Traffic & Transportation

Capped Robust Optimal Control Method to Improve Tram Operation Reliability Considering Random Number of Passengers at Station Serving Multiple Lines

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Original Scientific Paper
Submitted: 13 June 2023
Accepted: 9 Aug. 2023

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Publisher:
Faculty of Transport and Traffic Sciences, University of Zagreb

ABSTRACT

Tram signal priority control is a crucial approach for enhancing the reliability of tram operations and has been implemented in various cities. Nevertheless, unpredictable tram operations influenced by tram dwell time during station stops can cause signal priority control failure at intersections. It is challenging to precisely predict tram dwell time at stations that offer multiple lines. To address this issue, the proposed research presents a capped robust optimal control (CRC) technique for tram signal priority. This method entails considering the stochastic number of passengers boarding and alighting at stations with multiple lines. Furthermore, tram delay calculation models at intersections are established and integrated into an objective function. The main objective of this strategy is to enhance tram operation reliability and maximise tram operation efficiency while reducing the adverse impact of tram priority on other vehicles at the intersection. A case study was conducted to evaluate the effectiveness of the CRC method. The results indicate that the CRC technique significantly improves tram operation reliability and efficiency.

KEYWORDS

tram control; robust optimal; operational reliability; public transit.

1. INTRODUCTION

The tram system operates along a dedicated lane within the general roadway, yet must share the right-of-way with other vehicles at intersections. This semi-exclusive privilege necessitates interactions between trams and other vehicles, compounded by stochastic dwell times at stations, resulting in an unpredictable tram operation. Implementing signal priority control mechanisms at intersections constitutes an efficacious avenue towards enhancing the appeal and reliability [1] of tram services. Signal priority control methods include passive and active priority strategies. Passive priority control [2] employs signal control schemes based on historical data without requiring detection equipment at intersections, and its specific techniques include cycle length reduction and phase splitting [3]. However, static signal control schemes often fail to accommodate variable traffic environments, negatively affecting priority control efficiency and tram operational performance. In contrast, active priority [4] utilises intersection detectors to capture vehicle information, thereby enabling the adjustment of signal control schemes via priority strategies such as green extension, red truncation or phase insertion, which are more widely applicable.

The concept of active priority encompasses two priority types, namely unconditional and conditional priority. The former confers priority to every vehicle belonging to the target type, but an excessive number of priority requests may interfere with signal coordination and disrupt other vehicles' operation [5–7]. Thus, conditional priority has been introduced and is predominantly applied in the realm of public transit priority control to enhance network and single-line reliability, as demonstrated by previous studies [8–11]. Historically, scholars have put forward various methods to improve bus service quality, including signal priority control for late buses proposed by Furth and Muller [12], and intersection signal timing adjustment based on traffic data collected by loop detectors and vehicle location devices, resulting in maximised bus service reliability, as discussed by Chow et al. [13]. However, conflicts often arise due to multiple conflicting priority requests associated with bus signal priority, which has prompted further research. For example, various novel approaches have been proposed to address this issue, such as the multi-agent transit signal priority (TSP) control method for the network level problem presented by Xu et al. [14], and the regional coordinated signal priority control method accounting for pedestrian and passenger delay on the traffic network, presented by Li et al. [15]. A majority of research on bus priority has been conducted within the framework of bus lanes, where space priority can reduce passenger delay while the effectiveness of time priority hinges on the automatic adaptation level and signal setting, as demonstrated by Farid Yashar et al. [16]. To more accurately depict actual public transit operations, stochastic programming models have been constructed, taking into account the randomness of public transit operations, as suggested by Truong et al. [17], and heuristic approaches have been employed to solve these models, as explained by Peña et al. [18] and Behbahani and Poorjafari [19].

In recent years, numerous studies have utilised signal priority control methods to enhance the operational efficiency and dependability of trams. These methods include adjusting signal timing at intersections, emphasising on some quality of service indicators [20], such as minimising total tram travel time, which are used to reduce negative impacts of signal priority control on other vehicles [21, 22], and decrease passenger waiting times [23]. As computer technology evolves, modern technologies have been incorporated into signal control methods. For instance, Guo and Wang [24] and Zhang et al. [25] employ reinforcement learning techniques to investigate signal priority control in modern trams, thereby decreasing the number of stops required by trams [26]. Since trams are semi-exclusive right-of-way vehicles, their operation reliability is influenced by dwell time at stations. To enhance the reliability of TSP under stochastic tram dwell time [27], schedule compliance is employed as a reliability index to optimise multi-period tram schedules by adjusting signal timing at intersections [28].

Accurately predicting tram dwell time has been shown to enhance the reliability of tram operations through various studies such as those conducted by Wang et al. [29] and Liu et al. [30]. To predict tram dwell time at stations, Yang et al. [31] proposed a support vector machine model. Furthermore, Zhang and Guo [32] developed simultaneous tram schedule and dispatch optimisations to achieve multiple objectives, including minimising tram operation time. Zhou et al. [33] designed an integrated optimisation model based on tram schedule and signal priority to minimise delay between trams and other vehicles caused by extending or inserting green phases for trams at intersections. Additionally, coordinated signal control can decrease the number of vehicle stops, and Bai et al. [34] presented a coordinated signal control model for continuous intersections that accounts for unique features of both trams and vehicles. This model is applicable to tram signal priority control at non-stop intersections.

After conducting the aforementioned analysis, the majority of the research on tram signal control appears to disregard the stochastic nature of tram dwell time, which is a critical factor that influences the dependability of tram operations, ultimately affecting the effectiveness of signal priority control at intersections. Thus, it is essential to incorporate the stochastic tram dwell time when administering tram signal priority control. Additionally, in several studies, the number of passengers present at public transit stations is indeterminate, while this information is available through the swiping rules at tram stations for most trams. Thus, integrating the precise passenger count at the station during the prediction method of tram dwell time can significantly enhance the accuracy of dwell time calculation. Lastly, most of the research that pertains to tram signal priority control focuses on a solitary line, resulting in scenarios where a station may serve multiple lines being overlooked. Hence, there is a need to develop a signal priority control method for tram dwell time at stations that service multiple lines. The main contributions of this research are as follows:

1) To enhance the reliability of tram operations, new tram delay calculation models are formulated that consider stochastic tram dwell time at intersections where there is an upstream station serving multiple tram lines.
2) Using the newly formulated tram delay calculation models, a capped robust optimisation control method for tram signal priority is proposed. This method is designed to optimise tram operational efficiency while ensuring normal traffic operation at the intersection.
3) The effectiveness of the proposed tram delay calculation models is verified through simulation experiments using actual intersection data. The results of the experiments demonstrate the effectiveness of the method proposed in this paper.

The rest of this paper is organised as follows. The problem to be solved in this paper is described in Section 2. Section 3 formulated tram delay calculation models as function models for Section 4, which proposed the objective function and control process of the CRC method. In Section 5, the validation and availability of the CRC method are proven by a case study. The conclusion of this paper is given in Section 6.

2. PROBLEM DESCRIPTION AND ASSUMPTIONS

To make transfers more convenient and facilitate a safe access to stops by crosswalks, tram stations are typically located near intersections. This study focuses on the scenario where the tram station is located upstream of the intersection. However, to allow sufficient time for signal control adjustments, the tram detector must be placed far upstream of the tram station, as depicted in *Figure 1*. The tram lane is a closed lane, free from disturbances from other vehicles on the road, and is regulated by signals at intersections. The lane is a medium-sized lane, and the station is positioned in the middle of the road. Passengers enter and exit the station through the crosswalk at the intersection. The detector placed upstream of the station detects the time when the tram arrives at the detector. Additionally, the passenger swiping gate accurately records the number of passengers who swipe in and out of the station, while the tram can detect the number of boarding passengers at each station and record the total number of passengers onboard.

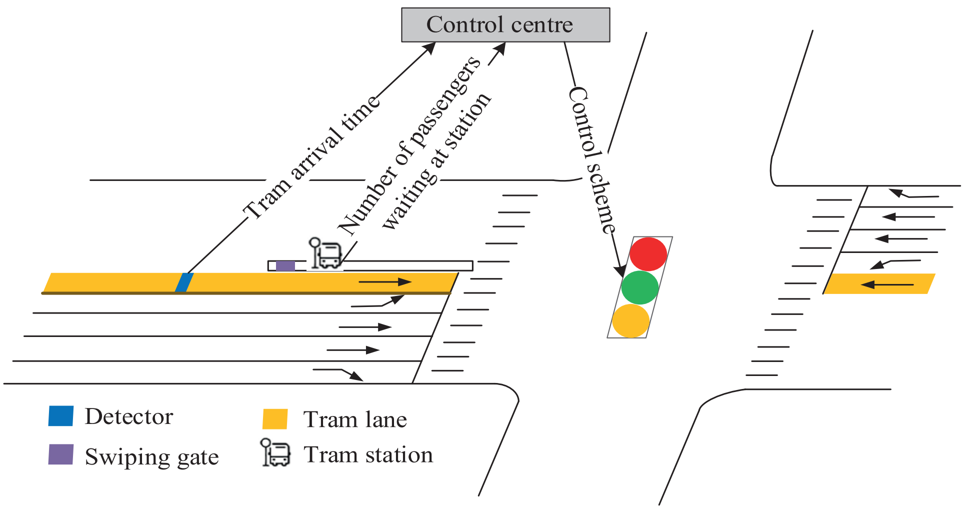

Figure 1 – Research scenario

The formulation of a capped robust optimal control model for trams is based on several key assumptions.

1) The detection device onboard the tram can accurately record the number of passengers boarding and alighting.
2) The equipment used to detect relevant information on the road is accurate and can transmit real-time data to the signal controller, including the arrival time of the tram and the total number of passengers boarding and alighting at the station.
3) The intersection experiences low tram traffic frequency with no conflicting priority requests.
4) Trams are prioritised during traffic flat peaks, and traffic flow is not congested at the intersection, so there are no queues for the tram.

3. TRAM DELAY CALCULATION MODEL

In order to accurately implement tram signal priority control, it is necessary to calculate the delay that the tram experiences at intersections. This information can then be used in combination with the signal control state to adjust signal timing.

3.1 Tram operation process analysis

The tram travel time from the detector to the stop line is calculated as the sum of the tram's driving time from the detector to the stop line without any stops and the dwell time at the station, as shown in *Figure 2*. The driving time from the detector to the stop line can be considered constant. However, the tram dwell time is determined by the stochastic number of passengers that get on and off. This section will analyse tram operation and formulate models to calculate the delay incurred by the tram at the intersection.

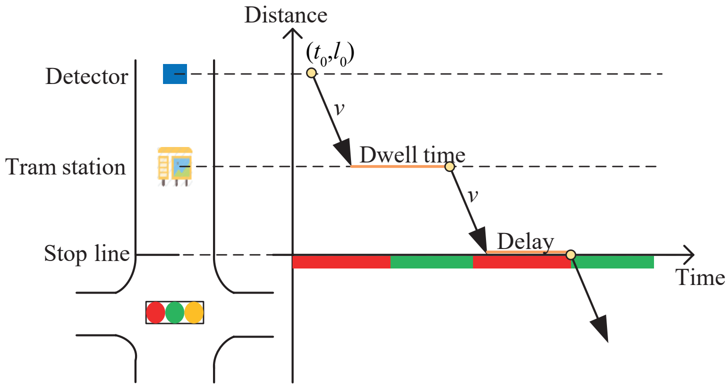

Figure 2 – Tram operation process

Tram driving time from detector to stop line without delay (t_r) can be calculated by *Equation 1*.

$$t_r = \frac{l_0}{v} \tag{1}$$

where l_0 is the distance from the detector to the stop line, v is the average operating speed of the tram on a section of the track.

The tram dwell time t_s at the station is determined by the time taken by passengers to get on and off, as well as the time taken for the doors to open and close. The door opening and closing time is generally considered a constant value q. This can be calculated using *Equation 2*:

$$t_s = a \cdot \left(n_{on} + n_{off}\right) + q \tag{2}$$

where α refers to the average time of a passenger getting on or off, n_{on} represents the number of passengers getting on, and n_{off} is the number of those who are getting off. The time of the tram arriving at the intersection (stop line) t_a can be obtained by tram driving time and dwell time shown in *Equation 3*.

$$t_a = t_0 + t_r + t_s \tag{3}$$

where t_0 is the time of the tram arriving at detector.

The tram delay at an intersection can be attributed to its arrival time and corresponding signal state. If the signal state is green when the tram arrives, it can pass through without any delay. However, if the tram arrives during a red signal, the delay can be calculated by subtracting the time of the tram arriving at the intersection from the end time of the red signal. It is important to note that each signal cycle, which spans a duration of C, starts with red and ends with green for trams as illustrated in *Figure 3*. The red signal lasts for r, and the green signal lasts for g. The signal cycle during which the tram arrives at the detector is considered the first signal cycle, with the sequence number of subsequent cycles incrementing over time.

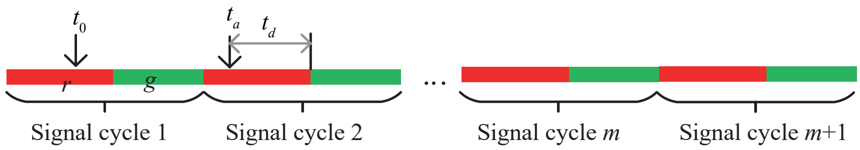

Figure 3 – Signal cycle

As shown in *Figure 3*, when the tram arrives at red in signal cycle 2, the tram delays t_d at the intersection, calculated by *Equation 4*.

$$= \begin{cases} 0, & \text{Tram arriving at intersection during green} \\ m \cdot C - g - t\ , & \text{else} \end{cases} \tag{4}$$

where m is the signal cycle number, with the signal cycle corresponding to the time of tram arrival at the detector as the first signal cycle.

3.2 Tram delay expectation and variance calculation model

For the case of a station serving a single line. When the station only serves a single tram line, passengers swipe their cards at the station. The number of passengers waiting to get on is known. Only the number of passengers getting off at the station is stochastic. The number of passengers getting off the tram obeys binomial distribution [23]. n is the number of passengers on the tram before arriving at the station. The probability of a passenger getting off at the station is γ, and the probability of n_{off} passengers getting off at the station is $P\{n_{off}\}$, shown in *Equation 5*.

$$P\{n_{off}\} = B(n, n_{off}) \cdot \gamma^{n_{off}} \cdot (1-\gamma)^{n-n_{off}}, n_{off} = 0,1,2... \tag{5}$$

where B represents the calculation method of permutation and combination. Based on tram delay and its probability, tram delay expectation can be calculated based on discrete number of passengers getting off shown as *Equation 6*.

$$E(t_d) = \sum_{n_{off}} t_d \cdot P\{n_{off}\} = \sum_{n_{off}} (m \cdot C - g - t_a) \cdot B(n, n_{off}) \cdot \gamma^{n_{off}} \cdot (1-\gamma)^{n-n_{off}} \tag{6}$$

According to the variance calculation method of discrete distribution, tram delay variance $D(t_d)$ calculation model at the intersection can be obtained by *Equation 7*.

$$D(t_d) = E^2(t_d) - E(t_d^{\ 2}) = \sum_{n_{off}} t_d \cdot P\{n_{off}\})^2 - \sum_{n_{off}} t_d^{\ 2} \cdot P\{n_{off}\} \tag{7}$$

For the case of a station serving multiple lines. When a station serves multiple tram lines, passengers waiting at the station may be waiting for different tram lines. As such, the number of passengers boarding each arriving tram from the station is uncertain. For a specific tram, the number of passengers arriving at the station follows a Poisson distribution [35]. The probability of k passengers waiting to board the target tram at the station is $P\{k\}$.

$$P\{k\} = \frac{(\lambda \cdot \Delta t)^k \cdot e^{-\lambda \cdot \Delta t}}{k!}, \ k = 0,1,2\ldots \tag{8}$$

where λ is the passenger arrival rate at the tram stops for a specific tram route, Δt is the headway between a specific tram and the preceding tram of the same route, measured by the time of arrival at the station.

The total number of passengers waiting at the station should be allocated reasonably among the various tram lines, taking into account the different possibilities of the number of passengers boarding each line. Tram dwell time should be calculated based on the number of passengers boarding in order to estimate tram delay expectation at the intersection.

Take the station serving two trams from different lines as an example to solve the number of passengers getting on target tram and its probability. Suppose the two trams are tram A and tram B, respectively. Before target tram A arrives at the station, N passengers are waiting at the station. Among them, i passengers are waiting for tram A with probability $P(A_i)$, and j passengers will get on tram B with the corresponding probability $P(B_j)$.

If the number of passengers entering and leaving the station cannot be recorded, resulting in the total number of passengers at the station being unknown, there are infinite possible number combinations for passengers to get on tram A and B, as shown in *Table 1*. However, when the accurate total number of passengers at the station can be transferred to the control centre, there are N+1 combinations for N passengers to get on. Only when the total number of passengers waiting for tram A and B is N, it is in line with the reality. The selection and corresponding possibility for the N passengers waiting at the station to get on trams is shown in the highlighted part of *Table 1*.

The N passengers waiting for the two trams is recorded as event O, and probability of event O occurring is $P(O)$. The arrival of passengers for the two trams is independent. The number of passengers arriving at the station for tram A and B obeys the Poisson distribution with two different parameters. The total number of passengers arriving at the station also follows the Poisson distribution due to Poisson distribution's addition, and its parameter is the sum of the Poisson distribution parameters of those two. It is considered that the number of passengers arriving at station for tram A is subject to Poisson distribution $P(\lambda_A \cdot \Delta t_A)$, and that for tram B it is

Table 1 – Selection of passengers and corresponding possibility

j \ i	0	1	…	N-1	N	N+1	...
0	$P(A_0)P(B_0)$	$P(A_1)P(B_0)$	…	$P(A_{N-1})P(B_0)$	$P(A_N)P(B_0)$	$P(A_{N+1})P(B_0)$	...
1	$P(A_0)P(B_1)$	$P(A_1)P(B_1)$	…	$P(A_{N-1})P(B_1)$	$P(A_N)P(B_1)$	$P(A_{N+1})P(B_1)$	...
…	…	…	…	…	…	…	...
N-1	$P(A_0)P(B_{N-1})$	$P(A_1)P(B_{N-1})$	…	$P(A_{N-1})P(B_{N-1})$	$P(A_N)P(B_{N-1})$	$P(A_{N+1})P(B_{N-1})$	...
N	$P(A_0)P(B_N)$	$P(A_1)P(B_N)$	…	$P(A_{N-1})P(B_N)$	$P(A_N)P(B_N)$	$P(A_{N+1})P(B_N)$	...
N+1	$P(A_0)P(B_{N+1})$	$P(A_q)P(B_{N+1})$	…	$P(A_{N-1})P(B_{N+1})$	$P(A_N)P(B_{N+1})$	$P(A_{N+1})P(B_{N+1})$	...
…	…	…	…	…	…	…	...

subject to $P(\lambda_B \cdot \Delta t_B)$. Therefore, the total number of passengers arriving at the station follows Poisson distribution $P(\lambda_A \cdot \Delta t_A + \lambda_B \cdot \Delta t_B)$. Then, $P(O)$ can be calculated by *Equation 9*.

$$P(O)=\frac{(\lambda_A \cdot \Delta t_A+\lambda_B \cdot \Delta t_B)^N \cdot e^{-(\lambda_A \cdot \Delta t_A+\lambda_B \cdot \Delta t_B)}}{N!} \tag{9}$$

where λ_A and λ_B are the arrival rates of passengers at the station for trams A and B, respectively. Δt_A represents the headway between tram A and last tram from the same line as tram A at the station. It has a similar meaning for Δt_B. According to the conditional probability theory, under event O, the probability of $i(i \leq N)$ passengers waiting for tram A and $j(j \leq N)$ passengers waiting for tram B is $P\{n_{on}\}$, written by *Equation 10*.

$$P\{n_{on}\}=\frac{P(A_i B_j)}{P(O)}=\frac{P(A_i)P(B_j)}{P(O)},\ (i+j=N) \tag{10}$$

The distribution and corresponding probability of boarding passengers at stations serving multiple lines can be derived according to the case of a station serving two lines. When a station serving y lines is marked as lines $D^h(h=1,2,3...y)$ and the probability of $n_{on}=i_h(i_h \leq N)$ passengers waiting for line $D^h(h=1,2,3...y)$ is $P(D_{i_h}^h)$, the probability of event O can be obtained by *Equation 11*.

$$P(O)=\frac{\left(\sum_{h=1}^{y}\lambda_{D^h}\cdot \Delta t_{D^h}\right)^N \cdot e^{-\sum_{h=1}^{y}\lambda_{D^h}\cdot \Delta t_{D^h}}}{N!} \tag{11}$$

There are $\sum_{N'=0}^{N+1} N'$ combinations for passengers getting on at the station. In the event O, the probability of $i_h(i_h \leq N)$ passengers waiting for line $D^h(h=1,2,3...y)$ at the station is $P\{n_{on}\}$, calculated by *Equation 12*, where $\sum_h i_h = N$.

$$P\{n_{on}\}=\frac{P\left(\prod_h D_{i_h}^h\right)}{P(O)}=\frac{\prod_h P(D_{i_h}^h)}{P(O)} \tag{12}$$

The expectation calculation model of the tram delay at the intersection when the upstream station is serving multiple lines can be obtained by *Equation 13*.

$$\begin{aligned} E(t_d) &= \sum_{n_{on}}\sum_{n_{off}} t_d \cdot P\{n_{off}\}\cdot P\{n_{on}\} \\ &= \sum_{n_{on}=0}^{N}\sum_{n_{off}}(m \cdot C - g - t_a)\cdot B(n,n_{off})\cdot \gamma^{n_{off}} \cdot (1-\gamma)^{n-n_{off}} \cdot \frac{\prod_h P(D_{i_h}^h)}{P(O)} \end{aligned} \tag{13}$$

Accordingly, the variance calculation model of tram delay at intersections $E(t_d)$ can be obtained through *Equation 14*.

$$D(t_d)=E^2(t_d)-E(t_d{}^2)=\left(\sum_{n_{on}}\sum_{n_{off}} t_d\cdot P\{n_{off}\}\cdot P\{n_{on}\}\right)^2-\sum_{n_{on}}\sum_{n_{off}} t_d{}^2\cdot P\{n_{off}\}\cdot P\{n_{on}$$

$$=\left(\sum_{n_{on}}\sum_{n_{off}}(m\cdot C-g-t_a)\cdot B(n,n_{off})\cdot\gamma^{n_{off}}\cdot(1-\gamma)^{n-n_{off}}\cdot\frac{\prod_h P(D_{i_h}^h)}{P(O)}\right)^2 \tag{14}$$

$$-\sum_{n_{on}}\sum_{n_{off}}(m\cdot C-g-t_a)^2\cdot B(n,n_{off})\cdot\gamma^{n_{off}}\cdot(1-\gamma)^{n-n_{off}}\cdot\frac{\prod_h P(D_{i_h}^h)}{P(O)}$$

4. CAPPED ROBUST OPTIMAL CONTROL METHOD

4.1 Objective function and constraints

This paper proposes a capped robust optimal control method to prioritise trams and enhance the reliability of tram operations, based on the available data on the number of passengers waiting at the station. The objective function of the proposed method comprises of three components. The first component aims to minimise the tram delay expectation, thereby ensuring minimal delays for trams at the intersection. The second component endeavours to minimise the variance in tram delay, which contributes to enhancing the robustness of the tram operation process. The third part involves signal timing adjustments which brings negative impacts on other vehicles at the intersection. This measure is imperative to restrict the duration of signal adjustment to mitigate these negative effects.

$$f=\min\beta_1\cdot E(t_d)+\beta_2\cdot D(t_d)+\beta_3\cdot(\Delta g+\Delta r) \tag{15}$$

$$\text{s.t. } \beta_1+\beta_2+\beta_3=1 \tag{16}$$

$$\Delta g\cdot\Delta r=0 \tag{17}$$

$$0\le\Delta g\le\Delta g_{\max},\Delta g\in N^* \tag{18}$$

$$0\le\Delta r\le\Delta r_{\max},\Delta r\in N^* \tag{19}$$

where $\beta_i(i=1,2,3)$ is the weight coefficient of objective function. *Equation 16* defines the weight assigned to each component of the objective function. To simplify the solution of multi-objective programming, a transformation is applied by weighting the components and treating it as a single-objective problem. *Equation 17* specifies that two priority control strategies, namely green extension and red truncation, are mutually exclusive. Furthermore, *Equations 18 and 19* impose constraints on the maximum and minimum signal adjustment times, respectively, where the time values are integers measured in seconds.

4.2 Control process

As illustrated in *Figure 4*, the control centre receives input from variable parameters, historical statistics and detectors. Signal priority is exclusively granted to trams that are scheduled to arrive later. The signal priority time intervals are determined based on different strategies, with priority given to shorter intervals. Various signal control schemes with differing signal adjustment times are utilised, and the delay expectation and variance for each scheme are computed. Both the expectation and variance are incorporated into the calculation of the objective function. The optimal control scheme is selected by comparing the objective function values, and the corresponding scheme with the minimum objective function is implemented at the intersection.

5. CASE STUDY

To investigate the efficacy of the CRC method proposed in this paper, a simulation platform was established for a real intersection located in Shanghai. The intersection includes tram lanes situated at the centre of the road, and the entire signal cycle lasts for 100 seconds with trams being allotted 30 seconds of green time and 70 seconds of red time. A tram station is positioned approximately 100 meters upstream of the intersection, wherein a tram detector is installed to record the arrival times of trams. In the simulation, the number of passengers waiting for the target tram, which serves multiple lines, conforms to a Poisson distribution with a parameter of 10. Additionally, the count of passengers alighting at the station for the target tram line follows a binomial distribution with an alighting rate of 10%.

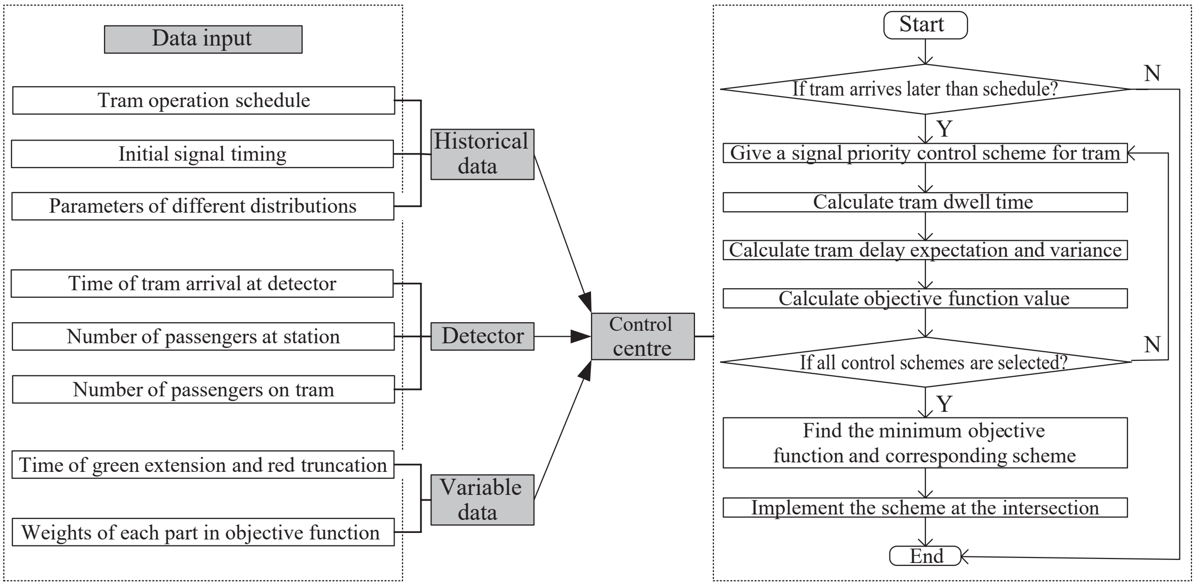

Figure 4 – Control process

5.1 Verifying the tram delay calculation model

To validate the proposed tram delay calculation models presented in Section 3, a series of experiments were conducted to simulate the operation process of trams at stations serving both a single line and multiple lines. The accuracy of the calculation models was verified by comparing the results obtained from the theoretical models with actual samples. The theoretical expectation and variance of tram delays were calculated using the tram delay calculation models outlined in Section 3. In the sample experiments, trams arrived at the detector every second throughout the signal cycle, with varying numbers of passengers disembarking at the station. This process was repeated 100 times to ensure statistical significance. Moreover, it was assumed that 15 passengers boarded the tram at the single-line serving station. For stations serving multiple lines, three scenarios were examined, with total numbers of passengers waiting at the station set to 5, 15 and 25, respectively. These scenarios aimed to capture different passenger loads at the station. By comparing the outcomes from the sample experiments with the theoretically calculated delays, the validity and accuracy of the tram delay calculation models can be assessed, thus verifying the proposed approach.

Figure 5 presents a comparison between the expected tram delay and its variance derived from the sample experiments and the theoretical calculations for trams at single-line serving stations. The results indicate that the average tram delay observed in the samples follows a similar trend to the theoretical tram delay expectation, with negligible differences in values. A similar trend is also observed for the delay variance between the sample experiments and the theoretical calculations. These observations provide strong evidence supporting the claim that the calculation models accurately predict tram delays at intersections for stations serving a single line.

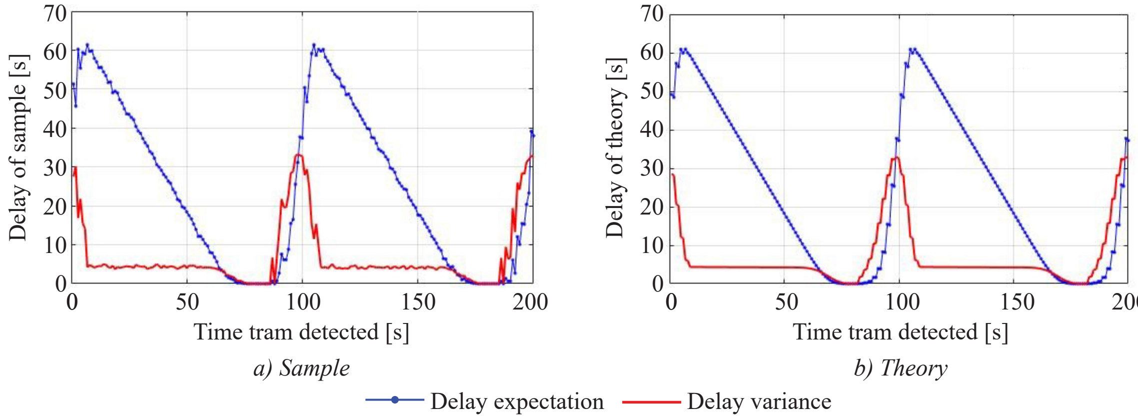

Figure 5 – Verification of the tram delay calculation model with a station serving a single line

At stations serving multiple lines, the number of passengers boarding and alighting from the target tram is a random process. *Figure 6* illustrates results of average delay obtained from sample experiments and the theoretical delay expectation for different numbers of waiting passengers (*N*). By comparing the results in *Figures 6a and 6b*, it is evident that the calculation model for tram delay expectation at intersections with stations serving multiple lines can accurately predict the tram delay expectation. The model takes into account the stochastic number of passengers boarding and effectively captures the variations in delay caused by differing numbers of waiting passengers.

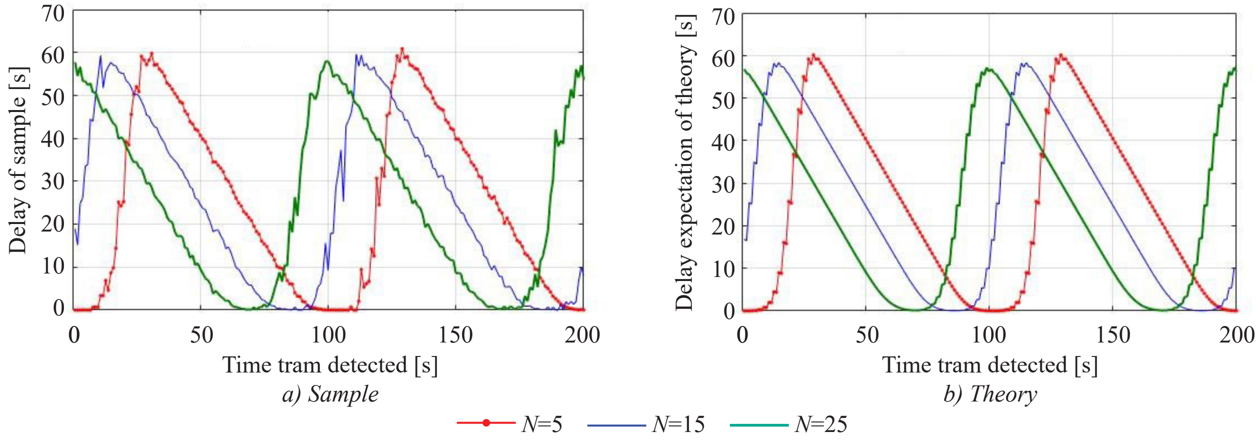

Figure 6 – Verify of tram delay expectation calculation model with station serving multiple lines

The study presents an empirical investigation on the variance of tram delays at intersections and compares them with theoretical results. The comparison is conducted under varying passenger loads at stations. The findings reveal that the calculated variances from the proposed model in this paper are consistent with those obtained through empirical observations, as depicted in *Figures 7a and 7b*. Thus, the results support the efficacy of the proposed model for accurately estimating tram delay variance at intersections.

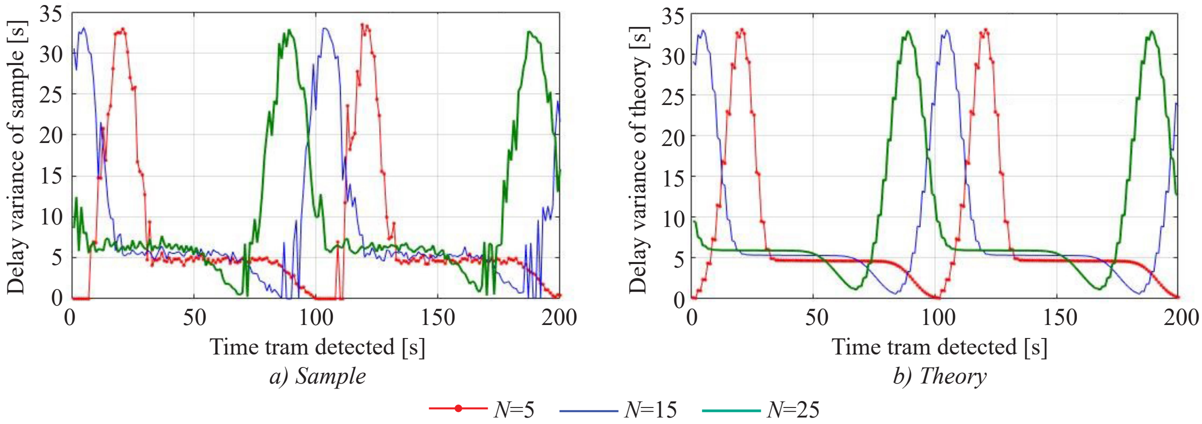

Figure 7 – Verification of tram delay variance calculation model with a station serving multiple lines

The study focuses on an empirical investigation of tram delay variances at intersections and compares them with the theoretical results obtained from the proposed model. The comparison takes into account different numbers of passenger loads at stations to capture real-world scenarios. The findings of the study demonstrate that the calculated variances from the proposed model align closely with the empirical observations of tram delay variances. This is illustrated in *Figures 7a and 7b*, where the trends in the empirical variances match those predicted by the theoretical model. These results provide strong support for the effectiveness of the proposed model in accurately estimating tram delay variances at intersections.

5.2 Vehicle operation analysis

To assess the efficacy of the proposed control method in this paper, two comparative tests have been devised in this section. The first test involves an experiment conducted with no optimal control (NC), wherein the signal control scheme remains unaltered. In the second test, the total number of passengers at the station is unknown, and the number of passengers boarding the target tram conforms to Poisson distribution without upper limit, thereby it is referred to as the infinite robust optimal control method (IRC).

For the case of a station serving a single line. *Figure 8* illustrates the delay expectation of trams under three distinct control methods. The data indicates that the CRC method exhibits superior efficiency in reducing tram delays, manifesting in nearly every second of the complete signal cycle. In contrast, the IRC method demonstrates limited effectiveness, solely operating within a truncated interval, that is, between the 100- and 120-second marks in *Figure 8*. The heightened accuracy of the CRC method in predicting the arrival time of trams at stop lines grants signal controllers greater capability to implement effective priority control schemes, which, in turn, serve to minimise tram delays.

Figure 8 showcases tram delay expectation under three different control methods. The data presented indicates that the CRC method surpasses the other methods in terms of efficiency and reduction of tram delays throughout the complete signal cycle. This is evident in the nearly continuous improvement in delay reduction observed with the CRC method. On the other hand, the IRC method demonstrates limited effectiveness, with its impact restricted to a specific time interval, as shown between the 100- and 120-second marks in *Figure 8*. This suggests that the IRC method may not be as effective in minimising tram delays compared to the CRC method. The heightened accuracy of the CRC method in predicting tram arrival times at stop lines plays a crucial role in enabling signal controllers to implement more effective priority control schemes. As a result, tram delays are minimised, leading to improved efficiency in tram operations.

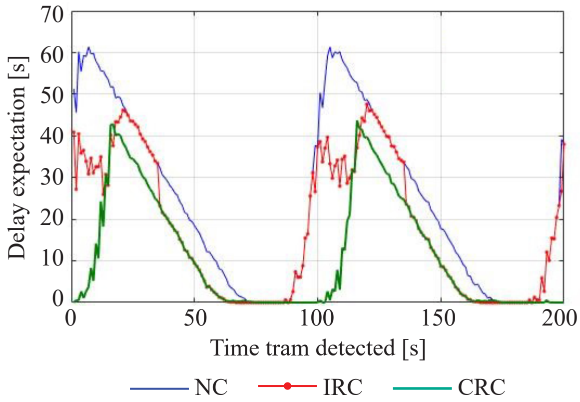

Figure 8 – Tram delay expectation when station serves a single line

The findings presented in *Figure 9* demonstrate the delay variance outcomes resulting from tram arrivals during successive signal cycles under three distinct control methods. Specifically, these results indicate that the implementation of the IRC method is associated with an increase in tram delay variance within the 100 s to 120 s period, while offering no meaningful reduction in delay variance across other periods. Conversely, employment of the CRC method yields a significant reduction in tram delay variance, with notable improvements observed within the 88 s to 103 s period. This outcome is attributed to the optimised control scheme which effectively reduces and shifts the peak of delay variance.

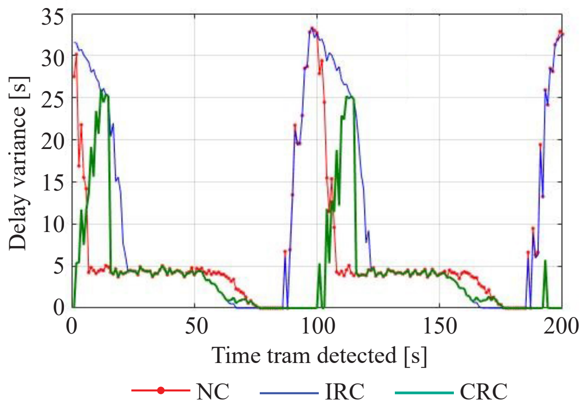

Figure 9 – Tram delay variance when station serves a single line

Table 2 delineates the signal control strategies and associated performance indices of various control methods in the time interval of 100 s to 140 s, presenting the information in a more comprehensible manner. The table demonstrates that during the period of 100 s to 115 s, a minor modification of signal timing utilising the CRC method can substantially diminish the expected delay and variance of tram transportation. For instance, at 105 s, employing a priority time of 15 s can reduce the anticipated delay by approximately 61 s. Conversely, under the IRC approach, the expected delay remains constant as the priority time increases, while the delay variance is even larger than that of the NC method. From 120 s to 135 s, the priority time of the CRC technique is significantly shorter than that of the IRC method, yet the use of IRC fails to mitigate the delay in comparison with the utilisation of NC, which indicates its ineffectiveness during this interval. Through calculating the average value of tram priority control within two consecutive signal cycles, the CRC method delivers a more substantial reduction effect on delay expectation and variance with less signal adjustment.

Table 2 – Control scheme and performance index

	$t_0(s)$	**100**	**105**	**110**	**115**	**120**	**125**	**130**	**135**	**140**	**Average(s)**
Green time [s]	NC	30	30	30	30	30	30	30	30	30	30
	IRC	30	35	40	45	44	45	44	45	39	38.9
	CRC	44	45	44	45	39	39	39	39	39	38.5
Red time [s]	NC	70	70	70	70	70	70	70	70	70	70
	IRC	70	65	60	55	56	55	56	55	61	61.1
	CRC	56	55	56	55	61	61	61	61	61	61.5
Priority time [s]	NC	0	0	0	0	0	0	0	0	0	0
	IRC	**0**	**5**	**10**	**15**	**14**	**15**	**14**	**15**	9	8.9
	CRC	**14**	**15**	**14**	**15**	**9**	**9**	**9**	**9**	9	8.5
Delay expectation [s]	NC	**37.4**	**61.4**	**58.2**	**53.1**	**48.7**	**43.2**	**37.5**	**33.7**	28.6	24.4
	IRC	**37.4**	**33.3**	**34.4**	**31.5**	**47.6**	**43.2**	**37.5**	**33.7**	19.6	17.8
	CRC	**0**	**0.5**	**13.1**	**31.5**	**39.7**	**34.2**	**28.5**	**24.7**	19.6	11.2
Delay variance [s]	NC	**32.7**	**11.5**	**4.4**	**3.9**	**3.9**	4.2	4.4	4.0	4.5	6.9
	IRC	**32.7**	**30.3**	**27.1**	**24.8**	**7.79**	4.2	4.4	4.0	4.5	9.8
	CRC	**0**	**5.3**	**22.9**	**24.8**	**3.9**	4.2	4.4	4.0	4.5	4.3

Figure 10 illustrates the delay incurred by other traffic participants and signal adjustments at the intersection under different control techniques. The employment of the CRC method results in a more substantial reduction effect on tram delay expectation (E) and variance (V), with less impact on other vehicles compared to the IRC method. Although the signal adjustment (SA) time for the CRC approach is slightly shorter than that of the IRC method, at 8.5 s and 8.9 s, respectively, their impact on car delay (CD) at the intersection is almost indistinguishable, increasing the average car delay by roughly 1.2 s, which is equivalent to 4.2%. As for passenger

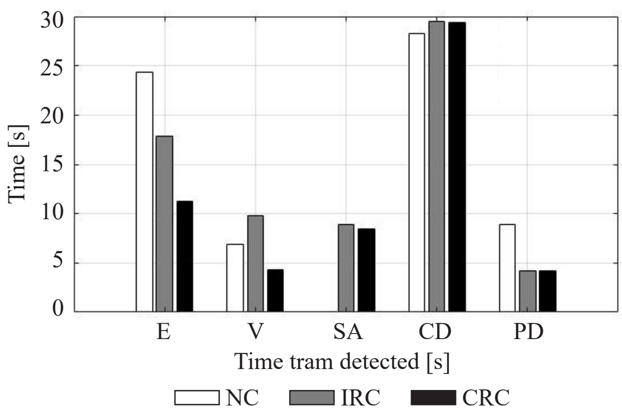

Figure 10 – Traffic operation of intersection

delay (PD), both control methods produce a similar beneficial effect, significantly decreasing passenger delay compared to using the NC.

For the case of a station serving multiple lines. In the simulation experiments involving stations that serve multiple tram lines, situations arise where two different trams, namely tram A and tram B, both from different lines, dwell at the station in succession. The Poisson distribution parameter for passenger arrivals at the station for tram A is denoted as 10, whereas the parameter for the total number of passengers arriving at the station for the two trams is represented by 12.75. *Figure 11* demonstrates the expected delay times of tram A under various control methods, given differing total numbers of passengers waiting at the station. It is apparent that when there are either 5 or 25 passengers waiting, the CRC approach is more effective in reducing tram delay expectations. Nevertheless, when the total number of passengers waiting at the station is closest to the parameter of the Poisson arrival distribution for tram A, which is 15, the IRC method performs optimally. In contrast, the IRC method fails to provide an accurate prediction of tram dwell time during certain time periods when the total number of passengers waiting is either 5 or 15.

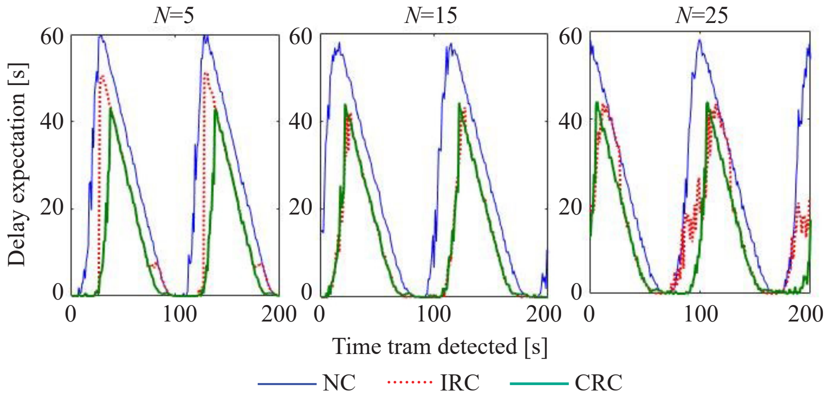

Figure 11 – Tram delay expectation with a station serving multiple lines

Figure 12 illustrates the variance of tram delay for tram A with respect to varying total numbers of passengers waiting at the station. It is evident that when there are only 5 passengers waiting, the peak of tram delay variance under the CRC method shifts backwards due to a reduction in travel time resulting from green light extension, which delays the meeting time between the tram and red light. Moreover, the use of the CRC method leads to a more significant decrease in extreme tram delay variance. Prior to the occurrence of maximum variance (which typically happens between 10 s and 30 s, and again between 110 s and 130 s), the control effect of the IRC approach is consistent with that of the CRC method. However, after the maximum value is reached, the red truncation of the IRC method only alters the delay expectation without affecting the variance, thus yielding results similar to those obtained with the NC method. When the total number of passengers waiting at the station is 15, the effects of the two control methods are quite similar. Finally, when there are 25 passengers waiting, the optimisation effect of the CRC method remains stable, whereas the use of the IRC method leads to partial failure, resulting in an increase in tram delay variance.

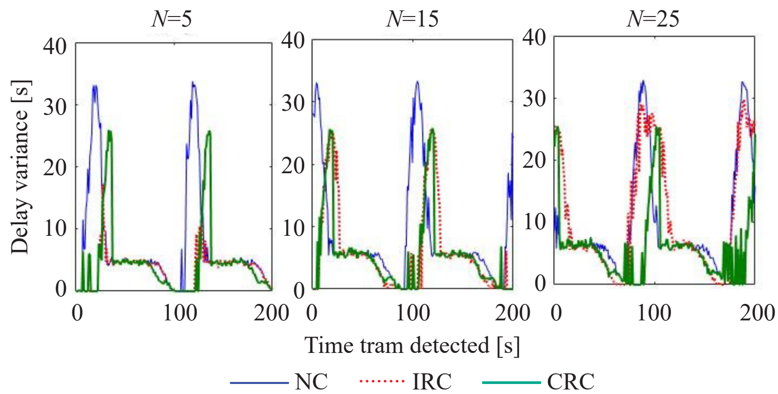

Figure 12 – Tram delay variance when station serves multiple lines

Figure 13 presents a comparative analysis of tram stop count at an intersection utilising different control methods, considering varying numbers of passengers waiting at the station. The results indicate that both optimal control techniques can diminish the lower quartile of tram stop count under diverse scenarios. When the count of passengers waiting at the station is at 15, the utilisation of IRC and CRC yields comparable distributions of tram stop count, significantly reducing the median number of tram stops. However, when the passenger count at the station alters to 5 or 25, the influence of IRC on median number of tram stops is indistinct. Unlike IRC, CRC proves to be more efficient in signal priority control for trams approaching the intersection, thus notably decreasing the median number of tram stops.

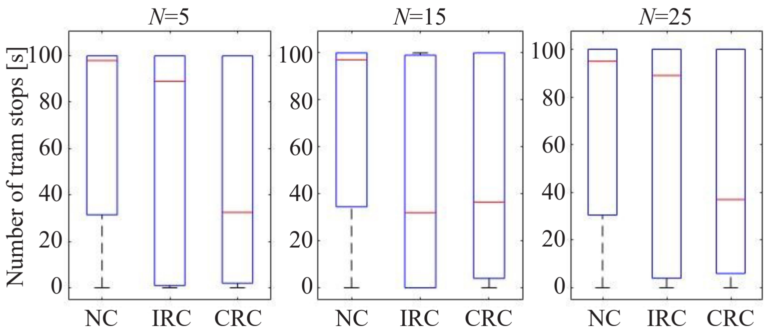

Figure 13 – Number of tram stops when a station serves multiple lines

Figure 14 presents a comparison of different priority control methods in terms of their impact on priority time and delay reduction. Our findings reveal that the CRC approach exhibits a horizontal comparison, indicating its ability to adjust signal priority control schemes while ensuring their effectiveness across various passenger traffic volumes at the station. Specifically, when the green extension time is consistently maintained at 15 s, the CRC method reduces tram delays by approximately 50 s to 60 s, with the red truncation time almost equivalent to the value of tram delay reduction. On the other hand, the IRC signal scheme remains unaltered even under varying passenger traffic volumes at the station, thereby becoming susceptible to failure scenarios where signal priority control fails to reduce intersection delays. For example, when the total number of passengers at the station is five, the use of IRC results in no significant reduction in tram delay from 85 s to 110 s, as shown in *Figure 14*.

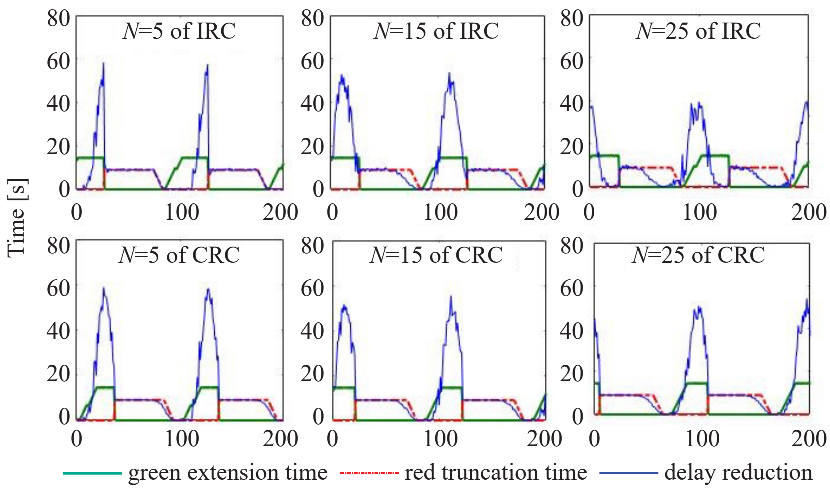

Figure 14 – Delay reduction and priority time when a station serves multiple lines

Table 3 illustrates the operational performance of trams and cars under varied control methods for different total waiting numbers at the station. The comparative analysis reveals that, when the total number of passengers is above or below the average threshold, the CRC method outperforms the IRC method in reducing tram delay expectation substantially. Car delay is pretty much close in all cases, which means the control methods have little negative impact on other vehicles at the intersection. Especially, both car delay and bus delay expectation under CRC are lower compared to IRC, proving that the CRC method performs better than the IRC method. At the same time, among the three different passenger volumes observed, the CRC approach yields greater delay reduction through minor signal adjustments, with minimal adverse effects on other vehicles at the intersection. In contrast, the invariant signal adjusting approach utilised by the IRC method suggests a lack of flexibility, making it difficult to cater to random factors that often occur during tram operations. In *Table 3*, values for delay expectation are very close in the three control methods for 5, 15 and 25 passengers, which is because the number of passengers waiting at tram station only influences the time of tram arriving at stop line and changes the time delay occurring. Delay expectation is the average tram delay at the intersection, thus they are close. Based on the above analysis, the CRC method, which generates signal control strategies based on varying passenger numbers at the station, ensures the reliability of trams at intersections.

Table 3 – Traffic operation of intersection when a station serves multiple lines

Number of passengers	Control method	Delay expectation [s]	Delay variance [s]	Priority time [s]	Car delay [s]
N=5	NC	24.34	7.60	0	28.21
	IRC	15.26	3.44	9.56	29.61
	CRC	11.36	4.78	8.34	29.38
N=15	NC	24.39	8.72	0	28.21
	IRC	11.24	6.06	9.56	29.61
	CRC	11.66	5.50	8.59	29.43
N=25	NC	24.44	9.74	0	28.21
	IRC	15.16	11.23	9.56	29.61
	CRC	11.74	6.05	8.68	29.44

6. CONCLUSION

This paper presents a capped robust optimisation control method to address the failure of tram signal priority control at an intersection that arises from the stochastic operation time of trams caused by random dwell time. The proposed method focuses on tram dwelling control at stations serving multiple lines. We have established models for calculating tram delay expectation and variance, which consider the stochastic number of passengers boarding and alighting at the station. The objective function incorporates the tram delay expectation and variance in solving the tram signal priority control scheme at the intersection. This approach aims to achieve accurate and efficient tram priority while minimising delay for other vehicles at the intersection. The case analysis verifies the accuracy of the tram delay expectation and variance calculation models and evaluates the optimisation effect of the capped robust optimisation control method on tram delay.

Despite its efficacy in addressing tram signal priority control problem, the proposed method has some limitations. For instance, it only considers the signal priority request of a single tram without accounting for multiple priority request conflicts. In addition, certain constants used in the model, such as the rate of passengers alighting from the tram are mainly obtained from historical data, which may not accurately reflect real-time situations. These limitations will be addressed in future studies.

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在固定信号控制的交叉口中，带有轨迹规划的互联和自动驾驶车辆的表现评估

互联和自动驾驶车辆（CAVs）被公认为交通工程领域的技术趋势。作为CAVs最受欢迎的功能之一，轨迹规划引起了学术界和工业界的广泛关注和兴趣。分段式轨迹规划因其在计算和部署方面的简易性和稳健性而越来越受欢迎。通过探索分段式轨迹在不同CAVs和交叉口设置下的影响，可以提供建设性的建议和指导。本研究提出了一种用于固定信号时序环境下分段式轨迹规划的控制策略。为了测试该控制策略的效果，本研究设计了简化的固定信号交叉口场景，并实现了CAVs在不同交通需求、距离和速度限制下的分段式轨迹规划功能。结果表明，所提出的控制策略在不同交通场景中表现出稳定的优越性，尤其在交通流量接近最大道路承载力时表现出色。

关键词:
互联和自动驾驶车辆，轨迹优化，固定信号控制交叉口，虚拟车队