Examining the Impact of Hysteresis on the Projected Adoption of Autonomous Vehicles

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ABSTRACT
This study explores the potential impact of per capita gross domestic product (GDP) changes on the adoption of autonomous vehicles (AVs). The level of adoption of AVs is anticipated to influence the benefits of future mobility, prompting numerous studies that forecast the market share of AVs using various methods. The influence of changes in the per capita GDP on vehicle ownership is crucial in assessing the challenges associated with reducing dependence on AVs in the future. This phenomenon, known as the hysteresis effect, implies that AV adoption estimates may differ when the GDP is rising as opposed to when it is falling. This research examines the effect of rising and falling GDP per capita on the anticipated AV diffusion in Hungary, utilising a scenario-based method to account for the variation in adoption rates in the literature. The study findings indicate that declines in GDP in the past will impact AV ownership, leading to a shift in future adoption patterns. The AV market is projected to reach saturation in the 2070s and the 2090s in favorable and moderate scenarios, respectively, while a pessimistic state would delay this outcome until after the year 2100.

KEYWORDS
autonomous vehicle; GDP; hysteresis; forecasting; adoption; diffusion.

1. INTRODUCTION
Autonomous vehicles (AVs) drive independently without the driver’s direct intervention [1]. They operate using the data collected from their sensors and are not necessarily connected to other vehicles in the flow or to the infrastructure [2]. AVs are classified into five levels of automation, from Level 0 (i.e. ordinary human-driven cars) to fully automated Level 5 vehicles [3]. This research considers Level 5 AVs, which do not require the user to take over driving. The potential benefits of AVs (e.g. increasing safety levels, reducing road accidents and decreasing traffic congestion [4–8]) vary considerably by their penetration into the market [9]. Some researchers tend to create several scenarios to cover the possible cases of integrating AVs into road networks as this new technology is not yet entirely developed (see e.g. [10–13]), which consumes time and effort.

Predicting AV ownership rates benefits transport planners, researchers and policymakers. To forecast transport demand, it is necessary to study and understand the historical travel patterns, the response to adjustments due to expected and unexpected events and the economic factors [14, 15]. Thus, this research considers the question of hysteresis, or asymmetry, in response to falling or rising gross domestic product (GDP) per capita on the expected diffusion of AVs in Hungary.

Income is a primary player in vehicle ownership. In the late 20th century, the increment in income allowed vehicle ownership to boost in almost all industrialised and developing countries [16]. Hungary is no exception, vehicle ownership in 1980 was less than 100 [veh/1,000 capita] while in 2021 this figure became approximately four times higher, and per capita GDP SPPP has risen from $6,300 to nearly $37,000 over the same period [17–19]. However, J. M. Dargay pointed out that the relationship between income and vehicle ownership does not follow a constant behaviour over time, and it was found that the increment in vehicle ownership was more than it is in GDP per capita until the seventies, but this was not the case after [16]. The long-run income elasticity of car ownership started declining to nearly one in 1992 in the OECD countries and continued to fall thereafter. This is crucial to understand that, according to the term of necessary and luxury goods, vehicles currently
are found to be a necessity rather than being categorised as luxury tools like in the past [16, 20]. The same case is anticipated with AV technology implementation. Many researchers consider AVs as luxury goods, e.g. [21]; however, once an AV is acquired, like other vehicle types, it becomes a necessary tool and difficult to dispose.

The concept of hysteresis, alongside the notion of GDP and ownership elasticity, explores the relationship between vehicle ownership and GDP fluctuations. This analysis investigates whether the decline in GDP leads to a proportional decrease in vehicle ownership and if the subsequent increase in GDP yields an equivalent rise in ownership. Thus, studying the hysteresis effect on AV adoption is essential in determining the difficulty of reducing dependence on AVs in the future.

In this study, we examine the effect of rising and falling GDP per capita on the anticipated AV diffusion rates in Hungary by a model that considers the hysteresis effect using the Gompertz function, and we develop the model using the method of proportion coefficient in order to predict the relationship between per capita GDP and AV adoption over time. We focus on GDP per capita as the sole predictor of AV adoption in this research to assess its individual impact and provide valuable insights into the relationship between economic factors and AV adoption patterns. This focused approach allows for a specific examination of the role of economic conditions in shaping the adoption dynamics of AVs. We investigate three AV growth scenarios: pessimistic, moderate and optimistic scenarios. Moreover, we estimate Hungary’s annual GDP per capita using the ARIMA model. The following section presents the literature review, next we discuss the methodology and data used to undertake this study. The section after presents the results, followed by the conclusion.

2. LITERATURE REVIEW

This study investigates the effect of rising and falling GDP per capita on the anticipated AV diffusion rates in Hungary using human-driven passenger vehicle trends from 1980 to 2021. To provide a comprehensive analysis, a thorough literature review is conducted to enhance the understanding of the association between AV adoption and hysteresis.

2.1 AV adoption

Driverless cars captured the attention of scientists, policymakers and auto companies due to the benefits they are expected to introduce, thus there are numerous studies that forecasted the adoption of this emerging technology in various methods and study cases. These AV adoption studies are widely held based on the estimation of sales, adoption patterns of other vehicle-related technologies and knowledge of professionals and experts [22]. Trommer et al. predicted the potential share of Level 4 and 5 AVs for the year 2035 in the US and Germany [17]. They developed a vehicle diffusion model to model two scenarios that differ by the initial implementation year and AV integration rate. Litman predicts that AVs are anticipated to form 40% of the car market share in 2050 based on past automobile technologies adoption trends [9]. The experts at The Institute of Electrical and Electronics Engineers (IEEE) introduced a study suggesting that the penetration rate of automated automobiles will reach a share of 75% in 2040 of the total vehicle market [22].

The aforementioned methods are the most used in the research; however, some researchers applied other methods and models. Some studies employed analytical modelling to predict the market share of AVs [23, 24]. Several studies also utilised the discrete choice modelling approach to understand AVs future demand trends [25–27], and additional researchers deployed more advanced models to estimate AV adoption using the diffusion of innovation (DOI) theory such as Lavasani et al. who employed the Bass diffusion model [28], and Talebian & Mishra who proposed a new approach to estimate the long-term adoption of AVs by coupling agent-based modelling with the theory of DOI [22].

A graphical map of keywords was performed in order to comprehensively investigate the bibliographic data associated with AV ownership predictions. Using the Web of Science (WoS) database, a co-occurrence network was built from 766 publications related to AVs and their adoption. Created using VOSviewer software [29], the network in Figure 1 visualises the relations amongst a set of 75 keywords from the selected articles, and they are represented in clusters with different colours to indicate the relatedness among them. Only keywords with a minimum of two occurrences and showing connections with other keywords are included in the network. The sphere size indicates the number of keyword occurrences and the links between them represent the relatedness strength [30]. The network diagram has a total of six clusters and 668 links with a total link strength of 918, and it depicts a close association between the adoption of AVs and user and public acceptance, innovations,
preferences, opportunities and others. However, the hysteresis effect on AV ownership caused by per capita GDP changes appears not to be taken under consideration in the adoption methods used in the literature.

Figure 1 – Co-occurrence network of keywords related to the adoption of AVs

2.2 Hysteresis

The term hysteresis is defined as “permanent effects of a temporary stimulus”, which originally comes from physics and magnetisation [31]. For example, Figure 2 illustrates the demand curve $D_R$, in which car ownership increases from $C_0$ to $C_1$ if the individual yearly income rises from $Y_0$ to $Y_1$, and if it falls back to $Y_0$ then the car ownership rate returns to $C_0$. This means that the demand curve $D_R$ is reversible based on the change in income. However, if the hysteresis effect is considered, the demand curve $D_R$ will not be entirely reversible but might follow the behaviour of the demand curve $D_f$, where a fall in the income from $Y_1$ to $Y_0$ would reduce

Figure 2 – Demand function with hysteresis effect (adopted from [16])
car ownership from $C_1$ to $C_2$, not to $C_0$, meaning that the demand is not completely reversed due to the fall in income. As a result, in contrast to most traditional demand analyses, the demand and income relationship does not follow a unique equilibrium relationship as seen in the $D_R$ curve. Instead, they follow the kinked curve of demand $D_R-D_F$, which is also known as a "hysteresis loop" [16].

VOSviewer was also used to thoroughly detect the literature related to hysteresis in the transportation field. The co-occurrence network in Figure 3 consists of 180 keywords from 229 publications from the WoS database and it has a total of 17 clusters and 1,123 links with a link strength of 1,244. Only keywords with a minimum of one occurrence and having a connection with other keywords were incorporated into the network. The network shows a strong connection between hysteresis and traffic flow, fundamental diagram and car following models, however, no connection was found with AV ownership.

According to the aforementioned literature and due to the lack of studies that investigate the hysteresis effect on AV adoption rates this research was performed. It examines the effect of rising and falling GDP per capita on the anticipated AV diffusion in Hungary using human-driven passenger car trends from 1980 to 2021.

Figure 3 – Co-occurrence network of keywords related to hysteresis in the transportation field

3. METHODOLOGY AND DATA

This section discusses the methodological framework for exploring the effect of rising and falling GDP per capita on the anticipated AV diffusion. It also presents the historical records of civilian vehicles in Hungary and their pattern over time, which will be utilised in the estimation process.

3.1 Methodology

In this research, we employed the Gompertz function to estimate the adoption of AV ownership in Hungary, as this function is relatively more flexible than the logistic model [32, 33]. The Gompertz model was named after Benjamin Gompertz in 1825 and was first deployed in biology. The model became, after approximately a century, more popular and was used to produce the life cycle analysis [34]. The general mathematical form of the Gompertz function is as follows:

$$A_t = \lambda \cdot e^{-ae^{-\beta t}} \quad \alpha, \beta > 0$$

where $A_t$ and $G_t$ are long-term vehicle ownership [veh/1,000 capita] and GDP per capita in year $t$, respectively, $\lambda$ is the upper limit of $A_t$ (i.e. the saturation level), $\alpha$ and $\beta$ are the Gompertz parameters that define the curvature of the function. A simple mechanism of partial adjustment was proposed by Dargay and Gately to consider the lags in the vehicle ownership adjustment to GDP and to investigate the question of hysteresis resulting from...
increasing and decreasing income [34]. The mechanism used is presented in Equation 2

\[ A_t = A_{t-1} + \delta \cdot (A_t - A_{t-1}) \quad 0 < \delta < 1 \]  

(2)

where \( A_t \) is the actual vehicle ownership [veh/1,000 capita] and \( \delta \) is the adjustment speed. By substituting the general form of the Gompertz formula shown in Equation 1 into the partial adjustment mechanism equation shown in Equation 2, we obtain the following equation:

\[ A_t = \lambda \cdot e^{-\alpha e^{-\beta G_t}} + \left( 1 - \delta \right) \cdot A_{t-1} \]  

(3)

In order to consider the asymmetric response of vehicle ownership rates to the rise and fall in per capita GDP, we allowed for the adjustment coefficient related to raising GDP (\( \delta_R \)) to be different than the adjustment coefficient when the GDP is falling (\( \delta_F \)) [16]. To deploy that into Equation 3, two dummy variables were defined:

\[ \delta = \delta_R \cdot F_t + \delta_F \cdot R_t \]  

(4)

if \( G_t - G_{t-1} > 0 \) then \( R_t = 1 \) otherwise \( = 0 \)

if \( G_t - G_{t-1} < 0 \) then \( F_t = 1 \) otherwise \( = 0 \)

By substituting Equation 4 into Equation 3, the modified Gompertz function that considers hysteresis will be as follows:

\[ A_t = \lambda \cdot \left( \delta_R \cdot F_t + \delta_F \cdot R_t \right) \cdot e^{-\alpha e^{-\beta G_t}} + \left( 1 - \left( \delta_R \cdot F_t + \delta_F \cdot R_t \right) \right) \cdot A_{t-1} \]  

(6)

The long-term GDP elasticity of car ownership is the same as in symmetrical models and can be determined as follows:

\[ E = \alpha \cdot \beta \cdot G_t \cdot e^{-\beta G_t} \]  

(7)

The short-term GDP elasticity, on the other hand, is affected by the speed of adjustment parameter [34] and is formulated as follows:

\[ E = \alpha \cdot \beta \cdot \left( \delta_R \cdot F_t + \delta_F \cdot R_t \right) \cdot G_t \cdot e^{-\beta G_t} \]  

(8)

Figure 4 shows the relation between car ownership levels and per capita GDP, which follows an S-shaped curve. The curve is segmented into four phases; (1) slow increase in car ownership rates, (2) fast growth, (3) slower growth compared with stage 2 and (4) the saturation state.

The inflection point in the elasticity curve occurs at a GDP level of \( \ln(\alpha)/\beta \), which is the maximum value for elasticity (i.e. maximum ownership rate) representing the transition point between the first and last two stages in the Gompertz curve. It can be noticed that the curve follows two mathematical behaviours around the inflection point, see Equation 9. In this regard, initial AV predictions from the literature can be incorporated into the human-driven ownership model using the method of proportion coefficient in a quadratic curve fitting in the accelerating growth stage and a logarithmic curve fitting in the slower growth stage. A Hungarian AV Gompertz curve can then be acquired considering the goodness of fit to the composite (i.e. quadratic and logarithmic) function.
As aforementioned, it is proved that the growth model of motorisation follows an S-shaped curve \[9, 22, 32, 34, 35\]. Thus, the ownership growth model of different types of automobiles, e.g. internal combustion engine vehicles, hybrid vehicles, electric vehicles and even autonomous vehicles, can be represented by the S-shaped curve. This research examined the effect of rising and falling GDP per capita on the anticipated AV adoption rates in Hungary and developed a model using the method of proportion coefficient in order to predict the relationship between per capita GDP and AV adoption over time.

### 3.2 Data

Civilian car market in Hungary is expected to reach a saturation level \((\lambda)\) of 570.2 [veh/1,000 capita] \[36\]. Figure 5 demonstrates the vehicle ownership rates and GDP per capita for Hungary from 1980 to 2021 \[17–19\].

Based on the collected statistical data shown in Figure 5, \(\alpha\) and \(\beta\) parameters were estimated using the ordinary least square method (OLS) method. We would expect that \(\delta_R\) is larger than \(\delta_F\) because vehicle ownership decrement pace when GDP falls is likely to be slower than the increment pace when GDP increases. The statistical optimisation tool, MS Excel Solver, was employed to estimate the adjustment parameters of the growth equation. The objective function was defined to minimise the discrepancy between the observed data and the predicted values generated by the growth equation. The optimisation algorithm iteratively adjusted the parameter values to find the optimal configuration that minimised the sum of squared residuals or maximised the likelihood of the observed data given the growth equation. By employing this statistical optimisation approach, the obtained speed adjustment parameters, \(\delta_R=97.7\%\) and \(\delta_F=33.5\%\), provided a goodness of fit, using the coefficient of determination \((R^2)\), of 0.94 to the real vehicle ownership data, which makes this model an adequate tool for analysing and forecasting future vehicle ownership growth patterns.

The growth model of vehicle ownership in Hungary based on per capita GDP considering the hysteresis effect can be described in Equation 10, where \(AR\) is the estimated vehicle ownership in rising GDP and \(AL\) is the estimated ownership in falling GDP. A graphical representation of the real and estimated car ownership trends is presented in Figure 6. The enlarged view in the figure highlights the presence of a hysteresis effect in the real vehicle ownership curve, with the estimated growth model successfully capturing this behaviour.

\[
AR = 570.2 \cdot (0.977) \cdot e^{-1.977e^{-0.016t}} + (1 - 0.977) \cdot AR_{t-1}
\]

\[
AL = 570.2 \cdot (0.335) \cdot e^{-1.977e^{-0.016t}} + (1 - 0.335) \cdot AL_{t-1}
\]

\[ (9) \]

\[ (10) \]
Knowing the value of $\alpha$ and $\beta$ parameters, the inflection point for the Hungarian car ownership model occurred at $13,687$ per capita GDP $\text{PPP}$. Figure 7 displays the elasticity of real car ownership data in Hungary based on GDP per capita and the long and short-term estimated elasticities. It is important to emphasise that the asymmetric response is related to the short-term GDP elasticity, while the long-term elasticity is symmetric.

It is important to clarify that Figure 6 and Figure 7 do not depict a time series; rather, they represent the variation in vehicle ownership based on the GDP per capita level. Multiple years can exhibit the same range of GDP values. When there is a decrease in GDP for a specific year or a consecutive set of years, followed by subsequent improvement, a noticeable “dent” in the curve may be observed. To illustrate, Table 1 presents the GDP per capita values for the years 1985 to 1995 in Hungary. It is evident that the per capita GDP showed an increasing trend until 1990, then experienced a sudden decline in the subsequent two years and began to rise thereafter. As a consequence of the decline in GDP, a corresponding decrease in vehicle ownership is expected, which is indeed observed. However, the decline in vehicle ownership does not align with the decline in GDP. As a result, occurrences can appear where different years with the same GDP show varying levels of vehicle ownership. For example, both the years 1987 and 1991 had an approximate GDP per capita level of $10,000$, yet they displayed vehicle ownership values of $157.95$ and $194.30$ [veh/1,000 capita], respectively.
Table 1 – Illustrative sample of vehicle ownership values and GDP per capita levels in Hungary [17–19]

<table>
<thead>
<tr>
<th>Year</th>
<th>Vehicle ownership</th>
<th>GDP per capita SPPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>135.48</td>
<td>9002.82</td>
</tr>
<tr>
<td>1986</td>
<td>145.73</td>
<td>9359.46</td>
</tr>
<tr>
<td>1987</td>
<td>157.98</td>
<td>10027.60</td>
</tr>
<tr>
<td>1988</td>
<td>171.02</td>
<td>10419.13</td>
</tr>
<tr>
<td>1989</td>
<td>166.24</td>
<td>10952.12</td>
</tr>
<tr>
<td>1990</td>
<td>187.43</td>
<td>11013.71</td>
</tr>
<tr>
<td>1991</td>
<td>194.30</td>
<td>10033.76</td>
</tr>
<tr>
<td>1992</td>
<td>198.41</td>
<td>9947.49</td>
</tr>
<tr>
<td>1993</td>
<td>201.80</td>
<td>10132.99</td>
</tr>
<tr>
<td>1994</td>
<td>210.33</td>
<td>10669.92</td>
</tr>
<tr>
<td>1995</td>
<td>217.22</td>
<td>11184.85</td>
</tr>
</tbody>
</table>

4. RESULTS

The results in this section present the future projections of civilian vehicle ownership rates in Hungary. They also illustrate the designed composite functions as well as the optimised Gompertz models that forecast the adoption rates of AVs based on GDP per capita and considering the hysteresis effect.

4.1 Future projections of passenger vehicle ownership rates

After discovering the growth function for private car ownership by using the historical data between 1980 and 2021 in Hungary and considering the hysteresis effect, we can apply Equation 10 to predict the future projections of passenger vehicle ownership rates. Nonetheless, Hungary’s GDP per capita estimation is required to carry out the growth model and the analysis considering these findings. To forecast the Hungarian GDP per capita, the autoregressive integrated moving average ARIMA (1,1,1) model was used taking into account its controlling factors, i.e. autocorrelation plot (ACF), partial autocorrelation plot (PACF), stationarity of the time series. Figure 8 shows the estimation of civilian vehicle ownership rates in Hungary. The figure also indicates that the Hungarian car market will reach saturation initially in the year 2070 at 570 [veh/1,000 capita] which is associated with a GDP per capita of approximately $110,000.
4.2 Formulating the composite functions

There are numerous studies that have forecasted the adoption of AV emerging technology by using various methods. Some studies anticipated that automated cars will form 1–2% of the car market share in 2030, 20% and 40% in 2040 and 2050, respectively [9]. More optimistic expectations found that the penetration rate of autonomous cars will reach a share of 75% in 2040 and a share of 90% in 2050 [22, 37]. As a result, we considered three distinct scenarios based on the literature to examine and model Hungary’s future AV adoption rates considering hysteresis in the per capita income, as shown in Table 2.

<table>
<thead>
<tr>
<th>Table 2 – The investigated scenarios to estimate Hungary’s AV adoption rates (%)</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pessimistic scenario (Scenario 1)</td>
<td>1.0</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>Moderate scenario (Scenario 2)</td>
<td>1.5</td>
<td>47.5</td>
<td>65</td>
</tr>
<tr>
<td>Optimistic scenario (Scenario 3)</td>
<td>2.0</td>
<td>75</td>
<td>90</td>
</tr>
</tbody>
</table>

Following the concept of necessary and luxury goods, vehicles are currently considered to be a necessity rather than being categorised as luxury tools, opposite to what they were four decades ago [16, 20]. The same situation is expected with AV technological adoption, where many studies consider AVs as luxury goods, however, once an AV is acquired, like other vehicle types, it becomes a necessary tool and difficult to dispose. Thus, similarly to civilian car adoption before, the inflection point for Hungary’s AV ownership is estimated to exist after a climb of nearly $14,000 in the per capita GDP. According to the estimations of the ARIMA model, the yearly income is expected to be approximately $57,000 and $71,000 in 2030 and 2040, respectively, which means that the ownership of AVs will be in the accelerating growth segment in this period, and in the decelerating growth stage just after 2040. As a result, the growth behaviour of AV ownership between 2030 and 2040 is expected to be quadratic and to follow a logarithmic behaviour after 2040. Considering the proportion coefficient method, AV diffusion can be estimated as follows:

\[ AV_t = \begin{cases} q_s \cdot A_t, & \text{if } G_t < \text{inflection point} \\ l_s \cdot A_t, & \text{if } G_t > \text{inflection point} \end{cases} \]  

where \( AV_t \) and \( A_t \) are AV ownership and civilian vehicle ownership [veh/1,000 capita] in year \( t \), respectively, \( q_s \) is AV occupation ratio in the fast growth stage for scenario \( s \), and \( l_s \) is AV occupation ratio in the stage of slow growth. The growth models for all scenarios can then be summarised by the composite functions in Equations 12–17.

**Scenario 1**

2030–2040:

\[ AV_t = \left( 0.01 + 0.72 \cdot 10^{-9} \cdot (G_t - 57,000)^2 \right) \cdot 570.2 \cdot e^{-1.979 \cdot e^{-0.4971 \cdot 10^{-9} G_t}} + (1 - \delta) \cdot A_{t-1} \]  

2040–2050:

\[ AV_t = \left( 1.19 \cdot \ln(G_t) - 13.13 \right) \cdot 570.2 \cdot \delta \cdot e^{-1.979 \cdot e^{-0.4971 \cdot 10^{-9} G_t}} + (1 - \delta) \cdot A_{t-1} \]  

**Scenario 2**

2030–2040:

\[ AV_t = \left( 0.015 + 1.77 \cdot 10^{-9} \cdot (G_t - 57,000)^2 \right) \cdot 570.2 \cdot \delta \cdot e^{-1.979 \cdot e^{-0.4971 \cdot 10^{-9} G_t}} + (1 - \delta) \cdot A_{t-1} \]  

2040–2050:

\[ AV_t = \left( 1.04 \cdot \ln(G_t) - 11.19 \right) \cdot 570.2 \cdot \delta \cdot e^{-1.979 \cdot e^{-0.4971 \cdot 10^{-9} G_t}} + (1 - \delta) \cdot A_{t-1} \]  

**Scenario 3**

2030–2040:

\[ AV_t = \left( 0.02 + 2.83 \cdot 10^{-9} \cdot (G_t - 57,000)^2 \right) \cdot 570.2 \cdot \delta \cdot e^{-1.979 \cdot e^{-0.4971 \cdot 10^{-9} G_t}} + (1 - \delta) \cdot A_{t-1} \]  

2040–2050:

\[ AV_t = \left( 0.89 \cdot \ln(G_t) - 9.25 \right) \cdot 570.2 \cdot \delta \cdot e^{-1.979 \cdot e^{-0.4971 \cdot 10^{-9} G_t}} + (1 - \delta) \cdot A_{t-1} \]  

where \( \delta = R \cdot \delta_s + F \cdot \delta_F \). Figure 9 demonstrates the composite growth model for AV ownership levels in Hungary considering the rising and falling income, calculated using Equations 12–17. The growth behaviour of the composite model depicted in the figure showcases a quadratic increase in AV ownership until reaching the inflection point at a GDP per capita level of approximately $57,000, corresponding to the year 2030. Subsequently, the growth model transitions into a logarithmic pattern after an incremental rise of approximately $14,000 in the GDP level. This composite model predicts the growth of...
AV ownership, simulating both the accelerating and the decelerating growth segments in an S-shaped growth curve. However, additional optimisation is needed to better align the composite model with the S-shaped trajectories observed in the Gompertz function, which will enhance the representation of the patterns and dynamics of the adoption models.

Further optimisation was performed to better represent the composite adoption models and match the S-shaped curves of the Gompertz function. We fitted the composite function (i.e. quadratic and logarithmic) into the Gompertz model considering the minimum squared error depending on the $\alpha$ and $\beta$ parameters. Thus, Hungary’s AV growth models, considering a speed adjustment of 97.7% in the increasing GDP and 33.5% in the falling GDP, as obtained in section 3.2, can be summarised in Equation 18.

$$AV_t = \begin{cases} 
570.2 \cdot (R_t \cdot \delta_R + F_t \cdot \delta_F) \cdot e^{-130.19 \cdot e^{-4.81 \cdot 10^{-4} \cdot t}} + (R_t \cdot \delta_R + F_t \cdot \delta_F) \cdot A_{t-1}, & \text{Pessimistic} \\
570.2 \cdot (R_t \cdot \delta_R + F_t \cdot \delta_F) \cdot e^{-430.52 \cdot e^{-5.31 \cdot 10^{-4} \cdot t}} + (R_t \cdot \delta_R + F_t \cdot \delta_F) \cdot A_{t-1}, & \text{Moderate} \\
570.2 \cdot (R_t \cdot \delta_R + F_t \cdot \delta_F) \cdot e^{-6099.73 \cdot e^{-12.9 \cdot 10^{-4} \cdot t}} + (R_t \cdot \delta_R + F_t \cdot \delta_F) \cdot A_{t-1}, & \text{Optimistic} 
\end{cases}$$

The accuracy of this optimisation was tested using several evaluation metrics. The selection of a specific metric to demonstrate accuracy can significantly influence the decisions about the overall performance of the method or data used [38]. Thus, Table 3 shows several optimisation performance measurements compared to the data from the composite function. Mean absolute error (MAE), mean absolute percentage error (MAPE) and root mean square error (RMSE) are well-known error metrics that range from 0 (best value) to $+\infty$ (worst value). Although the values of these metrics in the table are relatively small, it is evident that it is challenging to interpret them because they have an upper limit of $+\infty$, not a determined maximum value. The coefficient of determination ($R^2$) and symmetric mean absolute percentage error (SMAPE), on the other hand, have real upper limits. The positive rates of $R^2$ range between 0 and 1, with 0 meaning poor fitting, and 1 showing perfect fitting between the two curves. SMAPE values range between 0 and 200%, where 0 depicts a perfect fitting and 200% indicates the worst fitting possible [39]. Table 3 reveals that the three scenarios have an $R^2$ of higher than 0.96, and their SMAPE values do not exceed 15.60%, meaning that the optimised Gompertz model shown in Equation 18 is a very good fit for the composite function presented in Equations 12–17.

It is worth mentioning that we assumed the market saturation of automated cars would yield to 100% share of the motorisation sector due to their benefits and the introduced gains for the public and the overall transportation system.

Figure 10 indicates that in optimistic circumstances, 90% of Hungary’s passenger vehicles will be automated at a GDP of $85,000 and 100% at a GDP of approximately $111,000, corresponding to the year 2072. In a mild scenario, AVs will likely form 50% of Hungarian motorisation at a per capita income of $78,000 and will reach saturation in the 2090s. On the other hand, as per a pessimistic situation, Hungary is expected to reach a

\[\text{Figure 9 – Estimated AV ownership rates in Hungary based on per capita income}\]
market penetration of 50% of AVs in 2053, a percentage in moderate circumstances is estimated to occur nine years earlier. Moreover, the pessimistic scenario curve shows that a full emergence of AVs in Hungary will not happen before the year 2100. The data in Figure 11 also showcase the annual growth of automated car ownership per 1,000 capita in Hungary for the three different scenarios, which assists in understanding the deploying behaviour of this emerging technology over time.

Table 3 – Accuracy measures between the composite function and the optimised scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>MAE</th>
<th>MAPE</th>
<th>RMSE</th>
<th>SMAPE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pessimistic (Scenario 1)</td>
<td>7.76</td>
<td>9.92%</td>
<td>10.28</td>
<td>9.62%</td>
<td>0.9839</td>
</tr>
<tr>
<td>Moderate (Scenario 2)</td>
<td>19.40</td>
<td>17.54%</td>
<td>24.66</td>
<td>15.60%</td>
<td>0.9638</td>
</tr>
<tr>
<td>Optimistic (Scenario 3)</td>
<td>19.13</td>
<td>13.16%</td>
<td>23.98</td>
<td>11.85%</td>
<td>0.9831</td>
</tr>
</tbody>
</table>

AV ownership elasticities based on annual incomes follow a basic nonmonotonic pattern; going higher over low GDP per capita levels until reaching the inflection point, then decreasing over higher levels of income, but always remaining positive. Figure 12 presents the GDP elasticities of AV adoption in the long and the short run, where the long-run elasticity, as shown in Equation 7, is alike in symmetric models. In contrast, the short-run elasticity has a slightly lower inflection point than the long-run elasticity and has an asymmetric behaviour. This indicates that the adoption rate of AVs is certainly affected by the drop in GDP (i.e. the hysteresis effect), meaning that some non-AV owners who are considered potential buyers for the new technology will not consider buying AVs in the short run, or AV owners will sell their vehicles, or they will move to shared autonomous vehicle models. This shift in the adoption trends can be further investigated in future research.
5. CONCLUSION

The hysteresis effect on vehicle ownership caused by per capita GDP changes appears not to be taken under consideration in the AV adoption forecasting methods used in the literature. Therefore, this research examined the effect of rising and falling GDP per capita on the anticipated AV diffusion rates in Hungary using passenger car trends from 1980 to 2021 and the method of proportion coefficient. The short-run GDP elasticity to AV ownership showed a slightly lower inflection point than the long-run elasticity, which implied that the adoption rate of AVs is certainly affected by the drop in GDP causing a shift in the adoption trends.

Three distinct scenarios were adopted in this research: pessimistic, moderate and optimistic scenarios to account for the variation in the forecasted adoption rates in the literature. We found that the Hungarian auto market will reach saturation by AVs in the 2070s and 2090s in optimistic and moderate circumstances, respectively, and not before 2100 in a pessimistic state.

In this study, a mathematical approach is proposed to forecast the market penetration rate and saturation year of AVs. However, the approach has some limitations that should be noted. Firstly, the Level 5 automation technology is used for simplification and generalisation, and it is not distinguished from Level 4 automation technology, although both are considered to be highly advanced technologies. Moreover, Level 3 automation technologies are not included in the analysis since the driver is expected to occasionally take control of the car in specific circumstances, which significantly reduces the potential benefits of Level 3 automation. Additionally, this research focuses on GDP per capita as the sole predictor of AV adoption to assess its individual impact and provide valuable insights into the relationship between economic factors and AV adoption patterns. However, other factors beyond GDP per capita, such as infrastructure development, regulatory frameworks, cultural attitudes and technological readiness may also influence AV adoption. Therefore, future research may consider incorporating these additional variables to provide a more comprehensive understanding of the adoption process. Moreover, the data used in the study, such as vehicle ownership and GDP, are discrete data points, but a continuous regression model is employed to forecast future ownership rates. This decision is made because car ownership rates are represented based on GDP rather than as a time series, which makes it easier to comprehend the results of the model. Furthermore, the scenarios developed in this research do not consider the potential significant impact that AVs may have on the vehicle market saturation level, which could be a subject for future research.

The assumptions and the applied parameters can be effortlessly modified based on different case studies, and they can be updated due to technological breakthroughs in the field. Furthermore, the methodological framework of this study with the data collected and its initial predictions of the case study can be further developed in future studies using system dynamics, which offer a full system approach that considers time lags and feedback between the system’s elements.

REFERENCES


دراسة تأثير التباطؤ في الناتج المحلي الإجمالي للفرد على نسبة الاقتناء المتوقعة للمركبات ذاتية القيادة في المستقبل

الملخص:
تبحث هذه الدراسة في التأثير المحتمل للتغير الحاصل على الناتج المحلي الإجمالي للفرد على نسبة اقتناء المركبات ذاتية القيادة. بينما يؤكد أن نسبة اقتناء المركبات ذاتية القيادة تتأثر نظرًا بفوائد التنقل، الأمر الذي يحفظ العديد من الدراسات التي تتنبأ بالسوق السري للمركبات ذاتية القيادة بطرق تنبؤ مختلفة. يُؤثر التباطؤ والارتفاع في نصيب الفرد عن الناتج المحلي الإجمالي بشكل أساسي على نسبة اقتناء هذا النوع من المركبات. كما تظهر هذه الدراسة أن الناتج المحلي الإجمالي متقلبيًا بشكل كبير إلى نسبة اقتناء المركبات ذاتية القيادة. يتقلب الناتج المحلي الإجمالي للفرد بشكل كبير ضمن النطاق المتوقع لل sistem المركبات ذاتية القيادة في المستقبل. تشير هذه الدراسة إلى أن التباطؤ في الناتج المحلي الإجمالي يؤثر بشكل كبير على اقتناء المركبات ذاتية القيادة. يُبرز هذا التأثير الرافع للانخفاض، بدوره يُرجح لازمًا أكثر في النقطة بين الناتج المحلي الإجمالي والتأثير في التباين في معدلات الاقتناء في الدراسات السابقة. تشير هذه الدراسة إلى أن التأثير المحتمل للتغير في الناتج المحلي الإجمالي في مسار تقاعدي والمسار الحقيقي. في مسار أقل تفاوتًا، فإن الناتج المحلي الإجمالي يقل بشكل أكبر بين الناتج المحلي الإجمالي والتأثير في التباين في معدلات الاقتناء. من هذا الفصل، يمكن تقدير سوق المركبات ذاتية القيادة في المجر، باستخدام عدة مسارات لحساب الاقتناء في معدلات الاقتناء في الدراسات السابقة. تشير هذه الدراسة إلى أن التأثير المحتمل للتغير في الناتج المحلي الإجمالي في مسار تقاعدي والمسار الحقيقي. من هنا، يمكن تقدير طبقات ذاتية القيادة في المجر، باستخدام عدة مسارات لحساب الاقتناء في معدلات الاقتناء في الدراسات السابقة. تشير هذه الدراسة إلى أن التأثير المحتمل للتغير في الناتج المحلي الإجمالي في مسار تقاعدي والمسار الحقيقي. من هنا، يمكن تقدير طبقات ذاتية القيادة في المجر، باستخدام عدة مسارات لحساب الاقتناء في معدلات الاقتناء في الدراسات السابقة. تشير هذه الدراسة إلى أن التأثير المحتمل للتغير في الناتج المحلي الإجمالي في مسار تقاعدي والمسار الحقيقي. من هنا، يمكن تقدير طبقات ذاتية القيادة في المجر، باستخدام عدة مسارات لحساب الاقتناء في معدلات الاقتناء في الدراسات السابقة. تشير هذه الدراسة إلى أن التأثير المحتمل للتغير في الناتج المحلي الإجمالي في مسار تقاعدي والمسار الحقيقي. من هنا، يمكن تقدير طبقات ذاتية القيادة في المجر، باستخدام عدة مسارات لحساب الاقتناء في معدلات الاقتناء في الدراسات السابقة. تشير هذه الدراسة إلى أن التأثير المحتمل للتغير في الناتج المحلي الإجمالي في مسار تقاعدي والمسار الحقيقي. من هنا، يمكن تقدير طبقات ذاتية القيادة في المجر، باستخدام عدة مسارات لحساب الاقتناء في معدلات الاقتناء في الدراسات السابقة.