meta-heuristic algorithms. Although the former converges faster, the optimal solution performance of the latter is significantly higher in a comprehensive comparison. Liu [10] established a bi-level programming model based on elastic demand by using a simulated annealing algorithm for the upper level and the method of successive average (MSA) for lower level to solve them separately. The model solution achieved convergence in less than ten instances. Tafti [11] designed a bi-level congestion pricing problem and solved it using both the PSO and the GA. The final iterative results were the same, but the computation time of the PSO was 45% shorter than that of the GA, and the overall performance was better. Hao [12] applies a dynamic inertia factor to the particle swarm optimisation algorithm with a range of update rates. The merit is that it avoids slow iterations due to initial particles being too dispersed, and updates towards optimal solution and reaches convergence quickly to improve iteration efficiency. Generally, the particle swarm optimisation algorithm has a certain stability and accuracy to solve the bi-level programming model. PSO is applicable to a wide range of scenarios.

To our knowledge, there are as of yet no publications on time differential pricing models for urban rail transit based on passenger exchange coefficients for different time periods and route. In this paper, we will consider the impact of the passenger exchange coefficient on fare pricing from the perspective of urban rail transit and construct a bi-level programming model for urban rail transit from peak and off-peak hours, respectively. The upper level model serves as profit maximisation of an urban rail transit operating company, while the lower level model aims at generalised cost minimisation of passengers. Both passenger exchange coefficient

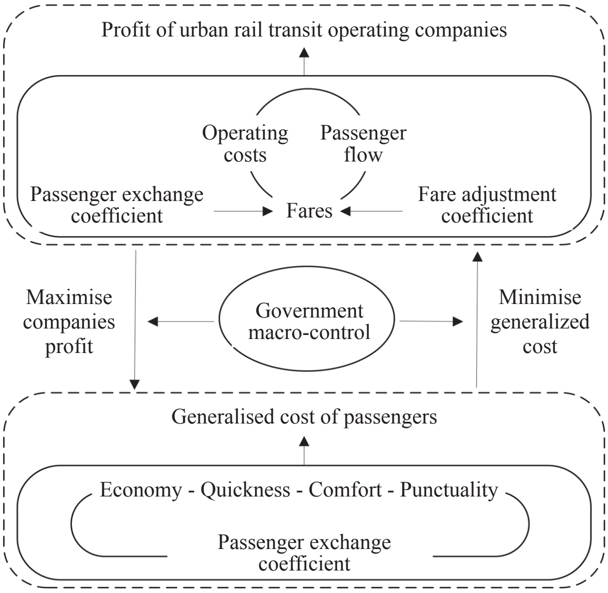

Figure 1 – Outline of the bi-level programming model considering passenger exchange coefficient

and fare adjustment coefficient are fed to models for governing fares. The outline of the model is shown in *Figure 1*.

2. METHODOLOGY

2.1 Generalised cost model with passenger exchange coefficient

In an urban rail transit network, key stations with larger passenger exchange volume, relative long-stay of train, high passenger loads and variation costs, which have great impact on the departure interval, train punctuality and operation, make constraints on the whole line. Passenger exchange volume is commonly measured by a passenger exchange coefficient which indicates the degree of passenger utilisation of trains within a certain transport interval [13]. It is assumed that the passenger exchange volume is characterised by the passenger exchange coefficient, which can be expressed as *Equation 1*

$$\eta_x = \frac{Q_t}{Q_t - \sum_{i=1}^{n} min(a_i, b_i)} \tag{1}$$

where Q_t denotes total number of boarding passengers along the line, $min(a_i,b_i)$ indicates the smaller value between the numbers of boarding passengers and alighting passengers at station i^{th} of the n stations, where a_i is the number of boarding passengers at a station and b_i is the number of alighting passengers at the same station. Change the form of *Equation 1* into *Equation 2*

$$\eta_x = \frac{\sum_{i=1}^{n-1} a_i}{\sum_{i=1}^{n-1} a_i - \left(\sum_{i=2}^{k} b_i + \sum_{i=k+1}^{n-1} a_i\right)} = \frac{Q_t}{Q_{max}} \tag{2}$$

where k indicates the station where the numbers of boarding and alighting passengers become equal (assuming more there are more boarding than alighting passengers at each station before the point k and more alighting than boarding passengers at each station after the point k), Q_{max} denotes the maximum cross-sectional passenger flow. For solving parameter of the passenger exchange coefficient η_x, we need three variables, namely, flow imbalance coefficient η_f, average distance L_a [14] and passenger turnover volume Q, which are expressed as *Equation 3–5*, respectively.

$$\eta_f = \frac{Q_{max}}{Q_a} \tag{3}$$

$$L_a = \frac{\sum_{i=1}^{n} l_i \cdot q_i}{\sum_{i=1}^{n} q_i} \tag{4}$$

$$Q = \sum_{i=1}^{n} l_i \cdot q_i \tag{5}$$

In *Equation 3*, Q_a denotes the average cross-sectional passenger flow. In *Equation 4 and 5*, l_i indicates the distance of the i^{th} station interval in a line and q_i is the cross-sectional passenger flow of the i^{th} station interval in a line. By substituting *Equation 3–5* into *Equation 2*, the relationship among η_x, η_f and L_a can be described as *Equation 6*

$$\eta_x = \frac{L}{\eta_f \cdot L_a} \tag{6}$$

where L indicates the total length of a route. Passenger exchange coefficient is an indicator to evaluate frequency utilised by passengers in a line at a certain period. The generalised cost function of the passenger travel characteristics (e.g. comfort, economy, safety and etc.) is commonly used in the lower level model for those current time differential pricing model studies. In this paper, we integrate passenger exchange coefficient η_x to the lower level model for increasing sensitivity of fare model to different lines in terms of passenger flow. To do this, the model has the ability to generate different fares accordingly by recognising the differences in passenger exchange. The lower level model proposed in this paper is expressed as follows:

$$\min Z(Q) = \sum_k \sum_{i\in n} \int_0^{Q_k^i} \{a(Q_k^i)^b - (\alpha_1 \varepsilon_m P_k^i + \alpha_2 S_k^i + \alpha_3 M_k^i + \alpha_4 T_k^i)\} dQ_k^i \tag{7}$$

$$s.t.\begin{cases} \sum_k \sum_{i\in n} Q_k^i = Q \geq 0 \\ \varepsilon_m = \left\{\frac{\eta_x^i}{\overline{\eta_x^i}}, 1\right\},\ m \in k \end{cases} \tag{8}$$

where the generalised cost function takes the form of a power function $f(Q_k^i)=a(Q_k^i)^b-V_k^i$. The parameter V_k^i denotes generalised utility of the k^{th} mode of transportation at time period i, and which includes P_k^i (economy, simplified for fares), S_k^i (quickness), M_k^i (comfort), T_k^i (punctuality). The parameter ε_m describes the exchange cost coefficient which indicates the ratio of the passenger exchange coefficient η_x^i of a line to the average value of the passenger exchange coefficient $\overline{\eta_x^i}$ in a time period i. Parameter ε_m is used to evaluate the relationship between the degree of passenger exchange of a line and the average level of the same time period. The exchange cost coefficient is set to 1 for all transportation modes except urban rail transit.

2.2 Time differential pricing model for peak hours

The main objectives are to control passenger interaction behaviour at stations and reduce the overload rate during peak hours when passenger flow is high. The upper level model in the bi-level programming model aims at maximising the interests of urban rail transit operator while taking into account certain social welfare. From the perspective of the publicity of urban rail transit, fares are limited under the guidance of the government. The lower level model serves as a generalised cost minimisation model which integrates passenger exchange coefficients. The time differential pricing model for peak hours is expressed as *Equation 9–11*.

$$max\ E_r^n(P,Q) = \sum_{i\in n} (\delta \varepsilon_m P_r^i - C_r^i) Q_r^i \tag{9}$$

$$\min Z(Q) = \sum_k \sum_{i\in n} \int_0^{Q_k^i} \{a(Q_k^i)^b - (\alpha_1 \varepsilon_m P_k^i + \alpha_2 S_k^i + \alpha_3 M_k^i + \alpha_4 T_k^i)\} dQ_k^i \tag{10}$$

$$s.t.\begin{cases} 1 \leq \delta\varepsilon_m \leq \delta_{max} \\ \sum_k \sum_{i\in n} Q_k^i = Q \geq 0 \\ \varepsilon_m = \left\{\frac{\eta_x^i}{\overline{\eta_x^i}}, 1\right\},\ m \in k \\ \varepsilon_m = \begin{cases} \varepsilon_m, & \varepsilon_m > 1 \\ 1, & \varepsilon_m \leq 1 \end{cases} \end{cases} \tag{11}$$

In the constraints, δ is the fare adjustment coefficient. The increase of δ during peak hours should not be higher than the limit δ_{max} set by the government. The exchange cost coefficient ε_m is integrated during peak hours only for lines with the passenger exchange coefficient higher than the average. The parameters P_r^i, C_r^i and Q_r^i indicate fare, operating cost per capita and passenger flow of urban rail transit in the i^{th} time period, respectively. Other parameters are explained in the previous sections.

2.3 Time differential pricing model for off-peak hours

During off-peak hours, passenger volume keeps a low status and the full load rate is also not high. The main goal of the urban rail transit operator is to attract passenger flow and improve income. We build the upper level model for maximising company benefits and the lower level model for minimising passengers' generalised costs to establish a bi-level programming model. They are described as follows:

$$max\ E_r^n(P,Q) = \sum_{i\in n} (\delta \varepsilon_m P_r^i - C_r^i) Q_r^i \tag{12}$$

$$min\ Z(Q) = \sum_k \sum_{i\in n} \int_0^{Q_k^i} \{a(Q_k^i)^b - (\alpha_1 \varepsilon_m P_k^i + \alpha_2 S_k^i + \alpha_3 M_k^i + \alpha_4 T_k^i)\} dQ_k^i \tag{13}$$

$$s.t.\begin{cases} \delta_{min} \leq \delta\varepsilon_m \leq 1 \\ \sum_k \sum_{i\in n} Q_k^i = Q \geq 0 \\ \varepsilon_m = \left\{\frac{\eta_x^i}{\overline{\eta_x^i}}, 1\right\}\ m \in k \\ \varepsilon_m = \begin{cases} \varepsilon_m, & \varepsilon_m < 1 \\ 1, & \varepsilon_m \geq 1 \end{cases} \end{cases} \tag{14}$$

Parameters in the model are defined as mentioned above and downward adjustment of δ during the off-peak hours should not be lower than the lower limit δ_{min} set by the government. The exchange cost coefficient ε_m is added during off-peak hours only for lines with the passenger exchange coefficient lower than the average.

2.4 Algorithm design

In this paper, we take the particle swarm optimisation (PSO) algorithm based on dynamic inertia factor ω to solve our pricing model for urban rail transit. The flowchart of the PSO algorithm is described in *Figure 2*. The description of the algorithm is as follows:

Step 1: *Initialisation*. Initialising each parameter in the PSO algorithm, setting population size, generating particle population with fare adjustment coefficient δ as the upper level model variable (initialise the position X_i^t and velocity V_i^t of each particle within the constraints), recording the current position of the particle, the optimal position of population g_{best} and corresponding fitness function value of each particle.

Step 2: *Calculating the lower level model*. Particle position X_i^t is brought to the lower level for computing, and passenger flow Y_i^t of different traffic modes at different periods is obtained as the optimal solution of the lower level model.

Step 3: *Obtaining fitness function value and updating it*. The parameters X_i^t, Y_i^t are fed to the upper level model to calculate the fitness function value and compare it with the previous fitness function value, update particle optimal position p_{best} and population optimal position g_{best}, and update inertia factor ω. The update rule for ω is: ω^t=(ω_{max}-ω_{min})(T-t)/T+ω_{min}. ω^t is the inertia factor of the t^{th} iteration, T is the maximum number of iterations and t is the current number of iterations.

Step 4: *Determining whether the convergence conditions are met*. If requirements are met, then move to Step 5; if not, then move to Step 2.

Step 5: *Optimal solution interference*. The interference is calculated for the population optimal position g_{best} and it is judged whether the corresponding fitness function value needs to be updated. If it needs to be updated, go to Step 3. If it does not require to be updated, go to Step 6.

Step 6: Output the optimal solution and the corresponding objective function value to end the algorithm.

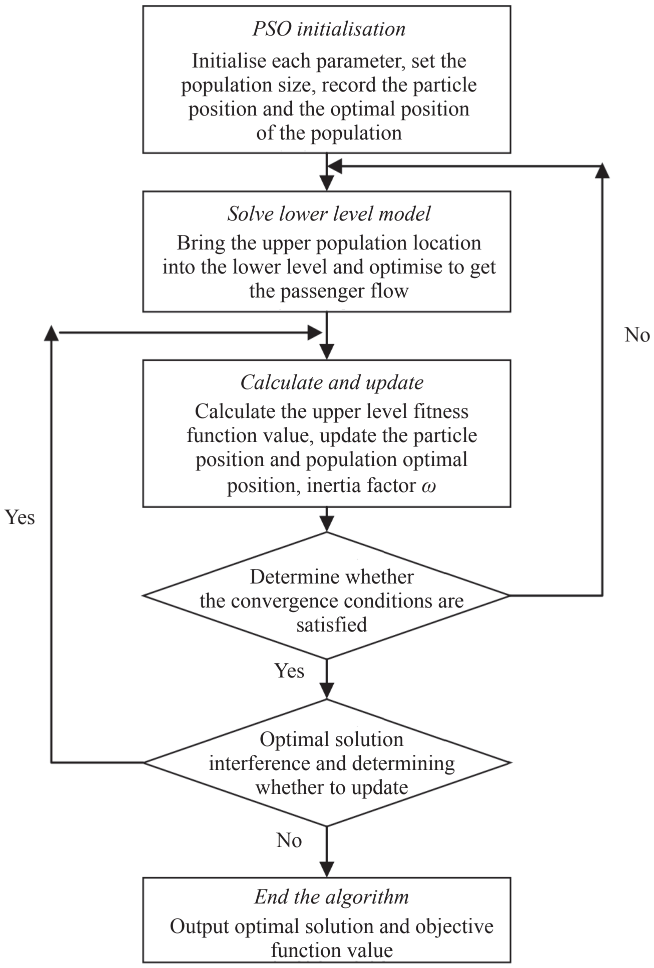

Figure 2 – Flowchart of improved particle swarm optimisation algorithm

3. ANALYSIS

The data used in the case study include data borrowed from the Beijing Transportation Development Annual Report 2020 [15] and data collected from our social survey and investigation means.

3.1 Case design

In 2019, passengers from both urban rail transit and conventional bus exceed 6 million in the metropolitan public transport. The rate is roughly 1:1. Here we just consider two public transports – urban rail transit and conventional bus. As shown in *Figure 3*, it is assumed that the peak hours are 7:00–9:00 a.m. and 5:00–7:00 p.m., and the rest of the hours are assumed as off-peak hours. The peak traffic volume of urban rail transit (including morning peak and evening peak) accounts for about 78.6% of the whole day traffic volume. Therefore, conventional bus accounts for 62.8%. The total number of passenger trips during peak hours is about 862,000, and the total number of passenger trips during off-peak hours is about 359,000 (all data are calculated on weekdays).

The survey utility value indicators, referring to the methodology of the literature [16], are calculated in conjunction with the current state of the market and summarised in *Table 1*.

The lower limit of the discount rate and the upper limit of the price increasing rate for urban rail transit are set to 0.5 and 1.5, respectively. For determination of parameters of the generalised cost function, a and b equal 1 and 0.25, respectively, in combination with the fitting results of previous data. The weight coefficients of utility values are determined by using the hierarchical analysis method AHP as shown in *Table 2*. The research individuals of the model are single lines in different time periods. Line 2 and Line 8 are selected as the objects of analysis in this case study and their initial values are shown in *Table 3*. After calculation, the average value of passenger flow exchange coefficient $\overline{\eta_x^i}$ of Line 2 and Line 8 equals 2.18. Assuming that the passenger flow exchange coefficient of the same line is equal to peak hours during off-peak hours. Combined with the Annual Statistics and Analysis Report of Urban Rail Transit 2019 [17], the discounted operation cost per capita is nearly \$1.

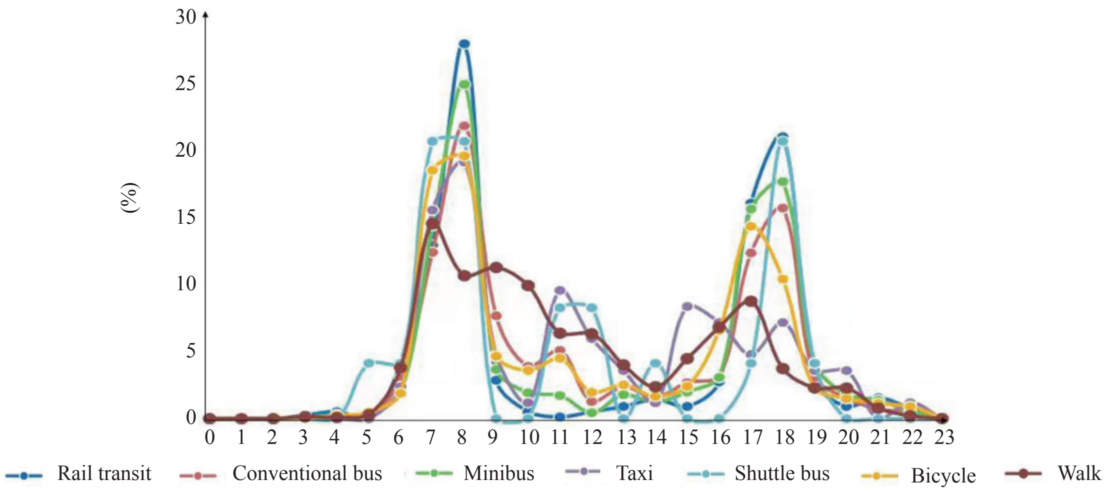

Figure 3 – The proportion of peak hours of each travel mode in the whole day in the central urban area

Table 1 – Index of generalized utility value

Transportation mode	Time period	Average fares (\$) P_k^i	Quickness S_k^i	Comfort M_k^i	Punctuality T_k^i
Rail transit	Peak hours	0.75	24.64	11.53	0.77
	Off-peak hours	0.75	15.36	8.05	0
Conventional bus	Peak hours	0.28	15.71	10.43	1.99
	Off-peak hours	0.28	9.60	6.06	0.68

Table 2 – Weight coefficients of each utility value index

Service indicators	Fares	Quickness	Comfort	Punctuality
Peak hours	-21.09%	-8.13%	-13.29%	-57.50%
Off-peak hours	-28.65%	-8.96%	-22.08%	-40.31%

3.2 Result

The two models are programmed separately using the MATLAB software. The initial parameters of the particle swarm algorithm are set (e.g. population size is 20, iterations are 50, c_1=2, c_2=2, r_1=0.6, r_2=0.3, ω_{max}=0.9, ω_{min}=0.3). After iterative calculations by the improved particle swarm optimisation algorithm, the change in passenger flow for urban rail transit and conventional bus after the implementation of time differential pricing can be obtained, as shown in *Table 4*.

The fare adjustment coefficients δ and fare variation for urban rail transit are calculated as shown in *Table 5*.

Figures 4 and 5 illustrate iterations of peak hours and off-peak hours, respectively. As seen in *Figures 4 and 5*, business profits reach peak value and keep stability at the duration of 20 and 30 iterations. From *Table 4*, it can be seen that the passenger flow of Line 2 and Line 8 during peak hours decreases by 9.94% and 19.48%, respectively, conventional bus increases by 10.56%. During off-peak hours, passenger flow of Line 2 and Line 8 increased by 32.23% and 44.96%, conventional bus decreased by 9.19%. *Table 5* shows fare of Line 2 and Line 8 increase by \$0.32 and \$0.09 during peak hours and decrease by \$0.11 and \$0.16 during off-peak hours. The results show that time differential pricing model integrated with passenger exchange coefficients can effectively differentiate Line 2 and Line 8, allowing them to take different fares depending on the different level of passenger exchange volume during peak hours and off-peak hours. Time differential pricing strategy guides passengers to arrange their travel mode and travel time rationally. This effectively diverts passenger flow to reduce traffic load in peak hours and increase traffic volume in off-peak hours.

Table 3 – Initial operation data of Line 2 and Line 8 of Beijing rail transit network

Line	Average daily passenger flow (10,000 passengers)	Peak hours' passenger flow (10,000 passengers)	Daily average distance [km]	Operating distance [km]	Peak hours' passenger flow exchange coefficient η_x^i
Line 2	98.6	77.5	5.07	23	2.78
Line 8	44.42	34.9	9.30	31	2.00

Table 4 – Public transport passenger flow after the implementation of time differential pricing

Transportation mode	Time period	Passenger flow (10,000 passengers)	Passenger flow change rate (%)
Line 2	Peak hours	69.8	-9.94
	Off-peak hours	27.9	+32.23
Line 8	Peak hours	28.1	-19.48
	Off-peak hours	13.8	+44.96
Conventional bus	Peak hours	95.3	+10.56
	Off-peak hours	32.6	-9.19

Table 5 – Adjustment coefficient δ of urban rail transit ticket price in each time period

Time period	Fare adjustment coefficient δ	Fare changes on Line 2 (\$)	Fare changes on Line 8 (\$)
Peak hours	1.12	+0.32	+0.09
Off-peak hours	0.86	-0.11	-0.16

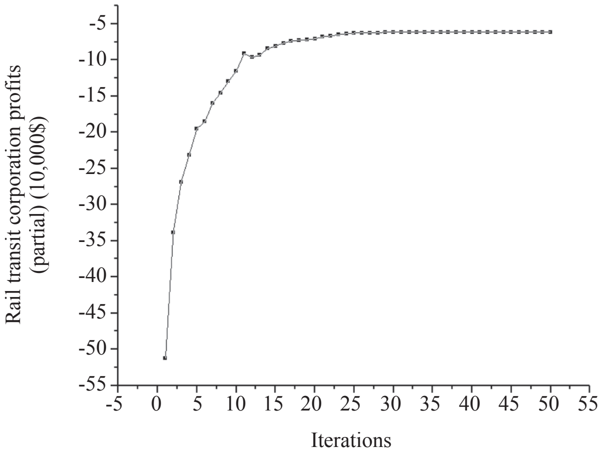

Figure 4 – Relations between profit of urban rail transit company and iterations in peak hours

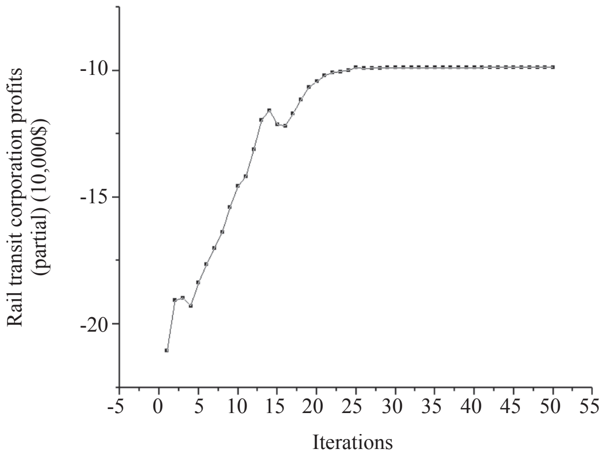

Figure 5 – Relations between profit of urban rail transit company and iterations in off-peak hours

3.3 Analysis

Here we compare our findings with the existing literature [16]. Based on the passenger flow characteristics of peak hours and off-peak hours, two bi-level programming models that aim at time differential pricing have been developed in literature [16]. In the lower level model, the generalised cost function is also used in literature [16] considering different service indicators. Due to the differences between these two cases, it is not possible to directly compare the results of our paper with [16]. However, we can compare the results by introducing a method to represent the ability between models indirectly.

We know that the larger the ratio of the passenger flow of peak hours and off-peak hours is, the more irrational the structure of passenger travel becomes. *Figure 6* clearly shows that after the calculation of the time differential pricing model proposed in this paper, the ratio (slope of the straight line in *Figure 6*) of passenger flow between peak and off-peak hours of urban rail transit has decreased a lot. In the case of this paper, the ratio changes from 3.67 to 2.35, a decrease of 1.32, while in the case of literature [16], the ratio changes from 2.37 to 1.55, a decrease of 0.82. It can be seen that the model proposed in this paper is more effective in solving the overcrowded problem during the peak hours, resulting in a more rational passenger travel structure. Meanwhile, with the addition of the passenger exchange coefficient, which is the biggest innovation of this paper, the model can set different fares according to the difference in passenger flow of the rail lines, so that time differential pricing can be achieved specifically for each rail line. The advantage of this is that both lines in core areas and lines in remote areas can generate more appropriate fares according to their characteristics. It achieves a balanced transportation system by shifting passenger flow to reduce full capacity during peak hours and attracting passenger flow to improve utilisation during off-peak hours. The limitation of the model is that the competition with other transportation modes in the region is not considered. Sensitivity analysis should be added to the values of some parameters in the next research to further verify the rationality.

Compared with the literature [16], the model proposed in this paper focuses more on the comparison between urban rail transit lines and divides the fare based on the passenger flow level of lines themselves, which improves the sensitivity of the model while forming a self-restraint. We know that passengers can be divided into elastic travellers and inelastic travellers based on their travel characteristics. Elastic travellers are those who can flexibly adjust their travel plans, such as tourists. The majority of inelastic travellers are workers and students.

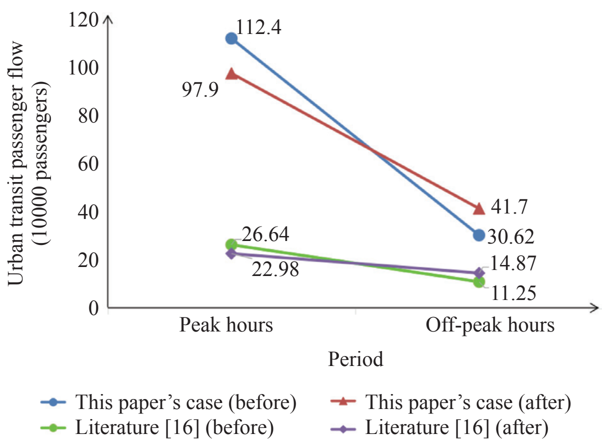

Figure 6 – Comparison of this paper and literature [16]

By reading the literature and surveys, we found that if the price model is implemented, the majority of the population being shifted are elastic travellers (they adjust their travel plans for a better price). In other words, inelastic travellers can trade a certain fare cost for better comfort during peak hours, but the total generalised cost is still optimal. Essentially, people always tend to travel with the lowest generalised cost. We know that transport systems are complex and that public transport pricing should take safety, social welfare, regional coordination and economy of the passengers into account. The model proposed in this paper still has some shortcomings in treating these aspects.

4. CONCLUSION

Compared with other time differential pricing models, the proposed model in this paper integrates passenger exchange coefficient into time differential pricing model, constructing a bi-level programming model for peak and off-peak hours, solving the models with an improved particle swarm optimisation algorithm. The simulation results conclude that the proposed model effectively distinguishes the degree of passenger exchange in different lines of urban rail transit network at different durations and improves flexibility of fares adjustment with time-variation. Thus, fares adjustment relying on this model are in accordance with real passenger flow characteristics.

Due to limited data, only a few lines are selected and assumptions are made on the passenger exchange coefficients during off-peak hours. The subsequent study will expand the sampling area and sample size, refine and enrich the generalised utility of passenger trips, delineate time periods in more detail and further investigate the changing patterns of rail transit more systematically.

ACKNOWLEDGEMENT

This study was supported by the Research on the Vulnerability of Beijing Urban Rail Transit System and Analysis of Countermeasures (11102JA1906).

张秋实，硕士研究生，第一作者[1]
zqstoan3@163.com
齐晶，讲师[2]
qijing400@vip.sina.com
麻永田，硕士研究生[1]
mytian_1996@163.com
赵家祥，硕士研究生[1]
1343306391@qq.com
方建军，教授，通讯作者[1]
jianjun@buu.edu.cn
[1] 北京联合大学城市轨道交通与物流学院， 北京朝区北四环东路97号
[2] 北京联合大学旅游学院，北京朝阳区北四环东路97号

考虑乘客交换系数的城市轨道交通分时定价模型

摘要：

乘客交换系数是一个重要因素，对城市轨道交通的定价模型有很大影响。本文通过分析客流特征的变化规律，将乘客交换系数引入到城市轨道交通分时定价的双层规划模型中。同时，交换代价系数也是定价模型中的一个限制性因素。应用改进的粒子群优化算法（*IPSO*）对模型进行求解，仿真结果表明，所提出的改进的定价模型能够有效地实现对不同时间段、不同线路的票价分层。以北京轨道交通*2*号线和*8*号线为例，仿真结果显示，*2*号线和*8*号线高峰时段的客流分别减少了*9.94%*和*19.48%*，在非高峰时段分别增加了*32.23%*和*44.96%*。该案例研究显示，通过票价调整疏导客流可以有效地实现削峰填谷。本文提出的城市轨道交通分时定价模型对于疏导客流，保证城市轨道交通的安全运营有很大作用，同时对其他城市的轨道交通运营公司也有参考价值。

关键字：城市轨道交通；分时定价；双层规划模型；乘客交换系数

REFERENCES

[1] Currie G. Quick and effective solution to rail overcrowding: Free early bird ticket experience in Melbourne. *Transportation Research Record*. 2010;2146(1): 35-42. doi: 10.3141/2146-05.

[2] Sharaby N, Shiftan Y. The impact of fare integration on travel behavior and transit ridership. *Transport Policy*. 2012;21: 63-70. doi: 10.1016/j.tranpol.2012.01.015.

[3] Kamel I, Shalaby A, Abdulhai B. A modelling platform for optimizing time-dependent transit fares in large-scale multimodal networks. *Transport Policy*. 2020;92: 38-54. doi: 10.1016/j.tranpol.2020.04.002.

[4] Yook D, Heaslip K. Determining appropriate fare levels for distance-based fare structure: Considering users' behaviors in a time-expanded network. *Transportation Research Record*. 2014;2415(1): 127-135. doi: 10.3141/2415-14.

[5] Borndörfer R, Hoang ND. Fair ticket pricing in public transport as a constrained cost allocation game. *Annals of Operations Research*. 2015;226(1): 51-68. doi: 10.1007/s10479-014-1698-z.

[6] Zhang XQ, Liu D, Wang B. Pricing methods for express rail freight under multi-modal competition (in Chinese). *Transportation System Engineering and Information*. 2016;16(05): 27-32. https://kns.cnki.net/kcms/detail/detail.aspx?FileName=YSXT201605004&DbName=CJFQ2016 [Accessed 15th Oct. 2016].

[7] Liu M, Wang J. Pricing method of urban rail transit considering the optimization of passenger transport

structure (in Chinese). *Journal of Transportation Systems Engineering and Information Technology.* 2017;17(03): 53-59. https://kns.cnki.net/kcms/detail/detail.aspx?FileName=YSXT201703009&DbName=CJFQ2017 [Accessed 15th June 2017].

[8] Cheraghalipour A, Paydar MM, Hajiaghaei-Keshteli M. Designing and solving a bi-level model for rice supply chain using the evolutionary algorithms. *Computers and Electronics in Agriculture.* 2019;162: 651-668. doi: 10.1016/j.compag.2019.04.041.

[9] Hajiaghaei-Keshteli M, Fathollahi-Fard AM. A set of efficient heuristics and metaheuristics to solve a two-stage stochastic bi-level decision-making model for the distribution network problem. *Computers & Industrial Engineering.* 2018;123: 378-395. doi: 10.1016/j.cie.2018.07.009.

[10] Liu XW. *Research on urban rail transit fare optimization method based on elastic demand.* PhD thesis. Beijing Jiaotong University; 2016.

[11] Fallah Tafti M, Ghane Y, Mostafaeipour A. Application of particle swarm optimization and genetic algorithm techniques to solve bi-level congestion pricing problems. *International Journal of Transportation Engineering*. 2018;5(3): 261-273. doi: 10.22119/ijte.2018.47767.

[12] Hao P, Cheng X. Application of particle swarm algorithm in railroad double-layer planning model solving (in Chinese). *Computer Knowledge and Technology*. 2017;13(26): 238-239+242. https://kns.cnki.net/kcms/detail/detail.aspx?FileName=DNZS201726106&DbName=CJFQ2017 [Accessed 15th Sep. 2017].

[13] Zong HX. *Research on cooperative flow limiting model and algorithm for multiple entry gates of subway during peak period.* PhD thesis. Beijing Jiaotong University; 2020.

[14] Zhang Y, Li X. Research on the calculation method of average distance of urban rail transportation (in Chinese). *Heilongjiang Transportation Science and Technology.* 2018;41(05):159-160. https://kns.cnki.net/kcms/detail/detail.aspx?FileName=HLJJ201805096&DbName=CJFQ2018 [Accessed 15th May 2018].

[15] Beijing Institute of Transportation Development. *Annual report on transportation development in Beijing.* 2020. https://www.bjtrc.org.cn/List/index/cid/7.html [Accessed 1st July 2020].

[16] Zhuang Y. *Research on urban public transportation time differential pricing model*. PhD thesis. Southeast University; 2016.

[17] China Urban Rail Transit Association. *Urban Rail Transit 2019 Annual Statistics and Analysis Report.* 2020. https://www.camet.org.cn/tjxx/5133 [Accessed 7th May 2020].