Traffic & Transportation

Optimising Electric Flex-Route Feeder Transit Service with Dynamic Wireless Charging Technology

Tianyang GAO[1], Dawei HU[2], Gang CHEN[3], Steven CHIEN[4], Bingshan MA[5]

Original Scientific Paper
Submitted: 17 Oct. 2023
Accepted: 30 Jan. 2024

[1] tianyang@chd.edu.cn, Chang'an University, School of Transportation Engineering

[2] corresponding author, dwhu@chd.edu.cn, Chang'an University, School of Transportation Engineering

[3] gc1991@chd.edu.cn, Zhejiang Scientific Research Institute of Transport

[4] chien@njit.edu, Chang'an University, School of Transportation Engineering; New Jersey Institute of Technology, Department of Civil and Environmental Engineering

[5] bingshanma@chd.edu.cn, Zhejiang Scientific Research Institute of Transport

Publisher:
Faculty of Transport and Traffic Sciences, University of Zagreb

ABSTRACT

The emergence of battery electric buses (BEBs) can alleviate environmental problems caused by tailpipe emissions in transit system. However, the high cost of on-board batteries and range anxiety hinder its further development. Recently, the advent of dynamic wireless power transfer technology (DWPT) has become a potential solution to promote the development of BEBs. Hence, this study focuses on the application of DWPT in flex-route transit system. A mixed integer non-linear model is proposed to simultaneously optimise the bus routing and the selection of corresponding bus types considering the constraints of passengers' travel time, battery size and bus capacity. The objective is to minimise both transit agency cost and passengers' travel time cost. A tangible hybrid variable neighbourhood search (HVNS) consisting of simulated annealing (SA) and variable neighbourhood search (VNS) is developed to solve the proposed model efficiently. Compared with GAMS (DICOPT solver) and VNS, the proposed algorithm can considerably improve computational efficiency. The results suggest that the proposed model can effectively determine the BEBs' routing and bus type for flex-route transit system powered by DWPT through a case study in Xi'an China. A comparative analysis shows the proposed model takes 12.97% less total cost than the alternative model with terminal charging technology (TCT).

KEYWORDS

flex-route feeder transit; electric bus; dynamic wireless charging; opportunity charging; hybrid variable neighbourhood search.

1. INTRODUCTION

The global transportation system is confronted with several environmental challenges (e.g. low energy efficiency and greenhouse gas emission). As reported in International Energy Agency 2019, the emissions from the transportation sector account for 27% of global emissions, which is expected to reach 30% by 2030 and to approach 50% by 2050 if no key measures are undertaken [1]. To mitigate the emission problems, many countries and agencies have adopted policies to electrify the transportation system in recent years, especially for the urban transit system. The battery-electric engine is considered the best alternative to replace the gasoline or diesel engine. Compared with the traditional fossil fuel counterpart, battery-electric buses (BEBs) provide immense potential for urban livability and sustainability, because the direct tailpipe emissions of BEBs is zero and it operates with higher efficiency [2]. With sufficiently large on-board battery, BEBs can have comparable driving range as a traditional diesel bus. However, the weight and cost of the battery will increase with the increase of battery capacity, and the on-board battery for a long-range BEBs can account for 25% of the total weight and 39% of the capital cost [3], which hinder its popularity [4].

Meanwhile, there are also some specific problems in the actual operation process, such as short mileage, long charging time, and unbalanced charging stack distribution during peak hours [5].

In the past few years, some fast-charging technologies (e.g. conductive fast charging and inductive fast charging) have been developed, and the peak charging rate can even reach 300–500 kW [3]. Thus, the opportunity charging, which allows BEBs to charge during the operation (especially when passengers get on and off), can be achieved. Recently, dynamic wireless charging technology (DWPT), allowing BEBs to charge in motion, has been proposed and deemed as the most potential way to achieve opportunity charging as well as to reduce battery capacity and costs, while alleviating the battery range limitation of BEBs [6]. The DWPT can effectively reduce the required battery size for BEBs by providing them frequent on-route charging opportunities. Several feasibility studies about DWPT have been conducted by Korea Advanced Institute of Science and Technology [7] and Texas USA [8]. The results suggested that DWPT charging devices can be deployed along the road in the closed experimental environment. While in the open environment (real urban bus system), they can only be deployed in the bus stop area due to the difficulties in their deployment and maintenance.

Several previous studies [6–12] have focused on the joint optimisation of battery size and the deployment of charging devices in the fixed-route bus system powered by DWPT. When passing through the bus stops installed with DWPT charging devices (recharging stops), the fixed-route BEBs can be recharged as required.

Unlike the fixed-route counterpart, the flex-route transit system (also known as demand responsive transit) can offer door-to-door service for passengers [13]. In the context of the internet, the flex-route transit system can greatly meet passengers' personalised and customised travel demands. The emergence of DWPT can promote the further development of BEBs, and alleviate the charging anxiety. The application of DWPT to the flex-route transit system can reduce battery costs while meeting passengers' demands, because the flex-route BEBs in the DWPT transit system can recharge energy during operation. As this study focuses on flex-route feeder transit, also known as demand-responsive feeder transit, the operational length and passengers' demands vary for each route. Therefore, employing different vehicle types for various routes is necessary and meaningful to meet the diverse passengers' demands. Thus, a fundamental question is correspondingly raised, "how to optimise the flex-route feeder bus routing as well as corresponding bus types in DWPT system?" To fill this gap, this study develops an electrified flex-route feeder transit service with DWPT, and the main contributions of this study are summarised as follows.

1) To minimise passengers' travel time cost, bus operating cost and bus depreciation cost, a novel mixed integer non-linear programming (MINLP) model is proposed to simultaneously optimise the flex-route bus routing as well as corresponding bus types (consisting of different battery sizes, capacity sizes, energy consumption rate and unit operating costs). The operating time and the charging time can be allocated to balance the transit agencies cost (including bus operating cost and bus depreciation cost) and the customer cost (passengers' travel time cost), which can improve efficiency and competitiveness.
2) To solve real-world cases, a tangible HVNS consisting of SA and VNS is developed to solve the problem efficiently.
3) Based on the proposed model and the case study from the real world, the results and sensitivity analysis suggested the impact of DWPT on the flex-route bus system, which can help decision-makers further assess its performance.

The remainder of the paper is organised as follows. An overview of the recent literature is provided in Section 2. Section 3 introduces the background information and states the formulation of the proposed model. The HVNS is detailed in Section 4. Section 5 covers the case study from the real world, where the experimental results and sensitivity analysis are discussed. Finally, Section 6 presents the concluding remarks and future research directions.

2. LITERATURE REVIEW

To our best knowledge, few previous studies have ever focused on the optimisation of the flex-route transit service based on the DWPT transit system. We will review the state-of-the-art of these problems from three aspects, i.e. (1) flex-route bus service optimisation; (2) application of wireless charging technology in transit bus service; and (3) solution algorithm.

The first subsection focuses on flex-route bus service optimisation. Quadrifoglio et al. [13] employed a simulation tool to study the optimisation of demand-responsive transit service considering zoning service. Later, they further designed a special flex-route transit mode "Mobility Allowance Shuttle Transit" (MAST) [14] and corresponding insertion heuristic algorithm [15]. Kim et al. [16 – 18] presented the closed-form equations to study the coordination between conventional and flex-route transit services from different perspectives (e.g. variable bus types [16], timed transfer at the transfer hub [17], and maximum net benefits [18]). Yu et al. [19] further extended the model to a bi-level nonlinear counterpart and constructed a tabu search with different local neighbourhood search strategies. Pan et al. [20] studied flexible feeder transit service optimisation for irregular shapes and presented a corresponding gravity heuristic algorithm. Pei et al. [21] considered passenger demand and willingness to pay into a flexible transit system. Wang et al. [22] proposed a multi-objective optimisation model to optimise centralised dispatching of flexible feeder transit, and then designed genetic algorithm to solve it. Cheng et al. [23] proposed a bi-level programming model to optimise flex-route transit service. Wu et al. [24] offered a global optimisation method for the flex-route service optimisation considering ride-matching and shuttle bus dispatching. Most of studies discussed in this section only focused on traditional flex-route service optimisation, and there are also some studies on the optimisation of flex-route service for BEBs [25 – 28]. However, the application of DWPT in the flex-route service was rarely considered in previous studies.

The second subsection relates to the application of wireless charging technology in transit bus service. Unlike the plug-in charging technology, the wireless charging technology can be installed in some bus stops and offer the opportunity to charge during BEBs operation. Ko and Jang [29] presented a mixed integer linear programming (MILP) model to simultaneously optimise the locations of DWPT devices and battery size for a single bus route. Later, they enhanced the model in both a closed experimental environment [7] and an open environment, which aimed to reduce DWPT installation and battery costs [9]. Liu et al. [8] extended the study of DWPT in transit service and presented a deterministic MILP model to optimise the locations of DWPT devices and battery size for the multi-route bus network. Then, Liu and Song [10] reformulated the model with a robust optimisation methodology and took the uncertainty of energy consumption and travel time into account. He et al. [30] further considered the energy storage system (ESS) in the model. Hence, the deployment of ESS and fast charging devices, as well as battery size, has been jointly optimised. Chen et al. [31] focused on the application of DWPT on feeder bus network planning. An MINLP model was proposed to comprehensively optimise feeder bus routing, DWPT locations and battery size. In the context of DWPT, Yıldırım and Yıldız [32] optimised the bus fleet composition and the schedules, which are critical for taking advantage of these emerging technologies and enabling the electrification of public transport in a cost-effective way. Luo et al. [33] proposed a model to jointly optimise the electric supply system and the transit service in the fixed-route bus system considering wireless charging facilities. The above studies only investigated the application of DWPT in the fixed-route bus system (e.g. conventional and feeder bus systems), but the impact of DWPT and bus types on the flex-route bus system was rarely considered in previous studies.

The third subsection reviews the solution algorithm of routing optimisation problems. Routing optimisation problems (e.g. dial-a-ride problem (DARP) and electric vehicle routing problem (EVRP)) have been proved to be combinatorial and hard to be solved in an acceptable time if the scale of the problem is large. For DARP, Kirchler and Wolfler Calvo [34] proposed a granular tabu search algorithm (GTS) and Muelas et al. [35] further designed a VNS algorithm, and the results showed that the performance of VNS outperforms GTS. Masmoudi et al. [36] combined local search strategy with genetic algorithm (GA) to solve the hybrid DARP considering different bus types. Then Masmoudi et al. [25] further proposed three enhanced evolutionary variable neighbourhood search (EVO-VNS) algorithm to solve the DARP with electric vehicles and battery swapping stations (DARP-EV). For EVRP, Felipe et al. [37] presented a hybrid VNS to solve EVRP with partial recharge and multiple recharging technologies. Goeke and Schneider [38] designed an adaptive large neighbourhood search algorithm (ALNS) to solve EVRP with mixed vehicle types. Hiermann et al. [39] proposed a branch-and-price algorithm as well as ALNS to solve EVRP with time windows, recharging stations and mixed vehicle types. Hof et al. [40] introduced an adaptive variable neighbourhood search (AVNS) algorithm to solve electric location routing problems (ELRP) with battery-swapping stations. For some variants of EVRP with many practical considerations, similar heuristics was also used, such as two-stage heuristics based on large neighbourhood search (LNS) [41], the approach based on VNS and variable neighbourhood descent (VND) [42], and an ALNS algorithm embedded in the speed optimisation subroutine [43, 44]. Previous research in this section illustrated that the neighbourhood search and its enhanced counterparts (e.g. VNS, AVNS, and ALNS) show good performance in optimising routing-related problems.

To address the research limitations that are discussed in the preceding paragraphs, this paper further expands the applicability of the traditional flex-route electric bus service optimisation model in the wireless charging mode, which can offer the opportunity to charge during BEBs' operation. Moreover, the bus type matching problem is also considered to explore more effects brought by the wireless charging mode. Regarding the solution method, a tangible HVNS combined SA with VNS is developed to solve the problem. The efficient search efficiency of the SA algorithm makes up for the deficiency of VNS. Therefore, we adopt HVNS as the general framework of the solution algorithm.

3. PROBLEM DESCRIPTION AND FORMULATION

In this section, the research background of the DWPT powered flex-route optimisation problem considering variable bus types is firstly introduced, followed by discussing the DWPT and its components. Then, the model, including assumptions, objectives and constraints, is detailed.

The electric flexible bus route transit system provides demand-responsive service for passengers, especially for commuters between home and transfer hub. As is illustrated in *Figure 1*, there are some DWPT devices on the fixed bus route, and some demand points with different locations. They all have the demand to transfer from the current location to the transfer hub. Therefore, flex-route feeder transit bus service aims to optimise their service routes. The demand requests can be collected in advance (one day or several days ahead of time). The passengers should provide their trip information, for instance, boarding point and arrival time. Then, the system will consider the traffic environment near the boarding point provided by the passengers and designate the final boarding point (which may or may not be the boarding point provided by the passenger), however, the final boarding point must be in the vicinity of the location provided by the passengers. This interesting work is not discussed in depth in this paper and will continue to be investigated subsequently. Taking the lowest cost as the goal, each bus route as well as its bus type is simultaneously determined and optimised, which starts and terminates at the transfer hub. Finally, the bus scheduling timetable and routing plan will be determined and fed back to passengers, subject to meeting their travel times. It is noted that some bus stops have been equipped with DWPT devices on the fixed bus route, which can be utilised by BEBs. Also, the flex-route BEBs can recharge as needed when passing through these recharging stops.

The electric bus and the recharging stop installed with DWPT devices are presented in *Figure 2*. A set of DWPT devices is composed of one inverter and several electric transfer pads beneath the ground. Compared with swap battery or plug-in charging technology, electric buses (e.g. fixed-route bus or flex-route bus) can be recharged while passing through or dwelling at those recharge stops without human intervention. In the case where the charging stops coincides with the passenger demand point, the bus can utilise the time passengers spend boarding or alighting for recharging, and all of this charging process may be completely unknown to passengers. Another benefit of DWPT technology is that it could substantially reduce on-board battery size. The battery downsizing not only makes electric buses more affordable, but also offers additional energy savings, due to reduced vehicle weight. These advantages of DWPT can reduce the capacity of on-board batteries while meeting passengers' demands. Thus, different bus types are considered in the model and the cost of on-board batteries can be greatly reduced without reducing the level of public transportation service. It is noted that in the open city transit system, the DWPT devices can only be installed in the bus stops area due to the high maintenance cost [9].

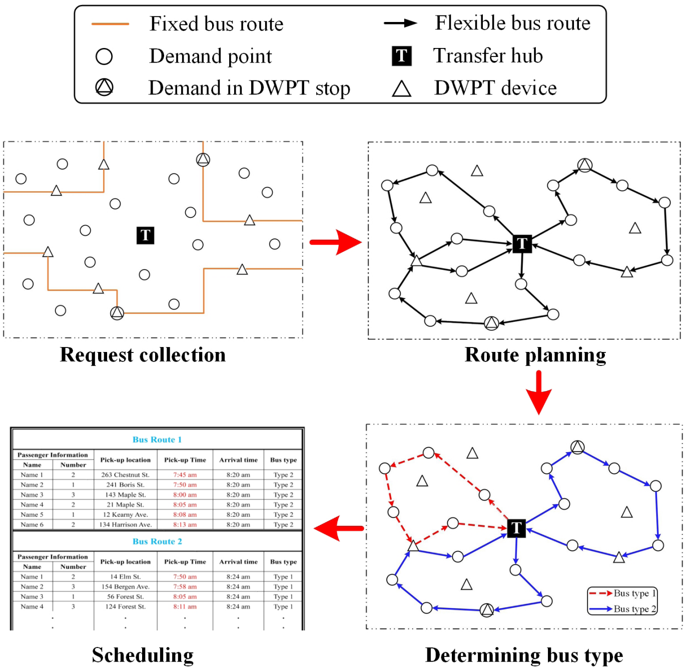


Figure 1 – Process of flex-route bus system planning

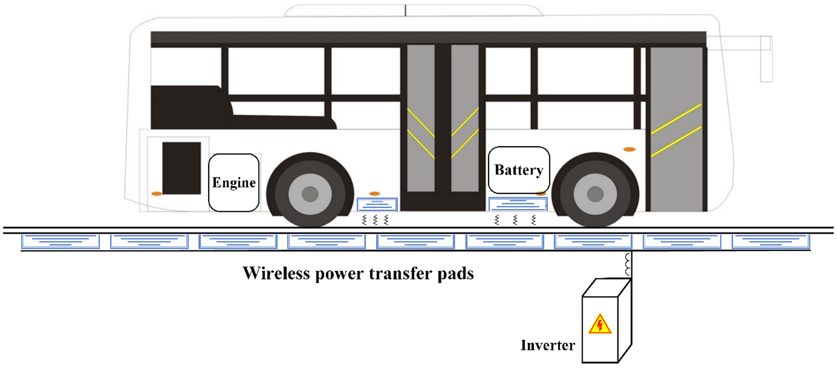


Figure 2 – The battery electric bus and DWPT devices

Based on the flex-route bus system and DWPT, we propose a novel model to simultaneously optimise bus routes and types. The model should meet the energy, bus capacity and passenger service time requirements. Then, we can minimise the total costs of transit agency (e.g. bus depreciation and operating costs) and passengers (e.g. travel time cost). As the wireless charging technology has great development potential, the model proposed in this paper is based on the fact that the technology has matured in the future. The proposed model is formulated based on the following assumptions:

1) Wireless charging facilities have covered some fixed bus stops near the transfer hub and the flex-route BEBs can recharge as needed when passing through these recharging stops.
2) A set of demand requests as well as the location of recharging stops are given.
3) Each request is served by only one bus and the recharging stop can be shared and revisited by several buses.
4) Each bus begins and terminates at the transfer hub and the number of buses in the transfer hub is sufficient for each type of bus.
5) Each bus is fully charged before conducting the first trip from the transfer hub.
6) There is only one bus service on one route, and after serving all passengers, the system will re-plan the bus route based on next booked requests.

7) Partial recharging is considered as the recharging scheme for flex-route BEBs.
8) The traffic congestion and bus bunching are not considered in the model.

Let $G(B,S,R)$ denote the flex-route bus network, where B is a set of bus types. Bus capacity, battery size, operating cost and fixed cost may vary with different types. $S=\{N\cup C'\cup 0\cup n+1\}$ is the union of all nodes in the network. N is a set of demand points. C' is the set of recharging stops. 0 and $n+1$ in the set S represent the same transfer hub, which are the starting and ending points of the bus route, respectively. It is noted that in the network, multiple visits to recharging stops are allowed, thus, some dummy recharging stops, also called the copies of recharging stops, are added into set C'. In the recharging stops, partial recharging is allowed, and the recharging time is given as an input parameter. R is the set of bus routes. The notations of all sets, variables and input parameters used in the developed model are listed in *Table 1*.

Table 1 – Notation list

Sets	**Definitions**
N	Set of demand points, i.e. pick-up points of passengers
C'	Set of recharging stops containing recharging stops and several copies of recharging stops.
$0,n+1$	Transfer hub
S	Set of all nodes $S=\{N\cup C'\cup 0\cup n+1\}$
S_0	Set of demand points, recharging stops and starting bus terminal, $S_0=\{N\cup C'\cup 0\}$
S_{n+1}	Set of demand points, recharging stops and ending bus terminal, $S_0=\{N\cup C'\cup n+1\}$
B	Set of bus types, $B=\{1,2,...,v\}$
R	Set of bus routes, $R=\{1,2,...,K\}$
Parameters	Definitions
K	Number of bus routes
M	A sufficiently large number
d_{ij}	Distance between point i and j
p_i	Passenger demand of request i, i.e. the pick-up passengers of node i
g_v	Unit depreciation cost for bus type v
o_v	Unit operating cost of bus type v
C_v	Bus capacity of bus type v
Q_v	Battery size of bus type v
w_v	Energy consumption rate of bus type v
r	Wireless electric charging rate
S_i	Dwell time of the bus at demand point i or recharging stop i
T_u	Latest time of arrival allowed at the transfer hub
T_s	Bus departing time at the transfer hub
υ	An average bus operating speed
φ_1	Upper bound of battery level
φ_2	Lower bound of battery level
λ_t	Value of passengers' time
τ	Slack time, i.e. the rest time of bus between two service routes
Variables	Definitions
x_{ijk}	$x_{ijk}=1$, request i precedes request j on bus route k; $x_{ijk}=0$, otherwise
y_{vk}	$y_{vk}=1$, bus type v is applied by route k; $y_{vk}=0$, otherwise
e_{ik}	Battery level of the bus arriving at point i of route
AT_i	Arrival time at the node $i\in S$, AT_0 indicates the departure time of the bus at the transfer hub
TT_k	Total travel time of route k

The objective of the developed model (denoted as Z) is divided into two parts, including passenger travel time and transit agency cost. As shown in *Equation 2*, passenger travel time consists of two sections (e.g. bus running time, and dwell time at demand point or recharging stop), which is transferred into travel cost denoted as Z_1 through multiplying the value of passengers' time λ_t. *Equation 3* represents the cost of a transit agency (denoted as Z_2), including bus operating cost and bus depreciation cost. It is worth noting that the depreciation cost of the recharging stops is not considered in this model, because it has been amortised and considered in the fixed-route DWPT bus system [30, 31]. Bus operating cost is determined by the unit operating cost of bus type v denoted as o_v and travel distance d_{ij}, while bus depreciation cost is only determined by the unit depreciation cost for bus type v. Note that these costs are amortised in hours.

$$\min Z = Z_1 + Z_2 \tag{1}$$

$$Z_1 = \lambda_t \sum_{k\in R} \sum_{i\in N} \sum_{\substack{j\in N\\ i\neq j}} p_i x_{ijk} \left(d_{ij}/v + s_i \right) \tag{2}$$

$$Z_2 = \sum_{k\in R} \sum_{\substack{i\in N\\ i\neq j}} \sum_{\substack{j\in N\\ i\neq j}} o_v y_{vk} d_{ij} x_{ijk} + \sum_{k\in R} \sum_{v\in B} g_v y_{vk} \tag{3}$$

The constraints of the proposed model consist of four sections (e.g. bus routing, energy, capacity and time constraints), which are discussed as follows.

Bus routing constraints: *Equation 4* ensures that each bus route should depart and terminate at the transfer hub. *Equation 5* restricts the number of dispatched electric buses cannot exceed the bus fleet number K. *Equation 6* guarantees the node flow conservation, ensuring that one bus route which enters into a node (e.g. demand request nodes and recharging stops) must leave from the same node. *Equation 7* enforces that each demand request can be served exactly once by one bus route. *Equation 8* ensures that each copy in the set C' including recharging stops and dummy recharging stops, is visited at most once by one bus route. *Equation 9* represents the bus type conformity constraint, ensuring that only the same type of bus is allowed to operate in one bus route.

$$\sum_{j\in N\cup C'} x_{j,n+1,k} = \sum_{j\in N\cup C'} x_{0jk} = 1 \quad \forall k\in R \tag{4}$$

$$\sum_{k\in R} \sum_{j\in N\cup C'} x_{0jk} \le K \tag{5}$$

$$\sum_{k\in R} \sum_{i\in S_0, i\neq j} x_{ijk} - \sum_{k\in R} \sum_{i\in S_{n+1}, i\neq j} x_{jik} = 0 \quad \forall j\in S \tag{6}$$

$$\sum_{k\in R} \sum_{j\in S, i\neq j} x_{ijk} = 1 \quad \forall j\in N \tag{7}$$

$$\sum_{k\in R} \sum_{j\in S, i\neq j} x_{ijk} \le 1 \quad \forall i\in C' \tag{8}$$

$$\sum_{v\in B} y_{vk} = 1 \quad \forall k\in R \tag{9}$$

Energy constraints: *Equations 10–11* link the battery level at nodes j and i denoted as e_{jk} and e_{ik}, respectivelyThe battery level drops due to the energy consumption of bus type v denoted as $\sum_{v\in B} w_v y_{vk} d_{ij}$ between nodes j and i. As formulated in *Equation 11*, if node i is the recharging stop, the bus can recharge energy $r\cdot s_i$ at the recharging stop i. The amounts of energy consumed and recharged are related to the energy consumption rate of bus type denoted as w_v and wireless charging rate r. *Equation 12* links the battery level between the transfer hub and the first node that the electric bus visits, enforcing that each bus is dispatched from the transfer hub with a fully charged battery. *Equations 13–14* restrict that the battery cannot be overcharged and over-discharged.

$$e_{jk} \le e_{ik} - \sum_{v\in B} w_v y_{vk} d_{ij} + M(1-x_{ijk}) \quad \forall i\in N, i\notin C', j\in S_{n+1}, i\neq j,\ \forall k\in R \tag{10}$$

$$e_{jk} \le e_{ik} + r\cdot s_i - \sum_{v\in B} w_v y_{vk} d_{ij} + M(1-x_{ijk}) \quad \forall i\in C', j\in S_{n+1}, i\neq j,\ \forall k\in R \tag{11}$$

$$e_{jk} \le \varphi_1 \sum_{v\in B} Q_v y_{vk} - \sum_{v\in B} w_v y_{vk} d_{0j} + M(1-x_{0jk}) \quad \forall j\in N\cup C', \forall k\in R \tag{12}$$

$$\varphi_2 \sum_{v\in B} Q_v y_{vk} \le e_{ik} + r\cdot s \le \varphi_1 \sum_{v\in B} Q_v y_{vk} \quad \forall i\in C', \forall k\in R \tag{13}$$

$$\varphi_2 \sum_{v\in B} Q_v y_{vk} \le e_{ik} \le \varphi_1 \sum_{v\in B} Q_v y_{vk} \quad \forall i\in \{S\backslash C'\}, \forall k\in R \tag{14}$$

Capacity constraint: *Equation 15* ensures that the number of passengers on route k cannot exceed the capacity of bus type k used on the same route denoted as C_v.

$$0 < \sum_{i \in N} \sum_{j \in N, i \neq j} p_i x_{ijk} \leq \sum_{v \in B} c_v y_{vk} \quad \forall k \in R \tag{15}$$

Time constraints: The time-related constraints are expressed in *Equations 16–19*. The starting time of the bus at the transfer hub denoted as AT_0 is set as T_s in *Equation 16*. *Equation 17* links the time between nodes j and i, and the time at node j denoted as AT_j is determined by the time at last node i denoted as AT_i and travel time between nodes j and i denoted as d_{ij}/v. *Equation 18* is formulated to calculate the total travel time of bus route k. *Equation 19* restricts that the bus must travel back to the transfer hub before the time T_u, where τ is the slack time at the transfer hub of flexible feeder bus.

$$AT_0 = T_s \tag{16}$$

$$AT_i + s_i + d_{ij}/v - M(1 - x_{ijk}) \leq AT_j \quad \forall i \in S_0, \forall j \in S_{n+1}, i \neq j, \forall k \in R \tag{17}$$

$$TT_k = \sum_{i \in S_0} \sum_{j \in S_{n+1}, i \neq j} x_{ijk} (d_{ij}/v + s_i) \quad \forall k \in R \tag{18}$$

$$TT_k + \tau \leq T_u - T_s \quad \forall k \in R \tag{19}$$

Decision variables: *Equations 20–21* ensure that binary variables y_{vk} and x_{ijk} pertaining to bus type and bus routing are either 0 or 1.

$$y_{vk} \in \{0,1\} \quad \forall v \in B, k \in R \tag{20}$$

$$x_{ijk} \in \{0,1\} \quad \forall i \in S_0, j \in S_{n+1}, k \in R \tag{21}$$

4. SOLUTION ALGORITHM

The proposed model is a mixed integer non-linear problem, which is the extension of the vehicle routing problem considering capacity constraints, energy constraints and mixed vehicle types. Several previous studies have proven its NP-hard complexity [39, 40], so it is difficult to find an exact solution in an acceptable time. To solve it efficiently, a tangible HVNS is developed in this section, which is a combination of VNS and SA. VNS was applied to solve routing-related combinatorial optimisation problems by many researchers, which shows good performance in searching for the best solution [39, 40]. SA can guide the solution algorithm to search the solution space efficiently [37]. The detailed process of the HVNS is discussed next.

4.1 Main algorithm

Algorithm 1 details the structure of the proposed HVNS. The input is the yielded feasible solutions by the initialisation algorithm while the output is the optimised solutions. It can be seen that the worst-case time complexity of HVNS is $O(i_{SA} \cdot p_{\max} \cdot n_{\max})$ where i_{SA} is the number of iterations, $p_{\max}$ is the number of shaking neighbourhood structures and $n_{\max}$ is the number of local search neighbourhood structures. The main algorithm is divided into four parts (e.g. initialisation, objective evaluation, VNS and SA), which are discussed next.

Algorithm 1 – Main Algorithm (Hybrid VNS with SA)

Input: Initial feasible solutions denoted as Z.
Output: Optimized solutions denoted as Z^*.
Set VNS shaking neighborhood structures p=1,2...,$p_{\max}$.
Set VNS local search neighborhood structures n=1,2...,$n_{\max}$.
Set number of iterations i=1,2...,i_{SA}
Set starting temperatures t_s.
Set a random number $a \in (0,1)$.
For ($i \leq i_{SA}$) *do*
  $t \leftarrow$ temperature $((i_{SA}-i/i_{SA})t_s$.
  $a \leftarrow$ randomly generated between 0 and 1.
  For (p=1,2...,$p_{\max}$) **do**
    Select a random solution $Z'=p(Z)$ from shaking perturbation.
    While $n \leq 1,2...,n_{\max}$ **do**

Find the best solution in $Z^{*}=n(Z)$ neighborhood structure n.
If ($S(Z^{*}) \leq S(Z)$) **then**
Set $Z=Z^{*}$ and n=1.
Else if ($\exp((-(S(Z^{*})-S(Z)))/t) \geq a$) **then**
Set $Z=Z^{*}$ and n=1
Else
Set $n=n+1$.
End if
End while
End for
End for

Initialisation: We aim to generate various initial feasible solutions in a short and acceptable time, further accelerating and improving the performance of the developed HVNS. The pseudocode of the initialisation algorithm is illustrated in *Algorithm 2*. The input is the collected information of passengers and some pre-set parameters, and the output is the yielded initial feasible solutions.

The initialisation algorithm starts from demand request i=0 and bus route k=1. Travel time, passenger load and energy are considered as the constraints to ensure the feasibility of the yielded solutions. We find h unserved demand requests which are closest to i. The large h means high diversification of yielded solutions while the initial solutions with high quality can be obtained if h is small. One unserved demand request is randomly selected from h and added into bus route k. As the recharging stops do not have to be used during the bus's operation, thus, only if the energy constraint is not satisfied, the nearest recharging stop is added to bus route k. Otherwise, a new bus route is created $k=k+1$. This process is repeated until all unserved demand requests are served. It is noted that the proposed initialisation algorithm mainly focuses on the feasibility and diversification of the yielded solutions. Therefore, only one bus type with the largest battery size is considered, which will be improved in the main algorithm.

Algorithm 2 – Initialisation Algorithm

Input: Information of bus network and demand request.
Output: Initial feasible solutions Z.
Set demand request i=0 and bus route k=1.
Repeat
Set travel time $AT_i=T_s$
Set bus passenger load p_i=0.
Find h unserved demand requests which are closest to demand request i and reachable according to capacity and time availability.
Find the recharging stop which is the closest to demand request i.
Randomly select one demand request from h denoted as j.
Apply the bus type v with the largest battery size to bus route k ($e_{ik}=Q_v$).
If (The bus can reach the transfer hub directly from j) **then**
demand request j is added into bus route k.
else if (The bus can reach the transfer hub from j by visiting a recharging stop in between) **then**
recharging stop j is added into bus route k.
else
transfer hub n+1 is added into bus route k.
set $k=k+1$ and i=0.
end if
until all demand requests are served.

Objective evaluation: In most previous studies, VNS allows infeasible solutions during the searching process. Thus, the objective function can be extended as $S(Z)$ in *Equation 22*, where Z denotes the original cost in *Equation 1*, $S_P(Z)$ is the bus capacity violation, $S_T(Z)$ is the travel time violation and $S_B(Z)$is the energy violation. θ, η and λ are their input penalty factors respectively, which are initialised from θ_0, η_0, λ_0, then dynamically updated in each iteration between lower limit $\theta_{\min}$, $\eta_{\min}$, $\lambda_{\min}$ and upper limit $\theta_{\max}$, $\eta_{\max}$, $\lambda_{\max}$.

$$S(Z) = Z + \theta S_P(Z) + \eta S_T(Z) + \lambda S_B(Z) \tag{22}$$

Equations 23 – 24 calculate the bus capacity and travel time violations of single bus route k denoted as $S_p(k)$ and $S_T(k)$ respectively, which are total passenger load and travel time TT_k (including slack time) minus the bus capacity C_v and latest time of arrival allowed at the transfer hub T_u correspondingly. The total violations of bus capacity and travel time can be obtained from *Equations 25 – 26*.

$$S_P(k) = \max\{\sum_{i\in S}\sum_{j\in S} p_i x_{ijk} - C_v y_{vk}, 0\} \tag{23}$$

$$S_T(k) = \max\{(TT_k + \mathrm{t}) - T_u, 0\} \tag{24}$$

$$S_P(Z) = \sum_{k\in R} S_P(k) \tag{25}$$

$$S_T(Z) = \sum_{k\in R} S_T(k) \tag{26}$$

Unlike violations of bus capacity and travel time, the energy violation is based on the recharging stops and the transfer hub. *Equation 27* calculates the energy violation of bus k before visiting the recharging stop j or back to the transfer hub. The total energy violation can be obtained by *Equation 28*.

$$S_B(j, k) = \max\{\mathrm{j}\ {}_2\, y_{vk} Q_v - e_{jk}, 0\} \quad \forall j \in C' \cup \{0\} \tag{27}$$

$$S_B(Z) = \sum_{j\in\{S\setminus N\}} \sum_{k\in R} S_B(j, k) \tag{28}$$

VNS: The procedure of the original VNS is described as follows. At first, a set of shaking perturbation neighbourhood structures $p_{\max}$ and local search neighbourhood structures $n_{\max}$ are preset. The initial feasible solution Z is input and passes through the shaking process, which generates temporary neighbouring solutions Z'. Then, the local search process n=1,2,...,$n_{\max}$ is applied on Z' to yield an optimized solution Z^*. If Z^* improves on Z', n=1; if Z^* is worse than Z', n=n+1. This process is repeated until the terminating condition is satisfied. It can be seen that neighbourhood structures play a vital role in VNS. In every iteration, $p_{\max}$ is to diversify the current solutions to avoid VNS being trapped into the local optima, and $n_{\max}$ facilitates VNS to search for better solutions.

In the shaking process of HVNS, the cyclic-exchange is introduced to diversify the solutions, which has been successfully applied in many routing-related problems. The sample cyclic-exchange used in this study is shown in *Figure 3*. Random nodes are exchanged between different bus routes. The neighbourhood structures of cyclic-exchange are illustrated in *Table 2*, where the number of bus routes involved varies from two to four while the number of exchanged nodes ranges between one and four. It is noted that the exchanged nodes may contain both demand requests and recharging stops.

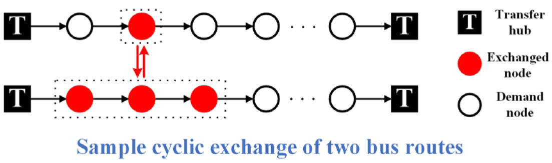


Figure 3 – Sample cyclic-exchange

Table 2 – Structure of cyclic-exchange

p^*	1	2	3	4	5	6	7	8	9	10	11	12
Selected bus routes	2	2	2	2	3	3	3	3	4	4	4	4
Maximum exchanged nodes	1	2	3	4	1	2	3	4	1	2	3	4

p^* is the number of the neighbourhood structures in the shaking perturbation process.

In the local search process of HVNS, six general neighbourhood structures are specially designed and used, among which 1) – 4) only exchange the nodes while 5) and 6) exchange the selected bus type as well.

1) Inter-route 2-opt: In *Figure 4*, similar to intra-route 2-opt, two edges in two different bus routes (dashed line) including four nodes (blue and red nodes) are randomly selected and inversed. Note that the two involved routes should use the same bus type.

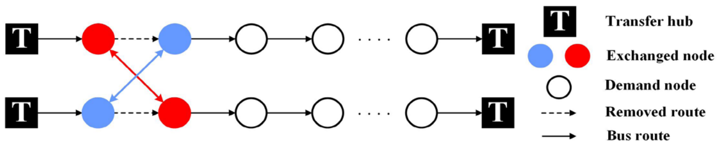


Figure 4 – Sample inter-route 2-opt

2) Intra-route 2-opt: As is shown in *Figure 5*, two edges (dashed line) in one bus route including four nodes are randomly selected and inversed.

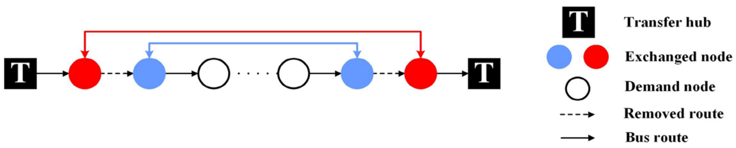


Figure 5 – Sample intra-route 2-opt

3) Relocate: In *Figure 6*, one node is removed from its bus route and inserted into another bus route, where the dashed lines represent the removed bus routes while the red lines represent the added routes

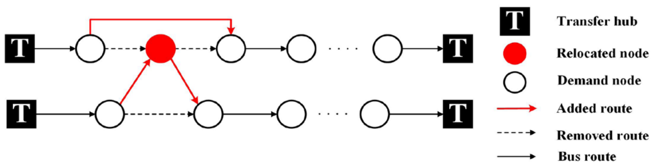


Figure 6 – Sample relocate

4) Recharging stop insertion and removal: In *Figure 7*, a recharging stop is randomly inserted or removed from one selected bus route, where the red triangle represents the added recharging stop while the triangle represents the removed recharging stop.

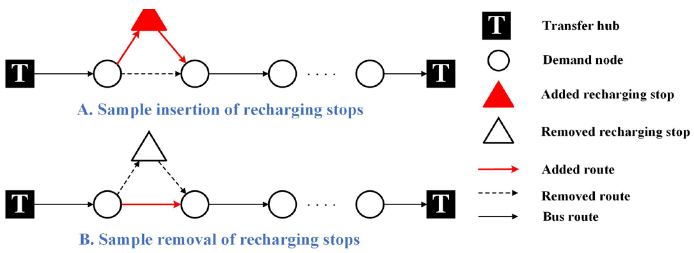


Figure 7 – Sample recharging stop insertion and removal

5) Resize: In *Figure 8*, the bus type of two different routes is selected and exchanged.

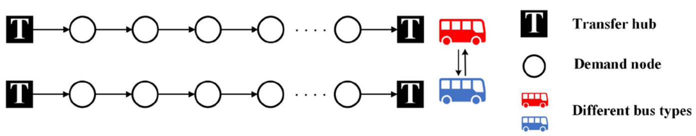


Figure 8 – Sample resize

6) Relocate and resize: In *Figure 9*, one node is removed from its bus route and inserted into another bus route. At the same time, the corresponding bus types are exchanged between two involved bus routes.

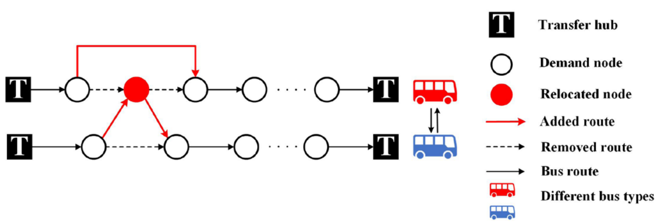


Figure 9 – Sample relocate and resize

SA: As is illustrated in *Algorithm 1*, Simulated Annealing is introduced as the framework to further diversify and intensify the search. The number of iterations i_{SA}, starting temperatures t_s and a random number $a \in (0,1)$ are pre-set. Different from the original VNS, the deteriorating solutions are also accepted according to the probability $\exp((-(S(Z^*)-S(Z)))/t) \geq a$. It is worth noting that at the beginning of the algorithm, the deteriorating solutions are accepted in high probability, which can diversify the search. The temperature as well as the probability of accepting the deteriorating solutions is continuously decreasing at the rate of $((i_{SA}-i)/i_{SA})t$ after each iteration, which gradually intensifies the search. Finally, only improved solutions are accepted.

4.2 Algorithm performance analysis

In this section, a set of instances based on the real case (from 5 to 20 demand requests) are randomly generated and solved by GAMS (DICOPT solver), VNS and HVNS. The obtained results are compared with the performance of the proposed HVNS. The upper bound of the running time is set as 7200s. The proposed HVNS and VNS are coded in JAVA on a personal laptop (Intel Core i5, 8GB, 2.4GHz). The penalty parameters θ_0, η_0 and λ_0 are all set as 10, $\theta_{\min}$, $\eta_{\min}$ and $\lambda_{\min}$ are all set as 0.5, and also $\theta_{\max}$, $\eta_{\max}$ and $\lambda_{\max}$ are all set as 5000. Some calculating parameters are determined experimentally in accordance with the scale of the instances, which yields the optimised solution with the least objective and minimum running time. The obtained results, along with the running times of GAMS, VNS and HVNS are the average of 30 runs and are compared in *Table 3* and *Figure 10*.

Table 3 illustrates the optimised results of the instances with the different numbers of requests and recharging stops. The difference between optimised results obtained by VNS and HVNS against those obtained by GAMS is denoted as $\Delta_{VNS}$and Δ_{HVNS}respectively. It can be found that GAMS, VNS and HVNS all can solve small-scale instances. When the number of demand requests increases to 15 (all nodes increase to 20), the solutions obtained by GAMS are worse than those obtained by VNS and HVNS. Because GAMS cannot calculate the optimal solution in the specified time and can only give a feasible solution. As the scale of the instances keeps enlarging, GAMS fails to find the solutions. It can be observed from *Figure 10* that the running time of GAMS exponentially increases while the times of VNS and HVNS are less than those of GAMS. Also, HVNS is generally superior to VNS, particularly as the scale of the instances keeps expanding. *Table 3* also indicates that HVNS shows good performance in searching for solutions.

$$\Delta_{VNS} = \frac{Z_{VNS} - Z_{GAMS}}{Z_{GAMS}}, \quad \Delta_{HVNS} = \frac{Z_{HVNS} - Z_{GAMS}}{Z_{GAMS}} \tag{29}$$

Table 3 – Optimised results of different instances with GAMS, VNS and HVNS

Instance	GAMS	VNS	Δ_{VNS}(%)	HVNS	Δ_{HVNS}(%)
7 (5, 2) *	102.32	102.32	0	102.32	0
8 (5, 3)	105.61	105.61	0	105.61	0
14 (10, 4)	185.23	186.61	0.75	185.23	0
15 (10, 5)	187.42	187.42	0	187.42	0
22 (15, 7)	227.15	217.74	-4.14	215.42	-5.16
23 (15, 8)	235.47	221.56	-5.91	219.81	-6.65
29 (20, 9)	—	274.16	—	270.21	—
30 (20, 10)	—	275.41	—	275.41	—
46 (35, 11)	—	432.19	—	432.19	—
47 (35, 12)	—	445.28	—	440.04	—
63 (50, 13)	—	787.62	—	772.42	—
64 (50, 14)	—	795.41	—	784.13	—

*a (b, c) *: a, b and c denote the number of all nodes, demand requests and recharging stops.*

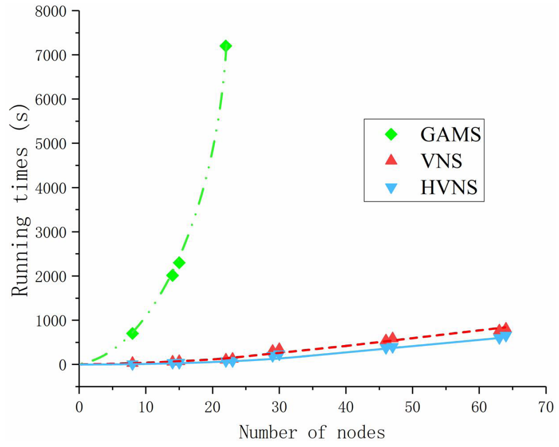


Figure 10 – Running times with GAMS, VNS and HVNS

5. CASE STUDY AND RESULTS

In this section, a case study from Xi'an, China based on Chen et al. [31] (see *Figure 11*) is employed to explore the relationship between the optimised solutions, decision variables and some input parameters. The developed model and HVNS are applied to simultaneously optimise the bus routing as well as corresponding bus types. As is shown in *Figure 11*, there are many fixed bus routes near Fengwulu Metro station, such as line 134, line 209 and line 234. We assume that near Fengwulu Metro station (seen as the transfer hub) some recharging stops are scattered in these fixed bus routes, which can also be utilised by flex-route bus during operation [31]. The number of stops on flexible feeder bus service routes is smaller than that of traditional bus service routes. In this case, there are 16 recharging stops and 22 demand points in the study area, some demand points are located at recharging stops. These demand requests are collected and assumed to be from their home to the transfer hub, which is estimated from historical data. Each electric bus starts from the transfer hub with fully charged, serves several demand requests, and travels back to the transfer hub within the time limit. Note that each demand request can be served by only one bus while the recharging stops allow multiple visits by buses.

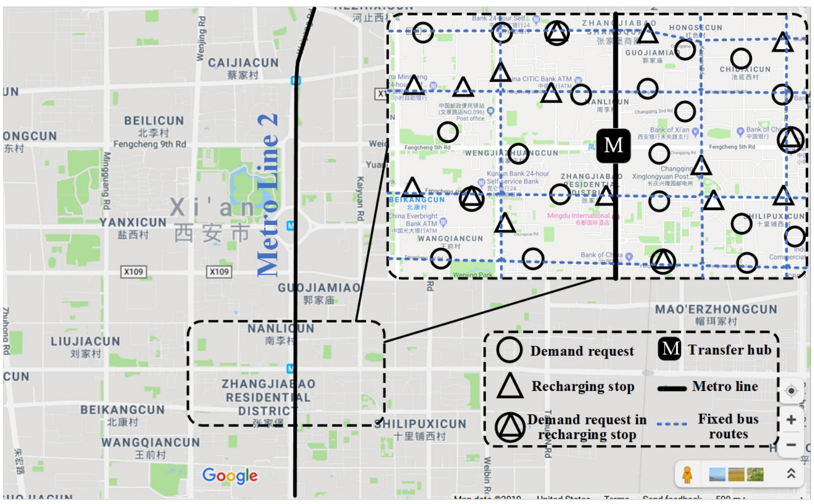


Figure 11 – The study area from Xi'an, China

5.1 Input parameters

As suggested in previous studies, the bus types used in this study are divided into six and listed in *Table 4*. It can be observed that the battery size varies from 10 to 60 kWh while the bus capacity ranges from 15 to 25 pass/veh [31]. The corresponding unit bus operating cost, bus depreciation cost and energy consumption rate are assumed to increase with the increase of battery size and bus capacity. The wireless charging rate is assumed to be 200kW [3]. The battery level is limited to between 20% and 80% [45]. Bus operating speed is assumed to be 35 km/h [46]. The value of passenger time is set as 8 $/h [47]. The slack time, starting time and latest time of arrival allowed at the transfer hub are assumed to be 0.1h [48], 7:45 a.m. and 8:30 a.m., respectively.

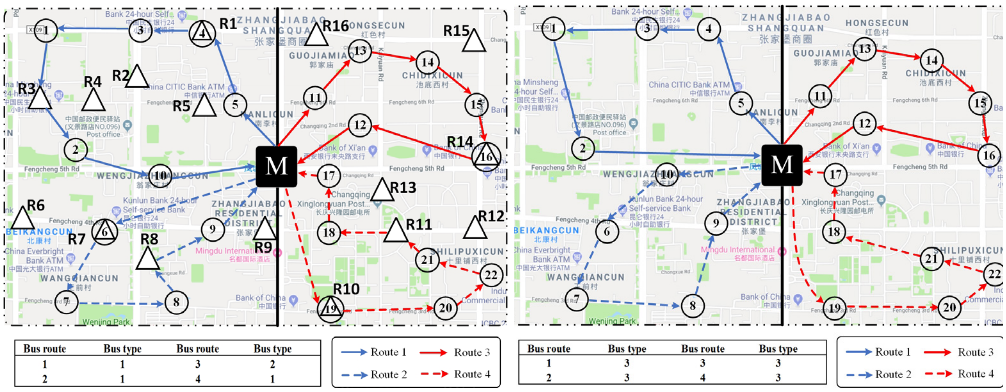
Table 4 – Detailed information of different bus types

Bus types	1	2	3	4	5	6
Battery size Q_v (kWh)	10	20	30	40	50	60
Bus capacity C_v (pass/bus)	15	15	20	20	25	25
Bus operating cost o_v ($/veh-km)	2.1	2.3	2.7	2.9	3.3	3.5
Bus depreciation cost g_v ($/veh-h)	8.9	9.6	11.3	12.0	13.7	14.4
Energy consumption rate w_v (kWh/km)	1.18	1.18	1.24	1.24	1.34	1.34

5.2 Results analysis

In previous studies, some scholars incorporated Terminal Charging Technology (TCT) into their models, meaning that bus charging could only occur at depots and no in-route charging is allowed [27, 38]. It is noteworthy that the prevailing charging method for BEBs in Xi'an mandates a return to the depot for charging, utilising plug-in charging, which aligns with the TCT approach. This section will present and compare the optimised results with both DWPT and TCT in the flex-route transit system, which may further help estimate the economic benefit of DWPT.

The optimised results (e.g. bus routing and bus type) with DWPT and TCT are graphically shown in *Figures 12a* and *12b* respectively. It can be observed that both systems (e.g. DWPT and TCT) contain four bus routes, and in the DWPT system, seven charging stations were selected, namely R1, R3, R7, R8, R10, R11, R14. The bus routing with DWPT slightly differs from that with TCT, because the bus system with TCT requires buses with larger battery sizes and bus capacity due to the non-use of DWPT devices. *Table 5* details the different optimised costs with DWPT and TCT. It was found that compared to the TCT system, the total cost with DWPT can be reduced from 393.44 $/h to 342.43 $/h (i.e. 12.97% reduction). Both bus depreciation and operation costs with DWPT decrease because of the small-battery bus employed in the DWPT system. It is noted that the passenger cost with DWPT is only 6.65% higher than that with TCT, which may indicate that battery recharging in the recharging stops has a slight impact on the passenger travel time in flex-route planning.



a) DWPT system *b) TCT system*

Figure 12 – The optimised results of different technologies in the flex-route transit system

Table 5 – Optimised costs with DWPT and TCT

	Total cost ($/h)	Passenger cost ($/h)	Bus operation cost ($/h)	Bus depreciation cost ($/h)
DWPT	342.43	80.52	133.17	128.74
TCT	393.44	75.16	150.24	168.04

5.3 Sensitivity analysis

The sensitivity analysis is conducted to identify the relative importance of input parameters and to explore the relationships among input parameters, decision variables and optimised results. The cost-related parameters (e.g. value of passengers' time and slack time) are varied in the probable range, and the output variables are analysed and illustrated in *Figures 13–14*.

As can be seen from *Figure 13*, with the value of passenger time increasing, the average travel time is greatly reduced. This is mainly because more buses have been operated in the network. Meanwhile, buses equipped with larger on-board battery capacity have been adopted, which further reduces the number of charging times of buses in wireless charging facilities, thereby reducing the travel time of the bus and the travel time of passengers.

In the real bus system, several uncertainties (e.g. traffic jams and passenger delays) may occur during operation. Thus, slack time plays a vital role in BEBs operation. If it is ignored by the policymakers, some issues may arise. *Figure 14* illustrates the output decision variables under different slack time. When slack time gradually increases, passenger travel time gradually decreases and more BEBs (from 3 to 6) are required to serve the passengers. This is due to the increase in slack time compressing the available bus running time, which also leads to a reduction in visits to charging facilities. Therefore, larger bus types and more buses will be selected to meet the travel needs of all passengers in the process of bus operation.

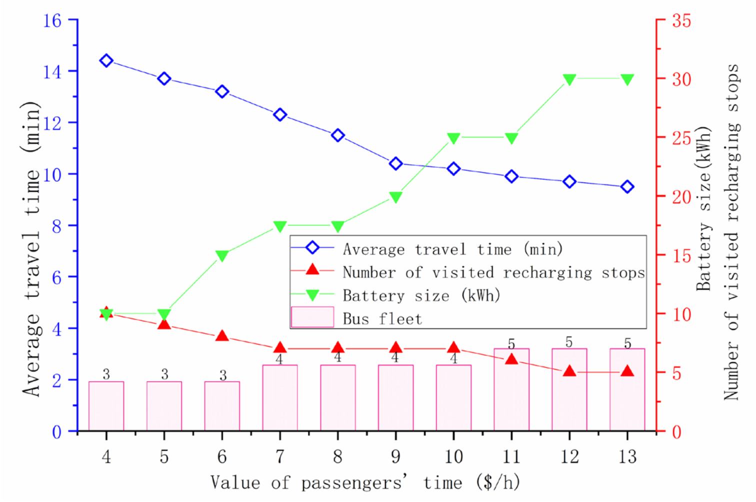


Figure 13 – Output results under different value of passenger time

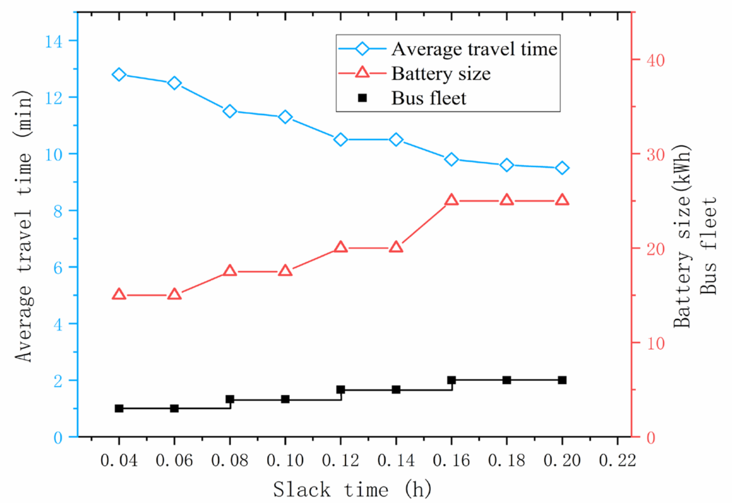


Figure 14 – Output results under different slack time

6. CONCLUSION AND FUTURE RESEARCH

This study focuses on the optimisation of flex-route transit services powered by DWPT. A MINLP model was formulated to simultaneously optimise the bus routing as well as corresponding bus type with the objective of minimising costs for both transit agency and passengers. The proposed model, which incorporates considerations for battery size, bus capacity and travel time constraints, can be applied to various electric bus networks. A tangible HVNS consisting of VNS and SA was designed to efficiently solve the model, and the performance of HVNS was demonstrated by the numerical experiments in comparison with the original VNS and GAMS (DICOPT solver) in terms of running time and obtained results. In order to demonstrate the practical applicability of the proposed model, a case study was conducted using a real-world bus network. The results suggested that the model can solve the optimal planning problem for DWPT-powered flex-route transit service optimisation efficiently. Additionally, the relationship between the output results and some input parameters was investigated through the result and sensitivity analyses. The key findings contain:

1) The comparison between the DWPT and terminal charging flex-route bus system demonstrates that DWPT can effectively reduce total costs, bus operation cost and depreciation cost, while the passenger cost slightly increases, and the total cost with DWPT can be reduced from 393.44 $/h to 342.43 $/h (i.e. 12.97% reduction).
2) The passenger travel time tends to decrease as the increase of value of passenger time, but the BEBs requires a larger battery and bus fleet.
3) More slack time reserved accounts for larger battery size and bus fleet, but less passenger travel time.

The proposed methodology (e.g. the model and corresponding algorithm) offers a wide range of applications, which can provide practitioners with a powerful tool to plan a DWPT-powered flex-route bus system. However, there are still some limitations to our model. Firstly, the recharging time at the recharging stops was predetermined as the input parameter for simplicity. Secondly, the model and algorithm proposed in this study are based on static demand requests, and dynamic demand requests is very complex, which would be discussed in future study. An immediate extension of this study may be the optimisation of the flex-route bus system based on many-to-many demand.

ACKNOWLEDGMENT

This paper was supported by the Natural Science Foundation of Shaanxi, China under grants 2021JZ-20, and the Fundamental Research Funds for the Central Universities (No. 310822150020 and No. 300102220102).

REFERENCES

[1] I.E. Agency, CO_2 Emissions from Fuel Combustion 2020. *International Energy Agency*, 2020. https://www.iea.org/reports/greenhouse-gas-emissions-from-energy-overview/data-explorer

[2] Lajunen A, Lipman T. Lifecycle cost assessment and carbon dioxide emissions of diesel, natural gas, hybrid electric, fuel cell hybrid and electric transit buses. *Energy.* 2016;106:329-342. DOI: 10.1016/j.energy.2016.03.075.

[3] Bi Z, et al. Plug-in vs. wireless charging: life cycle energy and greenhouse gas emission analysis of an electric bus system. *Applied Energy.* 2015;146:11-19. DOI: 10.1016/j.apenergy.2015.02.031.

[4] Bi Z, et al. A review of wireless power transfer for electric vehicles: Prospects to enhance sustainable mobility. *Applied Energy.* 2016;179(oct.1):413-425. DOI: 10.1016/j.apenergy.2016.07.003.

[5] He Z. Battery electric bus selection based on entropy weight method and road operation test: Using Nanjing bus company as an example. *Mathematical Problems in Engineering.* 2022;2022:1696578. DOI: 10.1155/2022/1696578.

[6] Jeong S, Jang Y, Kum D. Economic analysis of the dynamic charging electric vehicle. *IEEE Transactions on Power Electronics.* 2015;30(11):6368-6377. DOI: 10.1109/TPEL.2015.2424712.

[7] Jang Y, Jeong S, Ko Y. System optimization of the on-line electric vehicle operating in a closed environment. *Computers & Industrial Engineering*. 2015;80:222-235. DOI: 10.1016/j.cie.2014.12.004.

[8] Liu Z, Song Z, Yi H. Optimal deployment of dynamic wireless charging facilities for an electric bus system. *Transportation Research Record.* 2017;2647(1):128-139. DOI: 10.3141/2647-12.

[9] Jang Y, Suh E, Kim J. System architecture and mathematical models of electric transit bus system utilizing

wireless power transfer technology. *IEEE Systems Journal.* 2016;10(2):495-506. DOI: 10.1109/JSYST.2014.2369485.

[10] Liu Z, Song Z. Robust planning of dynamic wireless charging infrastructure for battery electric buses. *Transportation Research Part C: Emerging Technologies.* 2017;83:77-103. DOI: 10.1016/j.trc.2017.07.013.

[11] Mouhrim N, El Hilali Alaoui A, Boukachour J. Pareto efficient allocation of an in-motion wireless charging infrastructure for electric vehicles in a multipath network. *International Journal of Sustainable Transportation.* 2018;13(6):419-432. DOI: 10.1080/15568318.2018.1481242.

[12] Alwesabi Y, et al. Robust strategic planning of dynamic wireless charging infrastructure for electric buses. *Applied Energy.* 2021;307:118243. DOI: 10.1016/j.apenergy.2021.118243.

[13] Quadrifoglio L, Dessouky M, Ordónez F. A simulation study of demand responsive transit system design. *Transportation Research Part A: Policy & Practice.* 2008;42(4):718-737. DOI: 10.1016/j.tra.2008.01.018.

[14] Quadrifoglio L, Dessouky M, Ordónez F. Mobility allowance shuttle transit (MAST) services: MIP formulation and strengthening with logic constraints. *European Journal of Operational Research.* 2008;185(2):481-494. DOI: 10.1016/j.ejor.2006.12.030.

[15] Quadrifoglio L, Dessouky M, Palmer K. An insertion heuristic for scheduling Mobility Allowance Shuttle Transit (MAST) services. *Journal of Scheduling.* 2007;10(1):25-40. DOI: 10.1007/s10951-006-0324-6.

[16] Kim M, Schonfeld P. Conventional, flexible, and variable-type bus services. *Journal of Transportation Engineering.* 2012;138(3):263-273. DOI: 10.1061/(ASCE)TE.1943-5436.0000326.

[17] Kim M, Schonfeld P. Integration of conventional and flexible bus services with timed transfers. *Transportation Research Part B: Methodological.* 2014;68b(oct.):76-97. DOI: 10.1016/j.trb.2014.05.017.

[18] Kim M, Schonfeld P. Maximizing net benefits for conventional and flexible bus services. *Transportation Research Part A: Policy & Practice.* 2015;80A(OCT.): 116-133. DOI: 10.1016/j.tra.2015.07.016.

[19] Yu Y, Machemehl R, Xie C. Demand-responsive transit circulator service network design. *Transportation Research Part E: Logistics and Transportation Review.* 2015;76:160-175. DOI: 10.1016/j.tre.2015.02.009.

[20] Pan S, et al. Designing a flexible feeder transit system serving irregularly shaped and gated communities: Determining service area and feeder route planning. *Journal of Urban Planning & Development.* 2015;141(3):04014028.04014021-04014028.04014029. DOI: 10.1061/(ASCE)UP.1943-5444.0000224.

[21] Pei M, Lin P, Du J, Li X. Operational design for a real-time flexible transit system considering passenger demand and willingness to pay. *IEEE Access.* 2019;7:180305-180315. DOI: 10.1109/access.2019.2949246.

[22] Wang Y, et al. Optimizing centralized dispatching of flexible feeder transit considering transfer coordination with regular public transit. *Mathematical Problems in Engineering.* 2021;2021:6160321. DOI: 10.1155/2021/6160321.

[23] Cheng C, Wang T, Wang W, Ding J. Designing customised bus routes for urban commuters with the existence of multimodal network - A bi-level programming approach. *Promet–Trafic&Transportation.* 2022;34(3):487-498. DOI: 10.7307/ptt.v34i3.3980

[24] Wu M, et al. Joint optimization of timetabling, vehicle scheduling, and ride-matching in a flexible multi-type shuttle bus system. *Transportation Research Part C: Emerging Technologies.* 2022;139:103657. DOI: 10.1016/j.trc.2022.103657.

[25] Masmoudi M, et al. The dial-a-ride problem with electric vehicles and battery swapping stations. *Transportation Research Part E: Logistics and Transportation Review*. 2018;118:392–420. DOI: 10.1016/j.tre.2018.08.005.

[26] Bongiovanni C, Kaspi M, Geroliminis N. The electric autonomous dial-a-ride problem. *Transportation Research Part B: Methodological*. 2019;122:436-456. DOI: 10.1016/j.trb.2019.03.004.

[27] Li X, et al. Electric demand-responsive transit routing with opportunity charging strategy. *Transportation Research Part D: Transport and Environment*. 2022;110:103427. DOI: 10.1016/j.trd.2022.103427.

[28] Molenbruch Y, Braekers K, Eisenhandler O, Kaspi M. The electric dial-a-ride problem on a fixed circuit. *Transportation Science*. 2023;APR. DOI: 10.1287/trsc.2023.1208.

[29] Ko, Y, Jang, Y. The optimal system design of the online electric vehicle utilizing wireless power transmission technology. *IEEE Transactions on intelligent transportation systems.* 2013;14(3):1255-1265. DOI: 10.1109/TITS.2013.2259159.

[30] He Y, Song Z, Liu Z. Fast-charging station deployment for battery electric bus systems considering electricity demand charges. *Sustainable Cities & Society.* 2019;48:101530. DOI: 10.1016/j.scs.2019.101530.

[31] Chen G, Hu D, Chien S. Optimizing battery-electric-feeder service and wireless charging locations with nested

genetic algorithm. *IEEE Access.* 2020;8:67166-67178. DOI: 10.1109/ACCESS.2020.2985168.

[32] Yıldırım Ş, Yıldız B. Electric bus fleet composition and scheduling. *Transportation Research Part C: Emerging Technologies.* 2021;129:103197. DOI: 10.1016/j.trc.2021.103197.

[33] Luo X, Fan W. Joint design of electric bus transit service and wireless charging facilities. *Transportation Research Part E: Logistics & Transportation Review.* 2023;174:103114. DOI: 10.1016/j.tre.2023.103114.

[34] Kirchler D, Wolfler Calvo R. A granular tabu search algorithm for the dial-a-ride problem. *Transportation Research Part B: Methodological.* 2013;56:120-135. DOI: 10.1016/j.trb.2013.07.014.

[35] Muelas S, Latorre A, Pena J. A variable neighborhood search algorithm for the optimization of a dial-a-ride problem in a large city. *Expert Systems with Applications.* 2013;40(14):5516-5531. DOI: 10.1016/j.eswa.2013.04.015.

[36] Masmoudi M, Braekers K, Masmoudi M, Dammak A. A hybrid genetic algorithm for the heterogeneous dial-a-ride problem. *Computers & Operations Research.* 2017;81(MAY):1-13. DOI: 10.1016/j.cor.2016.12.008.

[37] Felipe Á, Ortuno M, Righini G, Tirado G. A heuristic approach for the green vehicle routing problem with multiple technologies and partial recharges. *Transportation Research Part E: Logistics & Transportation Review.* 2014;71:111-128. DOI: 10.1016/j.tre.2014.09.003.

[38] Goeke D, Schneider M. Routing a mixed fleet of electric and conventional vehicles. *European Journal of Operational Research.* 2015;245(1):81-99. DOI: 10.1016/j.ejor.2015.01.049.

[39] Hiermann G, Puchinger J, Ropke S, Hartl R. The electric fleet size and mix vehicle routing problem with time windows and recharging stations. *European Journal of Operational Research.* 2016;252(3):995-1018. DOI: 10.1016/j.ejor.2016.01.038.

[40] Hof J, Schneider M, Goeke D. Solving the battery swap station location-routing problem with capacitated electric vehicles using an AVNS algorithm for vehicle-routing problems with intermediate stops. *Transportation Research Part B: Methodological.* 2017;97(MAR):102-112. DOI: 10.1016/j.trb.2016.11.009.

[41] Pelletier S, Jabali O, Laporte G. The electric vehicle routing problem with energy consumption uncertainty. *Transportation Research Part B: Methodological.* 2019;126:225-255. DOI: 10.1016/j.trb.2019.06.006.

[42] Bac U, Erdem M. Optimization of electric vehicle recharge schedule and routing problem with time windows and partial recharge: A comparative study for an urban logistics fleet. *Sustainable Cities and Society.* 2021;70:102883. DOI: 10.1016/j.scs.2021.102883.

[43] Ma B, et al. The vehicle routing problem with speed optimization for shared autonomous electric vehicles service. *Computers & Industrial Engineering.* 2021;161:107614. DOI: 10.1016/j.cie.2021.107614.

[44] Ma B, et al. Time-dependent vehicle routing problem with departure time and speed optimization for shared autonomous electric vehicle service. *Applied Mathematical Modelling.* 2023;113:333-357. DOI: 10.1016/j.apm.2022.09.020.

[45] Bai Z, et al. A robust approach to integrated wireless charging infrastructure design and bus fleet size optimization. *Computers & Industrial Engineering*. 2022;168:108046. DOI: 10.1016/j.cie.2022.108046.

[46] Lu X, et al. Flexible feeder transit route design to enhance service accessibility in urban area. *Journal of Advanced Transportation.* 2016;50(4):507-521. DOI: 10.1002/atr.1357.

[47] Xiao M, Chien S, Hu D. Optimizing coordinated transfer with probabilistic vehicle arrivals and passengers' walking time. *Journal of Advanced Transportation.* 2016;50(8):2306-2322. DOI: 10.1002/atr.1460.

[48] Winter K, Cats O, Correia G, Van Arem B. Designing an automated demand-responsive transport system: Fleet size and performance analysis for a campus–train station service. *Transportation Research Record.* 2016;2542(1):75-83. DOI:10.3141/2542-09.

高天洋，胡大伟，陈刚，Steven CHIEN，马冰山

动态无线充电模式下电动灵活接驳公交服务优化研究

摘要

电动公交车的出现可以缓解交通系统中尾气排放造成的环境问题。然而，车载电池的高成本和里程焦虑阻碍了其进一步发展。近年来，动态无线充电技术的出现已成为推动电动公交车发展的潜在解决方案。因此，本文重点研究了动态无线充电技术

在灵活接驳公交系统中的应用。提出一种混合整数非线性规划模型，在考虑乘客出行时间、电池容量和公交车容量约束的情况下，同时优化公交线路和公交车型，以最大限度地降低车辆成本和乘客的时间成本。为了高效求解模型，本文提出了一种由模拟退火和变邻域搜索算法组成的混合变邻域搜索算法。与GAMS商业求解器和变邻域搜索算法相比，所提算法可以显著提高计算效率。求解西安市实际案例的结果表明，所提模型能够有效地确定动态无线充电模式下的灵活接驳公交路线和公交车型。与采用终端充电技术的模型相比，所提模型的总成本降低了12.97%。

关键词
灵活接驳公交；电动公交车；动态无线充电；机会充电；混合变邻域搜索算法