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Train Timetabling Optimisation Model Considering Headway Coordination between Mainline and Depot

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ABSTRACT

This paper proposes an optimisation model for an urban rail transit line timetable considering headway coordination between the mainline and the depot during the transition period. The model accounts for the tracking operation scenario of trains inserted from the depot onto the mainline and related train operation constraints. The optimisation objectives are the number of trains inserted, maximum train capacity rate and average headway deviation. Second-generation non-dominated sorting genetic algorithm is designed to solve the model. A case study shows that optimisation achieves a total of 25 trains inserted, a maximum train capacity rate of 0.975 and an average headway deviation of 9.5 s, resulting in significant improvements in train operations and passenger satisfaction. Compared with the current train timetable before optimisation, the average dwell time and the maximum train capacity rate at various stations have been reduced after optimisation. The proposed model and approach can be used for train timetabling optimisation and managing the operations of urban rail transit lines.

KEYWORDS

urban rail transit; optimisation model; timetable; train capacity rate; headway deviation.

1. INTRODUCTION

Evaluating the capacity of trains and train lines [1–4] is an important issue in the railway industry. Typically, the goal is to determine the maximum number of trains that can pass through a given railway infrastructure during a specific time interval. The time interval between two successive trains passing the same location is known as the headway; thus, the railway capacity is directly reflected by the train headway. The control and optimisation of train headway is an effective method of improving the reliability of transit services, producing benefits for both the transit user and operator. Abkowitz et al. [5] addressed a headway control strategy that can be implemented on high-frequency routes, and developed an algorithm for solving the headway variation model, while Ning et al. [6] developed an integrated real-time control method for optimising the train headway by adjusting the station arrival times of the trains, effectively reducing the average passenger waiting time and energy consumption. Ding et al. [7] proposed a real-time headway control model that maintains headway regularity by minimising the total headway variance, the simulation results demonstrated that the average passenger waiting time was significantly reduced by this control model. Zhao et al. [8] presented a dynamic headway system for positive train control based on active communications, which can be integrated in a dynamic dispatching model to improve track capacity and reduce the total travel time.

Optimising the train headway, a suitable train timetable is crucial for improving the operational efficiency of trains and passenger comfort. During the transition from the off-peak to peak period of urban rail transit line, trains are inserted from the depot to the mainline for passenger operation according to the planned timetable. In recent years, with the increasing passenger flow of urban rail transit, the headway of trains departing the depot often cannot meet the requirements of the mainline train tracking interval during peak hours, resulting in a mismatch in transportation capacity between the depot and the mainline trains. When trains from the depot need to be inserted onto the mainline, the headway of trains departing the depot must match the tracking interval of the mainline trains. Otherwise, the mainline trains will be delayed or the trains departing the depot will be unable to perform their planned service. Furthermore, the tracking interval and insertion time of trains from the depot must match the tracking operation of trains in relevant sections of the mainline at the micro level, otherwise it will reduce the efficiency of train operation.

The contribution of this paper is that we aim to propose an optimisation model of train timetabling for urban rail transit while considering headway coordination between mainline and depot, on the basis of effectively compressing the minimum headway for trains departing the depot and considering the micro tracking operation scenario of trains inserted from the depot to the mainline, and apply second-generation non-dominated sorting genetic algorithm (NSGA-II) to efficiently solve the proposed model.

The remainder of this paper is organised as follows. In Section 2, literature review is listed. In Section 3, the optimisation problem of train headway considering both the mainline and the depot is introduced, and a timetabling optimisation model is proposed. Section 4 and 5 evaluate the performance of the proposed model and verify its efficacy through a case study. Finally, Section 6 summarises the conclusions of this study and considers future research directions.

2. LITERATURE REVIEW

As an important optimisation target in railway transportation, various models and solution methods have been established for solving railway timetabling problems. For example, train timetable optimisation models [9–11] have been proposed with the purpose of minimising the passenger waiting time. Sparing et al. [12] proposed a periodic timetable optimisation model that ensures the maximum stability for various railway networks. Gong et al. [13] studied the robust train timetabling problem with consideration of energy-efficiency and dynamic passenger demand. Robenek et al. [14] presented a mixed-integer linear programming model with the purpose of maximising the benefits of the operating company and maintaining passenger satisfaction. Yue et al. [15] presented a train timetabling optimisation model that simultaneously accounts for both passenger service demands and train scheduling, and proposed a column-generation-based heuristic algorithm to solve the model. Jiang et al. [16] studied the train timetabling problem of a highly congested railway line with the optimisation goal of maximising the number of operation trains, whereas Zhang et al. [17] presented an integer programming timetabling optimisation model based on an extended time-space network for a double-track railway line. Niu et al. [18] considered the time-varying passenger demands and proposed a binary-integer programming timetabling optimisation model. Caprara et al. [19] established an integer linear programming timetabling model based on graph theory, while Cordone et al. [20] designed a mixed-integer nonlinear programming timetabling model to deal with the dynamic passenger demands, with consideration of the trade-off between the quality of timetable and passenger satisfaction. Lee et al. [21] presented an optimisation model that considers the maximum benefits for both train timetabling and train pathing. Robenek et al. [22] proposed a train timetabling model that prioritises the interest of passengers, and introduced the concept of a competitive market by considering the elasticity of passenger demand. Zhang et al. [23] proposed a joint optimisation model for both train timetabling and maintenance planning in which passenger travel time and maintenance costs are collaboratively minimised. Meng et al. [24] proposed an optimisation model to solve the robust single-track train schedules, while Ghoseiri et al. [25] developed an optimisation model suitable for single and multiple tracks with various train capacities. Zhang et al. [26] proposed an optimisation train timetabling model to cope with time-varying passenger flow for an urban rail transit line and used a simulated-annealing algorithm to effectively solve the model. Collaborative optimisation models for both train timetabling and train stop planning have also been proposed in previous research [27–30] to simultaneously obtain the optimal train timetables and train stop plans. Liu et al. [31] proposed a stochastic train timetabling model and constructed a hybrid heuristic branch and bound approach to solve the model, while Yang et al. [32] presented an optimisation timetabling model for the last train and proposed a dual decomposition algorithm to solve with it. Bababeik et al. [33] provided a train timetabling optimisation model for a single railway track with considering of related constraints such as train speed limit due to maintenance activities.

Existing studies did not consider the coordination of train headway between the mainline and the depot, nor did they consider the negative impact of irregular headway on passengers, so the micro operation of trains and passenger benefits cannot be accurately reflected in the existing timetable models. Therefore, the timetabling optimisation model constructed in this paper considers specific train tracking operation scenario and headway coordination between the mainline and the depot, and takes the average deviation value of train headway as one of the collaborative optimisation objectives, with the ultimate aim of jointly improving the train operational efficiency and passenger satisfaction.

3. MATERIALS AND METHODS

3.1 Optimisation of headway for trains departing the depot

As shown in *Figure 1*, only the down exit line of depot is used for train departing before optimisation. As the tracking of adjacent trains in the depot is based on fixed block, only one train can occupy the rail track within route D1A–CD01 under the current signalling system configuration.

Figure 1 – Signalling system configuration and train tracking diagram in the depot before optimisation

The minimum headway for trains departing the depot can be calculated as follows:

$$H_{min}^{ori}(depot) = T_{Route} + T_{Train\,B\text{-}Door} + T_{Driver} + T_{Clear\_Door} + T_{Clear\_Door\text{-}CD01} + T_{Clear\_TA1617} \quad (1)$$

where T_{Route} is the time required to set route D1A–CD01 for subsequent tracking of Train B after Train A has cleared transfer track TA1617; $T_{Train\text{-}B\text{-}Door}$ is the time required for Train B to run from its current position to the garage door; T_{Driver} is the time required for the driver to observe at the garage door; T_{Clear_Door} is the required running time for Train B to clear the garage door; $T_{Clear\_Door\text{-}CD01}$ is the required running time for Train B from clearing the garage door to signal CD01; and T_{Clear_TA1617} is the time required for Train B to clear transfer track TA1617.

Equation 1 indicates that the minimum headway for trains in the depot mainly depends on the time required for train operations in the entire throat area. Therefore, the minimum headway can be shortened by dividing the track section of the throat area into several sections. In addition, both up and down exit lines of the depot can be used simultaneously to increase the efficiency of train departure.

After optimisation, as shown in *Figure 2*, the original TA1 (down exit line) has been divided into three track sections, TA11, TA12 and TA13, by adding shunting signals S1, S2, DD01 and axle counting heads. Correspondingly, the original route D1A–CD01 has been divided to four routes: D1A–S1, S1–S2, S2–DD01 and DD01–CD01. As a result, there can be up to four trains in the throat area (Trains B–E) after optimisation. The up exit line has also been optimised accordingly.

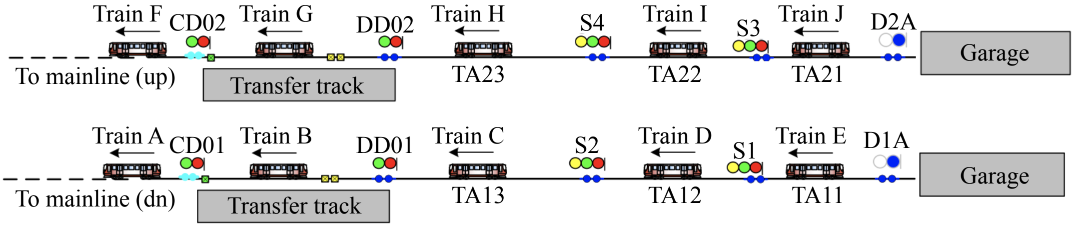


Figure 2 – Signalling system configuration and train tracking diagram in the depot after optimisation

Similar to *Equation 1*, the minimum headway of adjacent trains at S1, S2, DD01 and CD01 can be obtained separately, denoted as H_{min}(S1), H_{min}(S2), H_{min}(DD01), H_{min}(CD01), and the optimised minimum headway for down exit line trains is the larger of the above tracking intervals, namely

$$H_{min}^{opt}(dn) = max\{H_{min}(S1), H_{min}(S2), H_{min}(DD01), H_{min}(CD01)\} \tag{2}$$

The minimum headway for up exit line trains can also be obtained, denoted as $H_{min}^{opt}(up)$. Then, the minimum headway for trains departing the depot after optimisation is as follows:

$$H_{min}^{opt}(depot) = \frac{\max\{H_{min}^{opt}(up), H_{min}^{opt}(dn)\}}{2} \tag{3}$$

By comparing *Equations 1 and 3,* the conclusion can be drawn that the minimum headway for trains departing the depot can be effectively shortened through this optimisation procedure.

3.2 Consideration of headway coordination between trains

Taking the down exit line as an example, as shown in *Figure 3*, when Train 2 (depot departure train) leaves transfer track TA1617, passes through section TA1615 and enters section TA1613, the automatic train supervision system (ATS) triggers route S1607–S1602 based on the planned train timetable. According to the interlocking principle, if track section TA1602 is occupied by another train, route S1607–S1602 cannot be successfully set. Only after Train 1 (mainline train) has cleared TA1602 and switch P1601 turned to the right side can route S1607–S1602 be successfully set. Train 2 then runs along the turnout area, accelerates to reach the lateral speed limit of the turnout, and continues at a constant speed until it decelerates and stops at the up-direction platform of station 16.

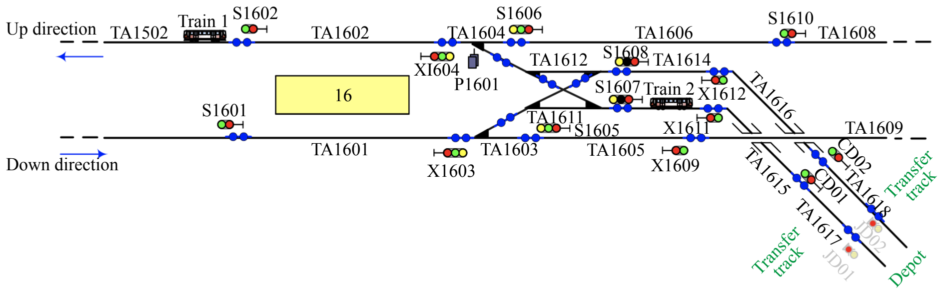


Figure 3 – Mainline Train 1 and depot departure Train 2 tracking operation scenario

The minimum headway is the minimum interval between adjacent trains without interference. After Train 2 enters section TA1613, if signal S1607 opens too late, Train 2 will have to brake and stop in front of signal S1607 due to the shortened movement authorisation. If signal S1607 opens too early, there will be a large tracking interval between Train 2 and Train 1. When signal S1607 opens at a suitable time that does not require any unnecessary deceleration by Train 2 and enables a new movement authorisation, Train 2 and Train 1 attain the minimum headway $H_{min}(1,2)$. To ensure safe operation, when signal S1607 opens, the distance between Train 2 and signal S1607 should be such that Train 2 could stop in front of signal S1607 at the maximum service braking rate.

The formula for the minimum headway between Train 2 and Train 1 is as follows:

$$H_{min}(1,2) = T_{Route} + T_{D\text{-}S1607} + T_{S1607\text{-}S1602} + T_{dwell} + T_{Clear\_TA1602} \tag{4}$$

In the formula, where T_{route} is time required to set route S1607–S1602 for subsequent tracking of Train 2 after Train 1 has cleared track section TA1602; $T_{D\text{-}S1607}$ is the time required for Train 2 to run from its current position to signal S1607; $T_{S1607\text{-}S1602}$ is the required running time for Train 2 to pass signal S1607 and stop accurately at the platform; T_{dwell} is the dwell time of Train 2 at the up-direction platform of the 16th station; and T_{Clear_TA1602} is the time required for Train 2 to depart from the platform and clear TA1602, during which process the operating speed cannot exceed the platform speed limit.

Only if $H_{min}^{opt}(depot) \leq H_{min}(1,2)$, then the train departure capacity of the depot satisfies the requirements for train insertion at the macro level.

Figure 4 shows the tracking operation scenario for Train 2 from the depot and Train 3 from the mainline. Similarly, the formula for the minimum headway between Train 3 and Train 2 is as follows:

$$H_{min}(2,3) = T_{Route} + T_{D\text{-}S1606} + T_{S1606\text{-}S1602} + T_{dwell} + T_{Clear\_TA1602} \tag{5}$$

According to the calculation results, $H_{min}(1,2)$=99.76 s, $H_{min}(2,3)$=81.24 s.

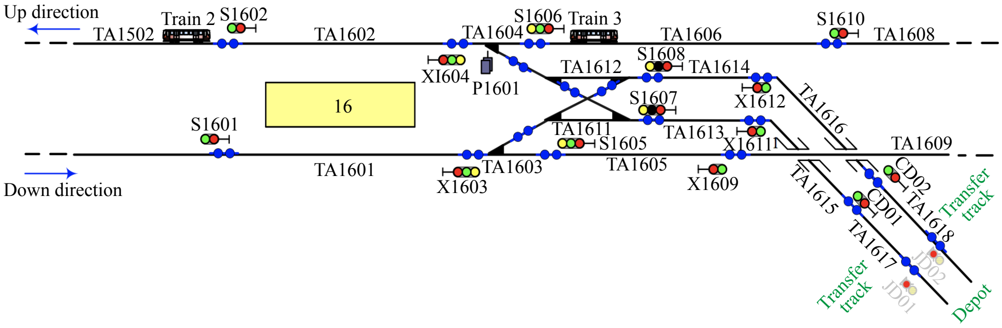


Figure 4 – Depot departure Train 2 and mainline Train 3 tracking operation scenario

3.3 Model formulation

Taking one urban rail transit line as an example, a timetabling optimisation model is proposed considering the headway coordination between the mainline and the depot during the transition period from off-peak to peak time. The purpose is to jointly improve the train operational efficiency and passenger satisfaction. *Figure 5* shows a schematic diagram of the rail transit line. There are $2I$ platforms, with platform F used for short-turn routing. The depot is connected to the down-direction platform M and up-direction platform ($2I$-M+1) through entrance and exit lines.

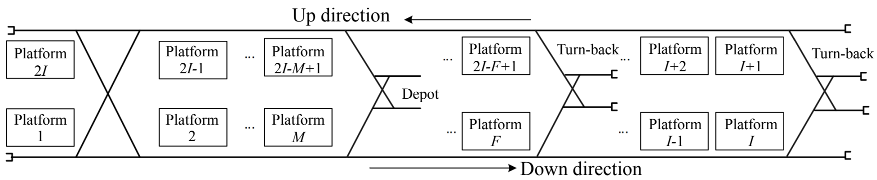


Figure 5 – Schematic diagram of one urban rail transit line

Based on the practical operation situation of the urban rail transit line, the following assumptions are made.

1) All trains running on the mainline or departing from the depot are of the same type and have the same parameters.
2) Each train service dwells at every station, and there is no operational scenario for overtaking on sections or at station.
3) Only after all passengers have alighted the train at the turn-back platform does the train start the turn-back operation.

Table 1 shows the set definitions.

From the definition of $Z_{l\text{-}up}$, $Z_{s\text{-}up}$ and $Z_{n\text{-}up}$ in *Table 1*, it can be concluded that: $Z_{l\text{-}up}=\{I+1,I+2,...,2I\}$, $Z_{s\text{-}up}=\{2I\text{-}F+1,2I\text{-}F+2,...,2I\}$, $Z_{n\text{-}up}=\{I+1,I+2,...,2I\text{-}F\}$.

Table 1 – Set definitions

Notation	Definition
K	Set of trains
Z	Set of platforms
T	Research period
T_{start}	Start time of research period
T_{end}	End time of research period
k	Index of trains
i, r, j	Index of platforms
K_l	Index of full-length routing trains
K_s	Index of short-turn routing trains
$Z_{l\text{-}up}$	Index of up-direction platforms for full-length routing
$Z_{s\text{-}up}$	Index of up-direction platforms for short-turn routing
$Z_{n\text{-}up}$	Index of up-direction platforms of non-collinear sections for full-length routing and short-turn routing

Table 2 shows the intermediate variables and corresponding parameters.

Table 2 – Intermediate variables and corresponding parameters

Notation	Definition
$t_{k,i}^{(w)}$	The dwell time of train service k at platform i
$t_{k,i}^{(d)}$	Time at which train service k departs from platform i
$t_{k,i}^{(a)}$	Time at which train service k arrives at platform i
β	Train dwell time coefficient
d_i	The minimum dwell time of train services at platform i
$n_{k,i}^{(al)}$	Number of passengers alighting at platform i for train service k
$n_{k,i}^{(b)}$	Number of passengers boarding at platform i for train service k
N_{flat}	Number of train services passing through the mainline before insertion
N_{peak}	Number of train services passing through the mainline after insertion has finished
N_{depot}	Number of trains that can be dispatched from the depot to the mainline
H_{flat}	Average headway of mainline trains before insertion
H_{peak}	Average headway of mainline trains after insertion has finished
$H_{min}^{opt}(depot)$	Optimised minimum headway for trains departing from the depot
$T_i^{(r)}$	Rated running time of the train service in section $[i,i+1]$
Δ_1	Maximum deviation between the actual and rated running times of the train service in section $[i,i+1]$
h_{min}	Minimum headway of short-turn routing train services
h_{max}	Maximum headway of short-turn routing train services
$l_k\left[t_{k,i}^{(d)}\right]$	Number of passengers aboard train service k when it departs from platform i
$l_k\left[t_{k,i}^{(a)}\right]$	Number of passengers aboard train service k when it arrives at platform i
$n_{k,i,j}^{(b)}$	Number of passengers boarding train service k at platform i with a destination of platform j
$t_{k,i}^{(c)}$	The arrival time at platform i of the last passenger boarding train service k
$n_{i,j}\left[t_{k^-,i}^{(c)},t_{k,i}^{(c)}\right]$	Number of passengers arriving at platform i with a destination of platform j during time period $\left[t_{k^-,i}^{(c)},t_{k,i}^{(c)}\right]$
$\tau_{i,j}(t)$	The passenger arrival rate at platform i with a destination of platform j at time t
$l_k(t)$	Number of passengers aboard train service k at time t
δ_{max}	Maximum passenger capacity coefficient of the train
c_k	Rated passenger capacity of the train
$n_{k,i}^{(t,t+1)}$	Number of arriving passengers assigned to train service k at platform i during time period $[t,t+1]$
$n_{i,j}^{(t,t+1)}$	Number of passengers arriving at platform i with a destination of platform j within time period $[t,t+1]$
$\delta_{k,i}$	Capacity rate of train service k between platforms i and $i+1$

Suppose that train services $k+$ and k are continuously tracked on the mainline before train service $k(e)$ enters the mainline from the depot. Train service $k(e)$ runs between train services k^+ and k after its insertion, then the three train services continuously tracked on the mainline section are k, $k(e)$, and k^+. To enhance the robustness of the train diagram, a 10% margin is considered for the train operation intervals. The constraints are as follows:

$$h_{k,i} = t_{k(e),i}^{(d)} - t_{k,i}^{(d)} \geq 1.1 \cdot H_{min}(1,2),\ \ \forall k,k(e) \in K_s;\ \ \forall i \in Z_{s\text{-}up}\ \text{ or }\ \forall k,k(e) \in K_l;\ \ \forall i \in Z_{l\text{-}up} \tag{6}$$

$$h_{k(e),i} = t_{k^+,i}^{(d)} - t_{k(e),i}^{(d)} \geq 1.1 \cdot H_{min}(2,3),\ \ \forall k,k(e) \in K_s;\ \ \forall i \in Z_{s\text{-}up}\ \text{ or }\ \forall k,k(e) \in K_l;\ \ \forall i \in Z_{l\text{-}up} \tag{7}$$

where $h_{k,i}$ is one of the integer decision variables and represents the headway between train services k and

k^+ at platform i.

To fulfil the requirements of train insertion, the optimised minimum headway for trains departing from the depot should be less than $H_{min}(1,2)$:

$$H_{min}^{opt}(depot) \leq H_{min}(1,2) \tag{8}$$

The numbers of trains N_{flat}, N_{peak} and N_{depot} are calculated using *Equations 9–11*, respectively. See *Table 2* for the meanings of the various symbols.

$$N_{flat} = \left\lfloor \frac{T_{end} - T_{start}}{H_{flat}} \right\rfloor + 1 \tag{9}$$

$$N_{peak} = \left\lfloor \frac{T_{end} - T_{start}}{H_{peak}} \right\rfloor + 1 \tag{10}$$

$$N_{depot} = \left\lfloor \frac{T_{end} - T_{start}}{H_{min}^{opt}(depot)} \right\rfloor + 1 \tag{11}$$

N_{depot} represents the train insertion capacity from the depot to the mainline, which should be greater than the number of trains that need to be dispatched. The relevant constraint is

$$N_{depot} > (N_{peak} - N_{flat}) \tag{12}$$

Equation 13 gives the constraint of actual number of inserted train services.

$$\sum_{k \in K} (1 - a_k) \leq (N_{peak} - N_{flat}) \quad \forall k \in K \tag{13}$$

where a_k is a binary 0–1 variable and denotes whether train service k comes from the depot. If a_k=0, the train service k comes from the depot; if a_k=1, the train service k comes from the mainline. $\sum_{k \in K} (1 - a_k)$ represents the actual number of trains inserted to the mainline after optimisation.

According to the definition of train headway and operation rules of adjacent tracking trains, the constraints for the train headway, train dwell time, train arrival time, train departure time and train running time are as follows:

$$h_{k,i} = t_{k^+,i}^{(d)} - t_{k,i}^{(d)}, \quad \forall k \in K_s; \quad \forall i \in Z_{s\text{-}up} \text{ or } \forall k \in K_l; \quad \forall i \in Z_{l\text{-}up} \tag{14}$$

$$t_{k,i}^{(w)} = \beta\left[n_{k,i}^{(al)} + n_{k,i}^{(b)}\right] + d_i, \quad \forall k \in K_s; \quad \forall i \in Z_{s\text{-}up} \text{ or } \forall k \in K_l; \quad \forall i \in Z_{l\text{-}up} \tag{15}$$

$$t_{k,i}^{(d)} = t_{k,i}^{(a)} + t_{k,i}^{(w)}, \quad \forall k \in K_s; \quad \forall i \in Z_{s\text{-}up} \text{ or } \forall k \in K_l; \quad \forall i \in Z_{l\text{-}up} \tag{16}$$

$$t_{k,i+1}^{(a)} = t_{k,i}^{(d)} + t_{k,i}^{(r)}, \quad \forall k \in K_s; \quad \forall i \in Z_{s\text{-}up} \text{ or } \forall k \in K_l; \quad \forall i \in Z_{l\text{-}up} \tag{17}$$

where $t_{k,i}^{(r)}$ is one of the integer decision variables and represents the running time of train service k between platforms i and i+1.

The research period constraint is as follows:

$$T_{start} \leq t_{k,2I-M+1}^{(d)} \leq T_{end}, \quad \forall k \in K_s \text{ or } \forall k \in K_l \tag{18}$$

The deviation value of train running time and headway should be within a certain range to meet the line and train conditions as well as passenger satisfaction. The constraints are as follows:

$$\left| t_{k,i}^{(r)} - T_i^{(r)} \right| \leq \Delta_1, \quad \forall k \in K_s; \forall i \in Z_{s\text{-}up} \text{ or } \forall k \in K_l; \forall i \in Z_{l\text{-}up} \tag{19}$$

$$\begin{cases} h_{min} \leq h_{k,i} \leq h_{max}, & \forall k \in K; \forall i \in Z_{s\text{-}up} \\ h_{min} \leq h_{k,i} \leq h_{max}, & \forall k \in K_l; k = 3n+1 (n \in \mathbf{N}); \forall i \in Z_{n\text{-}up} \\ 2h_{min} \leq h_{k,i} \leq 2h_{max}, & \forall k \in K_l; k = 3n+2 (n \in \mathbf{N}); \forall i \in Z_{n\text{-}up} \end{cases} \tag{20}$$

The constraint related to the number of passengers on a train is

$$l_k\left[t_{k,i}^{(d)}\right] = l_k\left[t_{k,i}^{(a)}\right] - n_{k,i}^{(al)} + n_{k,i}^{(b)}, \quad \forall k \in K_s; \forall i \in Z_{s\text{-}up} \text{ or } \forall k \in K_l; \forall i \in Z_{l\text{-}up} \tag{21}$$

As shown in *Equations 22 and 23*, the number of passengers boarding and alighting at platform i for train service k should be calculated separately according to the route of the train service and the platform served.

$$\begin{cases} n_{k,i}^{(b)} = \sum\limits_{j \in Z_{l\text{-}up}, i<j} n_{k,i,j}^{(b)} = \sum\limits_{j \in Z_{l\text{-}up}, i<j} n_{i,j}\left[t_{k^-,i}^{(c)}, t_{k,i}^{(c)} \right] \\ n_{k,i}^{(al)} = \sum\limits_{r \in Z_{l\text{-}up}, r<i} n_{k,r,i}^{(b)} = \sum\limits_{r \in Z_{l\text{-}up}, r<i} n_{r,i}\left[t_{k^-,r}^{(c)}, t_{k,r}^{(c)} \right] \end{cases}, \quad \forall k \in K_l; \forall i \in Z_{l\text{-}up} \tag{22}$$

$$\begin{cases} n_{k,i}^{(b)} = \sum\limits_{j \in Z_{s\text{-}up}, i<j} n_{k,i,j}^{(b)} = \sum\limits_{j \in Z_{s\text{-}up}, i<j} n_{i,j}\left[t_{k^-,i}^{(c)}, t_{k,i}^{(c)} \right] \\ n_{k,i}^{(al)} = \sum\limits_{r \in Z_{s\text{-}up}, r<i} n_{k,r,i}^{(b)} = \sum\limits_{r \in Z_{s\text{-}up}, r<i} n_{r,i}\left[t_{k^-,r}^{(c)}, t_{k,r}^{(c)} \right] \end{cases}, \quad \forall k \in K_s; \forall i \in Z_{s\text{-}up} \tag{23}$$

The number of passengers arriving at platform i with a destination of platform j during time period $\left[t_{k^-,i}^{(c)}, t_{k,i}^{(c)} \right]$ is calculated according to the following equation:

$$n_{i,j}\left[t_{k^-,i}^{(c)}, t_{k,i}^{(c)} \right] = \sum_{t_{k^-,i}^{(c)}}^{t_{k,i}^{(c)}} \tau_{i,j}(t) \quad \forall i,j \in Z, i<j \tag{24}$$

The arrival time at platform i of the last passenger boarding train service k is calculated as follows:

$$t_{k,i}^{(c)} = \min\left\{ t_{k,i}^{(d)}, \max\left\{ t \middle| l_k(t) < \delta_{max} c_k \le l_k(t) + n_{k,i}^{(t,t+1)} \right\} \right\}$$
$$\forall k \in K_s; \forall i \in Z_{s\text{-}up} \text{ or } \forall k \in K_l; \forall i \in Z_{l\text{-}up} \tag{25}$$

The number of arriving passengers assigned to train service k at platform i during time period $[t,t+1]$ is calculated as follows:

$$\begin{cases} n_{k,i}^{(t,t+1)} = \sum\limits_{j \in Z_{s\text{-}up}, i<j} n_{i,j}^{(t,t+1)} \quad \forall k \in K_s; \forall i \in Z_{s\text{-}up} \\ n_{k,i}^{(t,t+1)} = \sum\limits_{j \in Z_{l\text{-}up}, i<j} n_{i,j}^{(t,t+1)} \quad \forall k \in K_l; \forall i \in Z_{l\text{-}up} \end{cases} \tag{26}$$

The number of passengers arriving at platform i with a destination of platform j within time period $[t,t+1]$ is expressed as follows:

$$n_{i,j}^{(t,t+1)} = \sum_{t}^{t+1} \tau_{i,j}(t) \quad \forall i,j \in Z, i<j \tag{27}$$

The capacity rate of train service should not exceed the maximum passenger capacity coefficient, with the following constraint:

$$\delta_{k,i} = l_k\left[t_{k,i}^{(d)} \right] / c_k \le \delta_{max}, \quad \forall k \in K_s; \forall i \in Z_{s\text{-}up} \text{ or } \forall k \in K_l; \forall i \in Z_{l\text{-}up} \tag{28}$$

The optimisation objectives of the proposed model concern the number of trains inserted to the mainline from the depot, the maximum train capacity rate and the average deviation value of train headway. To achieve a short train headway during peak time, the number of trains inserted should be as high as possible. This gives the single objective function shown in *Equation 29*. To ensure passenger satisfaction, the maximum train capacity rate and average deviation in the train headway should be as small as possible. This gives the single objective functions shown in *Equations 30 and 31*, respectively.

$$\max f_1 = \max\left\{ \sum_{k \in K} (1 - a_k) \right\}, \forall k \in K \tag{29}$$

$$\min f_2 = \min\left\{ \max(\delta_{k,i}) \right\} \quad \forall k \in K_s; \forall i \in Z_{s\text{-}up} \text{ or } \forall k \in K_l; \forall i \in Z_{l\text{-}up} \tag{30}$$

$$\min f_3 = \min\left\{ \frac{\sum\limits_{k \in K, i \in Z_{s\text{-}up}} \left| h_{k,i} - H_{peak} \right| + \sum\limits_{k \in K_l, k=3n+1 (n \in \mathbf{N}), i \in Z_{n\text{-}up}} \left| h_{k,i} - H_{peak} \right| + \sum\limits_{k \in K_l, k=3n+2 (n \in \mathbf{N}), i \in Z_{n\text{-}up}} \left| h_{k,i} - 2H_{peak} \right|}{|F| \cdot \left(N_s^{peak} - 1 \right) + |I| \cdot \left(N_l^{peak} - 1 \right)} \right\} \tag{31}$$

In the *Equation 31*, $|F|$ and $|I|$ are the corresponding numbers for platforms F and I, respectively.

Obviously, *Equation 29* is equivalent to

$$\min f_{1\text{-}1} = \min\left\{ N_{peak} - \sum_{k \in K} (1 - a_k) \right\}, \forall k \in K \tag{32}$$

Then, the goal of collaborative optimisation is as follows:

$$\min f = \min\{\lambda_1 f_{1\text{-}1} + \lambda_2 f_2 + \lambda_3 f_3\} \tag{33}$$

In the *Equation 33*, λ_1, λ_2, and λ_3 are the coefficients of the objective function, $\lambda_1+\lambda_2+\lambda_3=1$.

4. CASE STUDY

We take one urban rail transit line as an example. The total length of the line is 41.57 km, and there are 23 stations giving a total of 46 platforms set for both directions. The average headway of the trains is 240 s during off-peak times (from 05:30:00–07:30:00) and 120 s during peak times (from 07:30:00–09:00:00). The relevant train and model parameters are given in *Table 3*.

Table 3 – Parameters of the train and model

δ_{max}	λ_1	λ_2	λ_3	c_k	Δ_1 (s)	h_{min} (s)	h_{max} (s)
1.2	0.3	0.6	0.1	1,440	6	108	180

At present, trains operate at regular intervals with a headway of 240 s before train insertion. Before optimisation, according to *Equation 1*, the minimum headway for trains departing the depot $H_{min}^{ori}(depot)$ is 255.13 s, which is much greater than $H_{min}(1,2)$, so it is only possible for the depot to perform 2:1 train insertion to the mainline. That is, for every two trains passing through the up-platform of the 16th station, the depot can insert one train service.

The time needed to insert all trains from the depot to the mainline exceeds 150 min before optimisation, with only 13 train services inserted from 05:50:00–07:30:00, resulting in a significant deviation between the actual and planned train operations. In this case, the dispatcher must manually adjust or even cancel some train services, reducing the punctuality rate of mainline trains.

As the headway of trains during the morning peak time should be compressed to 120 s, it is necessary to insert 25 trains before the start of the morning peak time (07:30:00). Before optimisation, 12 trains are still waiting to be inserted to the mainline at 07:30:00, resulting in insufficient mainline train capacity. To complete all train insertion tasks before 07:30:00, the insertion must be performed in advance, or other depots (if any) should be used to simultaneously insert trains to the mainline.

5. RESULTS AND DISCUSSION

The optimisation model established has many independent variables and constraints and belongs to large-scale NP-hard problem, and the computational complexity is mainly determined by the number of train services and platforms. Specifically, the maximum number of decision variables $h_{k,i}$, $t_{k,i}^{(r)}$ and a_k are $|K\text{-}1|\cdot|Z|$, $|K|\cdot|Z\text{-}1|$ and $|K|$, respectively, and there are also numerous computations required for processing constraints *6–28.*

NSGA-II heuristic algorithm outperforms the original NSGA algorithm in terms of objective function convergence and computation time [34], and is widely used in the field of railway transportation because of its excellent performance in solving large-scale optimisation problems [35–37].

We use NSGA-II to solve the proposed model. The parameter settings are as listed in *Table 4*.

Table 4 – Parameter settings related to NSGA-II

Name of parameter	Definition	Value
PN	Population size	80
P_c	Crossover probability	0.85
P_m	Mutation probability	0.1
$Gen(max)$	Maximum number of iterations	500

Matlab R2019b was used to program the algorithm on a personal computer with an Intel(R) Core(TM) i3-9100 CPU 3.60 GHZ, 8.00 GB memory and Windows10 64-bit operating system. The objective function reaches a minimum value of 9.34. After optimisation, a total of 25 trains can be inserted (f_1=25, $f_{1\text{-}1}$=26), with a maximum train capacity rate of 0.975 (f_2=0.975) and an average headway deviation value of 9.5 s (f_3=9.5) during the research period.

The algorithm takes 232 s to execute. As shown in *Figure 6*, as the number of iterations increases, the optimal individual value continuously decreases from an initial value of 12.12 to 9.34. After the 210th iteration, the optimal individual value tends to stabilise and its value no longer changes.

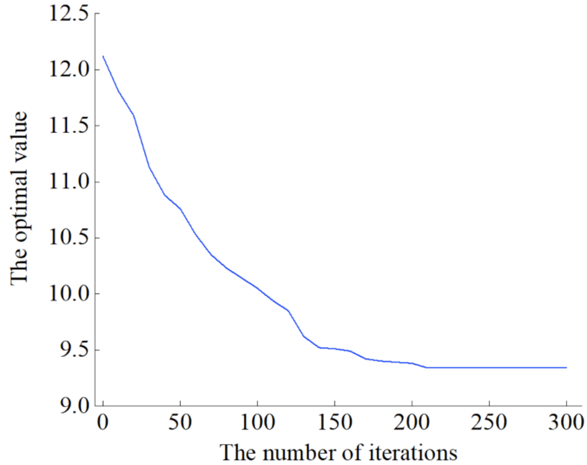


Figure 6 – Evolution of optimal value in NSGA-II

The results obtained from NSGA-II have been compared with those obtained from the Simulated Annealing algorithm (initial temperature of 500, termination temperature of 1, attenuation coefficient of 0.85 and Markov chain length of 1000) and CPLEX solver on the same computer. The results are shown in *Table 5*: the Simulated Annealing algorithm terminates after 1875 seconds and obtains an optimal value of 9.73 (relatively poor compared to the results obtained by NSGA-II), while CPLEX solver (Gap set as 0.05) fails to find a solution within 10 hours. The comparison of these algorithms proves the effectiveness and efficiency of NSGA-II in solving the proposed model.

Table 5 – Comparison of algorithms

Algorithm	**Computation time (s)**	**Returned optimal value**
NSGA-II	232	9.34
Simulated Annealing	1875	9.73
CPLEX (Gap set as 0.05)	36000	/

According to *Equation 3*, the optimised minimum headway for trains departing the depot $H_{min}^{opt}(depot)$ is 90.93 s. Because $H_{min}^{opt}(depot)<H_{min}(1,2)$ and the corresponding train headway satisfies *Equation 34*, 1:1 train insertion from the depot onto the mainline can be achieved after optimisation.

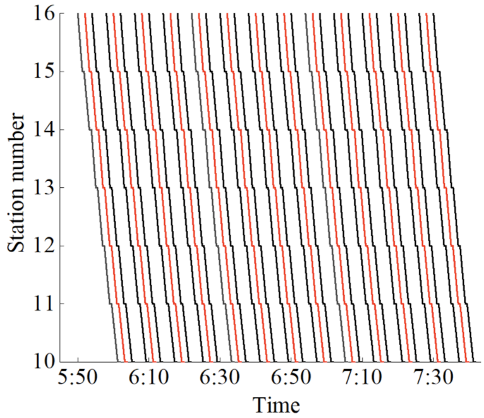
$$1.1\cdot[H_{min}^{opt}(depot)+H_{min}(2,3)]\leq H_{flat} \tag{34}$$

After optimisation, all train insertions have been completed before 07:30:00. The optimised train departure times at the 16th station's up-direction platform are presented in *Table 6*. A total of 25 trains have been inserted within 96 min – the first inserted train (service number 2) departs at 05:52:43 and the last inserted train (train service 50) departs at 07:28:04, so the efficiency of train insertion has been improved and the workload of dispatchers and operators has been greatly reduced after optimisation.

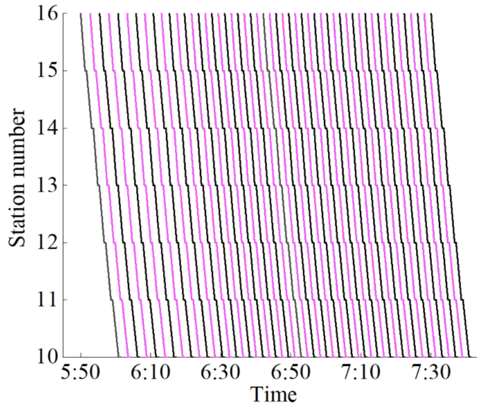
Figure 7a shows train operation lines before optimisation, where black solid lines represent mainline trains and red solid lines represent the trains inserted onto the mainline from the depot. *Figure 7b* shows that after optimisation, where black solid lines represent mainline trains and the magenta solid lines represent trains inserted. *Figure 7* visually shows that compared to a 2:1 proportional train insertion before optimisation, it allows for a 1:1 proportional train insertion after optimisation, which greatly improves the efficiency of train operation.

Table 6 – Train departure times at 16th station's up-direction platform after optimisation

Service no.	Departure time	Service no.	Departure time	Service no.	Departure time	Service no.	Departure time
1	05:50:00	14	06:21:39	27	06:45:43	40	07:09:21
2	05:52:43	15	06:23:37	28	06:47:33	41	07:11:09
3	05:55:24	16	06:25:32	29	06:49:21	42	07:12:59
4	05:58:03	17	06:27:25	30	06:51:11	43	07:14:50
5	06:00:41	18	06:29:17	31	06:52:59	44	07:16:42
6	06:03:17	19	06:31:08	32	06:54:49	45	07:18:34
7	06:05:49	20	06:32:59	33	06:56:37	46	07:20:27
8	06:08:18	21	06:34:49	34	06:58:27	47	07:22:20
9	06:10:44	22	06:36:39	35	07:00:15	48	07:24:14
10	06:13:04	23	06:38:27	36	07:02:05	49	07:26:09
11	06:15:19	24	06:40:17	37	07:03:53	50	07:28:04
12	06:17:30	25	06:42:05	38	07:05:43	51	07:30:00
13	06:19:37	26	06:43:55	39	07:07:31	--	--



a) Before optimisation



b) After optimisation

Figure 7 – Train operation lines after train insertion finished (up-direction from station 16 to station 10)

The train headways 05:50:00–07:30:00 are shown in *Figure 8*. The red dots represent the headway between adjacent trains after the completion of train insertion before optimisation. Due to the minimum headway required for trains leaving the depot, only a total of 13 trains can be inserted, and the headway alternates in cycles of 120 s, 120 s and 240 s. The black dots represent the headway between adjacent trains after optimisation, with a total of 25 trains inserted during the research period.

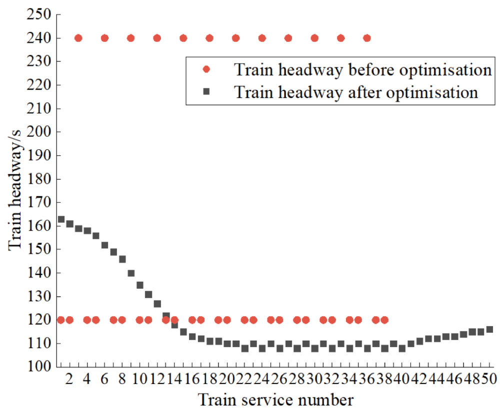


Figure 8 – Train headway for each train service before and after optimisation

After optimisation, train services adapt well to unevenly distributed passenger flows via irregular train headways, enabling better responses to fluctuating passenger arrival rates. As can be found from *Table 7*, the headway of the first 21 trains (departure times of 05:50:00–06:34:49) gradually decreases from 163 s to 110 s to deal with the high-density passenger flow during the morning peak time. The constraint of *Equation 6* means that the headway between the inserted train and the preceding train is compressed to a maximum of 110 s. Therefore, the headway between trains 22–42 (departure times of 06:36:39–07:12:59) fluctuates between 110 s and 108 s: the headway between the inserted train and the preceding train is 110 s, and the headway between the inserted train and the following train is 108 s. As the passenger flow in the relevant sections decreases, the train headway gradually increases from the 43rd train (departure time of 07:14:50), and the headway of the 50th and 51st train reaches 116 s.

Table 7 – Mainline train headway at the 16th station after optimisation

Service no.	Headway (s)	Service no.	Headway (s)	Service no.	Headway (s)	Service no.	Headway (s)
1, 2	163	14, 15	118	27, 28	110	40, 41	108
2, 3	161	15, 16	115	28, 29	108	41, 42	110
3, 4	159	16, 17	113	29, 30	110	42, 43	111
4, 5	158	17, 18	112	30, 31	108	43, 44	112
5, 6	156	18, 19	111	31, 32	110	44, 45	112
6, 7	152	19, 20	111	32, 33	108	45, 46	113
7, 8	149	20, 21	110	33, 34	110	46, 47	113
8, 9	146	21, 22	110	34, 35	108	47, 48	114
9, 10	140	22, 23	108	35, 36	110	48, 49	115
10, 11	135	23, 24	110	36, 37	108	49, 50	115
11, 12	131	24, 25	108	37, 38	110	50, 51	116
12, 13	127	25, 26	110	38, 39	108	--	--
13, 14	122	26, 27	108	39, 40	110	--	--

Table 8 indicates that the average dwell time of the train service at each station has decreased compared with that before optimisation, with an optimisation ratio of 9.3%–19.7%. The total dwell time of one train at all stations has been reduced by 114.4 s, with an optimisation ratio of 15.2%. The compressed dwell time has a good effect on reducing passenger travel times.

Table 8 – Average dwell time of the train service before and after optimisation

Station no.	Dwell time before optimisation (s)	Dwell time after optimisation (s)	Optimisation ratio (%)	Station no.	Dwell time before optimisation (s)	Dwell time after optimisation (s)	Optimisation ratio (%)
1	--	--	--	13	30	24.6	18.0
2	30	25.3	15.7	14	35	30.2	13.7
3	30	25.5	15.0	15	30	24.7	17.7
4	35	29.2	16.6	16	30	24.5	18.3
5	35	28.9	17.4	17	35	29.9	14.6
6	35	29.5	15.7	18	30	24.9	17.0
7	45	39.2	12.9	19	40	34.7	13.3
8	50	42.6	14.8	20	30	24.1	19.7
9	40	36.3	9.3	21	30	25.3	15.7
10	40	35.7	10.8	22	30	24.9	17.0
11	30	26.8	10.7	23	35	29.3	16.3
12	30	24.5	18.3	--	--	--	--

According to *Table 9*, the maximum train capacity rate at each station have significantly decreased by 18.7%–35.6% compared with that before optimisation, which is beneficial for improving passenger comfort.

Table 9 – Comparison of maximum train capacity rate before and after optimisation

Station no.	Maximum train capacity rate before optimisation	Maximum train capacity rate after optimisation	Optimisation ratio (%)	Station no.	Maximum train capacity rate before optimisation	Maximum train capacity rate after optimisation	Optimisation ratio (%)
1	0.623	0.441	29.2	12	1.135	0.731	35.6
2	0.642	0.458	28.7	13	0.889	0.594	33.2
3	0.746	0.518	30.6	14	0.876	0.572	34.7
4	0.969	0.658	32.1	15	0.802	0.539	32.8
5	1.179	0.785	33.4	16	0.764	0.505	33.9
6	1.200	0.975	18.7	17	0.737	0.513	30.4
7	1.200	0.968	19.3	18	0.763	0.506	33.7
8	1.200	0.897	25.3	19	0.887	0.615	30.7
9	1.200	0.874	27.2	20	0.736	0.519	29.5
10	1.200	0.863	28.1	21	0.617	0.446	27.7
11	1.195	0.799	33.1	22	0.536	0.397	25.9

6. CONCLUSIONS

This paper presents an optimisation train timetabling model for an urban rail transit line, which considers headway coordination between the mainline and depot during the transition period from off-peak to peak period based on the compressed minimum headway of depot departing trains and the tracking operation scenario of trains inserted. The aim is to jointly improve the train operational efficiency and passenger satisfaction in consideration of constraints, i.e. the requirements of train insertion, train headway, train dwell time and running time, and the number of passengers assigned. NSGA-II algorithm is designed to solve the model, and the comparison with computational results of other algorithms are used to demonstrate the effectiveness and efficiency of the NSGA-II algorithm.

A case study has shown that the optimised timetable better matches with variable passenger flows, which significantly improves the train operation efficiency and passenger satisfaction, fulfills the headway requirements of mainline trains during peak hours, and reduces the workload of dispatchers and operators. Compared with that before optimisation, the average dwell time of trains at each station is reduced after optimisation, with an optimisation ratio of 9.3%–19.7%. The total dwell time of one train at all stations is reduced by up to 114.4 s, with an optimisation ratio of 15.2%, and the maximum train capacity rate at various stations is reduced by 18.7%–35.6%. The proposed model and approach can provide a theoretical basis for improving the operational efficiency and organisational management of urban rail transit trains.

It is assumed that all trains running on the mainline or departing from the depot have the same parameters, it needs to consider the mixed operation of various length and marshalling trains to achieve better overall capacity and economic benefits in future research. In addition, as resource sharing between different lines has led to a trend towards networked train operation across urban rail transit lines, the optimisation of headway between trains from different lines should be investigated in future. Finally, if multiple depots are considered simultaneously to be used for departing trains, more constraints need to be added to expand the model, which will also be studied in the future.

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考虑正线与车辆段追踪间隔协同的列车时刻表优化模型

摘要：

基于列车从车辆段向正线追踪运行场景及列车运行限制条件，本文构建了考虑过渡时段正线与车辆段追踪间隔协同的城市轨道交通时刻表优化模型。模型优化目标为插入列车数量、列车最大满载率以及追踪间隔平均偏离。设计了第二代非支配排序遗传算法对模型进行求解。案例分析表明：优化后可插入25列车次，列车最大满载率为0.975，追踪间隔平均偏离为9.5s，极大地提高了列车运行效率及乘客满意度。相较于优化前时刻表，优化后列车在各个车站平均停站时间及最大满载率均有所减少。本文所构建模型及方法可为城市轨道交通列车时刻表优化及运营管理提供依据。

关键词：

城市轨道交通；优化模型；时刻表；列车满载率；追踪间隔偏离