Traffic & Transportation

Online Model Predictive Control for Energy-saving Train Operation in Passenger-Freight Mixed Lines

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Original Scientific Paper
Submitted: 4 Jan 2024
Accepted: 3 Jun 2024

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Publisher:
Faculty of Transport and Traffic Sciences, University of Zagreb

ABSTRACT

This paper focuses on the online energy-saving operation control problem for passenger and freight trains running in a single-track railway line. Firstly, we design a centralised optimisation method to generate energy-saving reference profiles for both passenger and freight trains, in order to improve the punctuality of passenger trains and to reduce the total running time of freight trains in a central way. Secondly, we propose the distributed model predictive control (DMPC) based online trajectory optimisation problems for both types of trains, subject to their respective operational constraints including safety, punctuality, static speed limits and temporary speed restrictions. Then we formulate an online train operation control algorithm based on the centralised optimisation method for the initialisation of train trajectories and the DMPC method for the online trajectory planning. Finally, the proposed algorithm is applied to case studies of passenger and freight trains in a single track railway, and the numerical simulation results show that the proposed algorithm can realise online control for energy-saving train operation in the presence of input disturbances and temporary speed restrictions.

KEYWORDS

optimal train control; energy-saving operation; distributed model predictive control; passenger and freight mixed lines.

1. INTRODUCTION

Rail transportation is becoming increasingly important in public transportation systems due to its colossal transportation capacity, passenger comfort, punctuality and energy efficiency. The Automatic Train Operation (ATO) system, which represents a key component of train control systems, enables automatic control and regulation of train operation. This not only alleviates the labour burden on drivers but also decreases operational energy consumption, ultimately ensuring operation safety and reliability [1]. In recent decades, significant efforts have been made to come up with advanced ATO control algorithms for achieving accurate, fast and stable tracking control in response to the enlarging railway networks and increasing train speeds.

The ATO system computes a speed profile for the upcoming journey prior to the train's departure from the station. This profile is the reference signal for the ATO system and sets the target position and speed at a given time. It is crucial for ensuring punctuality and energy efficiency during automatic train operation. The generation of the recommended speed profile is usually modelled as an optimisation problem, and the optimisation problem can be solved by a variety of methods, which can be roughly classified into two groups: direct and indirect methods. Indirect methods, including Pontryagin's Maximum Principle [2], have been widely used for train optimal control problems. Using Pontryagin's Maximum Principle, Ichikawa [3] derived optimal regimes for energy-efficient train driving on level tracks. Subsequent studies applied this principle to generate optimal train driving strategies with considering varying slopes, speed limits and force constraints [4–5].

One of the main challenges of these indirect methods mentioned above is the complexity involved in deriving switching conditions for the optimal regimes. In contrast, the direct method works by transforming the original optimal train control problem into constrained mathematical programming problems. In recent years, the pseudospectral method, a type of direct method, has gained popularity in solving train optimal control problems. This is largely due to its faster convergence speed and better computational accuracy [6–10]. Wang and Goverde [6] proposed a novel approach for optimising multi-train trajectories on single-track lines, where the multi-train trajectory optimisation is formulated as a multiple-phase optimal control problem and solved by a pseudospectral method. Wang and Goverde [7] studied the train trajectory optimisation problem with consideration of general operational constraints as well as signalling constraints, in which the train trajectory optimisation problem is also formulated as a multiple-phase optimal control model and solved by a pseudospectral method. Su [8] focused on the development of an automatic train driving strategy by utilising the pseudospectral method, aiming to minimise traction energy consumption for single train operations. Ye and Liu [9] considered a combined train control and scheduling problem involving multiple trains, and they successfully solved this problem by using the Matlab software package GPOPS Version 5.1, which is based on the Radau pseudospectral method. Li et al. [10] introduced a model for optimising the vertical alignment of lines with the objective of minimising both energy consumption and running time deviation, in which an exact solution approach, known as the Gaussian Pseudospectral Method, is utilised.

However, most of these literatures mentioned above are limited to offline computation of the train's reference trajectory, relying on fixed operational parameters such as train resistance coefficients and static speed limits. If the ATO system of a certain train still follows the offline-determined reference profile in the presence of input disturbances or Temporary Speed Restriction (TSR), it may cause unnecessary energy consumption or even lead to disorder of the whole line. Therefore, it is crucial to investigate online control strategies for energy-efficient train operation, which has attracted increasing attention. Zhao et al. [11] proposed a hybrid approach for determining priority weights of emergency alternatives, using the Weighted Ordered Weighted Averaging operator to aggregate preference matrices based on the emergency response task model. Yan et al. [12] introduced a moving horizon optimisation scheme to dynamically determine the reference speed profile for trains under changeable situations. This method employs the immune differential evolution algorithm in each moving horizon to achieve optimal results. Then Yan et al. [13] extended this approach to handle cooperative trajectory planning for multiple trains using the ant colony optimisation algorithm. However, it is important to acknowledge that intelligent or heuristic algorithms cannot always confirm the optimal and converging solutions [14–15], leading to the implementation difficulty in real-time scenarios. For comprehensive reviews on train online control, we refer to [16–20].

Moreover, to support regional economic growth and enhance the competitiveness of railways in freight transportation, some railways are actively pursuing a passenger-freight mixed operation mode, in which both passenger and freight trains coexist and operate in a common line. Obviously, passenger and freight trains have significant differences in operation demands and constraints, bringing challenges to traffic regulation and management. However, the existing work on optimal train control problem for passenger-freight mixed lines is still limited. Liu and Dessouky [21] studied the joint problem of scheduling passenger and freight trains for complex railway networks and presented a novel heuristic optimisation algorithm. According to the author's most recent knowledge, the online control for energy-efficient train operation in passenger-freight mixed lines with considering operational disturbances or TSR is still open.

Inspired by the aforementioned discussions, this paper proposes an online control algorithm based on distributed model predictive control (DMPC) for energy-saving train operation in passenger-freight mixed lines with taking into account input disturbances or TSR. Firstly, we propose a centralised optimisation method to generate reference trajectories for multiple passenger and freight trains in a single-track railway line, which is executed before all the trains depart from their initial stations. Next, we construct the DMPC based online trajectory optimisation problems for both types of trains, subject to their respective operational constraints including safety, punctuality, static speed limits and temporary speed restrictions. Then we formulate an online train control algorithm by applying the centralised optimisation method at initialisation and solving the DMPC based optimisation problems at each time step during operation. Finally, we present numerical simulation examples to validate the efficiency of our proposed algorithm, in which the optimisation problems are solved by using GPOPS-II [22–23] based on the pseudospectral method.

The remaining part of this paper is outlined as described below. Section II introduces the energy-saving operation control problem for passenger and freight trains running in a single-track railway line. An online train operation control algorithm is proposed in Section III, including a centralised optimisation method for the initialisation of train trajectories and a receding horizon optimisation method based on the DMPC. Section IV illustrates the efficiency of our proposed algorithm through numerical case studies. Section V concludes the paper.

2. PROBLEM FORMULATION

2.1 Single-track railway with passenger and freight trains

We study the energy-efficient train control problem for both types of passenger and freight trains running in a single-track railway. As shown in *Figure 1*, consider a single-track railway line that features intermediate stations with multiple tracks, allowing trains to meet and overtake each other. The total number of stations is M. Let $z_m, m = 1,2,\dots,M$ denote the position of station m. Trains operate from the starting point z_1 to the endpoint z_M. Passenger trains stop at every station while freight trains are not required to stop at intermediate stations. Let $Q_t = Q_p \cup Q_f$, where Q_t represents the collection of trains, Q_p represents the collection of passenger trains and Q_f represents the collection of freight trains.

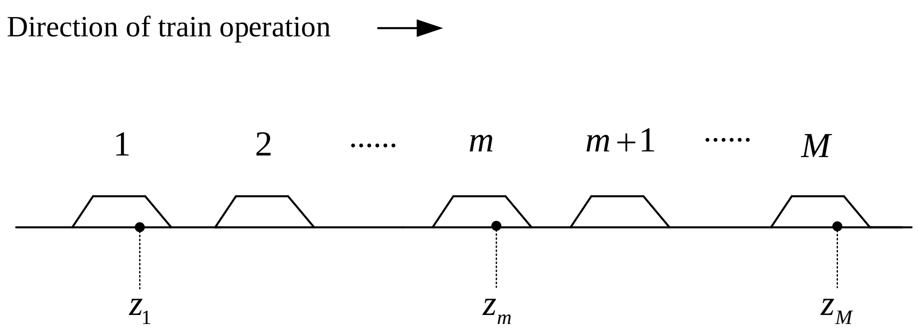


Figure 1 – Illustration of a single-track railway with M stations, in which z_m denotes the position of the m[th] station with $m = 1,2,\dots,M$

2.2 Basic train dynamic model

For the operation of train $i \in Q_t$, the dynamic model with position s as the independent variable can be written as follows:

$$\begin{cases} \dfrac{dv_i(s)}{ds} = \dfrac{u_{i1}(s) + u_{i2}(s) - R_i^{train}(v_i) - R_i^{line}(s)}{m_i v_i(s)} \\ \dfrac{dt_i(s)}{ds} = \dfrac{1}{v_i(s)} \end{cases} \tag{1}$$

where $v_i(s)$ is the velocity of train i at position s, $t_i(s)$ is the time of train i at position s, m_i is the mass of train i, $u_{i1}(s)$ and $u_{i2}(s)$ are the traction and braking forces of train i respectively. The maximum traction and braking forces of freight trains are greater than those of passenger trains, and they have different traction characteristic curves. $R_i^{train}(v_i)$ is the fundamental resistance resulting from both mechanical and aerodynamic friction, typically represented by the Davis equation:

$$R_i^{train}(v_i) = a_i + b_i v_i + c_i {v_i}^2 \tag{2}$$

where a_i, b_i, c_i are positive coefficients that depend on the particular train. Furthermore, $R_i^{line}(s)$ is the linear resistance arising from the track slope:

$$R_i^{line}(s) = m_i g \sin\alpha(s) \approx m_i g \tan\alpha(s) \tag{3}$$

where $\alpha(s)$ represents the angle of inclination of the track at position s and g denotes the acceleration due to gravity. The approximation in *Equation 3* holds when $\alpha(s)$ is small.

Due to complex environmental factors, the train will inevitably be subject to external disturbance during operation. In this paper, we take input disturbances into account, and then the train dynamic model (1) can be reformulated as:

$$\begin{cases} \dfrac{dv_i(s)}{ds} = \dfrac{u_{i1}(s) + u_{i2}(s) + rand_i - R_i^{train}(v_i) - R_i^{line}(s)}{m_i v_i(s)} \\ \dfrac{dt_i(s)}{ds} = \dfrac{1}{v_i(s)} \end{cases} \tag{4}$$

where $rand_i$ denotes a random number to represent the input disturbance of train i.

2.3 Train operation control objective

For passenger train $i \in Q_p$, the timetable indicates scheduled arrival and departure times at every station. Each train departs from the starting point z_1 with speed 0 at a given time $T_{i,1}^d$, then runs along the track and arrives at the endpoint z_M with speed 0 at a given time $T_{i,M}^a$. Thus, we have:

$$\begin{aligned} &t_i(z_1) = T_{i,1}^d, v_i(z_1) = 0, \\ &t_i(z_M) = T_{i,M}^a, v_i(z_M) = 0 \end{aligned} \tag{5}$$

Moreover, to facilitate passenger boarding and alighting, the passenger train must adhere to the timetable and stop at each intermediate station:

$$v_i(z_m) = 0, m = 2,3,\ldots,M-1 \tag{6}$$

$$t_{i,m}^a(z_m) = T_{i,m}^a, m = 2,3,\ldots,M-1 \tag{7}$$

$$t_{i,m}^d(z_m) = T_{i,m}^d, m = 2,3,\ldots,M-1 \tag{8}$$

where $t_{i,m}^a(z_m)$ is actual arrival time of train i at station m, and $t_{i,m}^d(z_m)$ is actual departure time of train i at station m. Moreover, $T_{i,m}^a$ and $T_{i,m}^d$ denote scheduled arrival and departure times of train i at station m.

During train operation, it is essential to consider the constraints on traction and braking forces, velocity and inter-train spacing:

$$0 \le u_{i1}(s) \le u_{i1}^{max}(v_i) \tag{9}$$

$$-u_{i2}^{max}(v_i) \le u_{i2}(s) \le 0 \tag{10}$$

$$0 \le v_i(s) \le V_{max}(s) \tag{11}$$

$$t_i(s) - t_{i-1}(s) \ge T_{min} \tag{12}$$

where $u_{i1}^{max}(v_i)$ and $u_{i2}^{max}(v_i)$ are the maximum traction and braking forces respectively, $V_{max}(s)$ is the train speed limit at position s and T_{min} denotes the minimum safe spacing for train operation.

The control objective for passenger trains is to enhance the punctuality and minimise traction energy consumption, hence we define the following cost function for passenger train i:

$$J_i^p = \sum_{m=2}^{M} \left\{ w_{ip} \cdot \left[t_{i,m}^a(z_m) - T_{i,m}^a\right]^2 + \int_{z_{m-1}}^{z_m} u_{i1}(s)ds \right\} \tag{13}$$

where $w_{ip} > 0$ is a weight coefficient utilised to strike a balance between the punctuality and energy efficiency for passenger trains.

For freight train $i \in Q_f$, it follows the same constrains as a passenger train at the initial and terminal stations, while passing through the intermediate stations without requirement of stops. The freight train is bound by the same operation constraints as the passenger train. Therefore, the freight train adheres to constrains (5) and constrains (9)-(12).

The control objective for freight trains is to minimise total running time and traction energy consumption, hence we define the following cost function for freight train i:

$$J_i^f = \int_{z_1}^{z_M} \left[w_{if} \cdot \frac{1}{v_i(s)} + u_{i1}(s) \right] ds \tag{14}$$

where $w_{if} > 0$ is a weight coefficient utilised to strike a balance between the time-saving objective and energy efficiency for freight trains.

Since the presence of input disturbances or TSR would bring propagating delays, simply tracking the offline-determined reference profile may be infeasible or cause unnecessary energy consumption. Therefore, this paper aims at solving the online control problem for passenger and freight trains running in a single-track railway line, taking into account constraints (5)–(12) for passenger trains and constraint (5), (9)–(12) for freight trains, with the goal of being energy-efficient, punctual and time-optimal (13)–(14).

3. DMPC BASED ONLINE TRAIN OPERATION CONTROL

We firstly consider the centralised optimisation problem to determine the optimal state trajectories and arrival times for both passenger and freight trains, before they depart from the initial station. Then we construct the DMPC based trajectory optimisation problems for both passenger and freight trains. Finally, we formulate an online train operation control algorithm by applying the centralised optimisation method at initialisation and solving the DMPC-based optimisation problems at each time step during operation.

3.1 Centralised optimisation of train trajectories

Centralised optimisation of train trajectories is to improve the punctuality of passenger trains, reduce the total running time of freight trains and save the total operation energy consumption. The centralised optimisation problem for all trains is written as follows:

$$min \sum_{i \in Q_p} J_i^p + \sum_{i \in Q_f} J_i^f \tag{15}$$

subject to the dynamic constraints as *Equation 1* for train $i \in Q_t$, the operation constraints for train $i \in Q_t$:

$$\begin{cases} 0 \le u_{i1}(s) \le u_{i1}^{max}(v_i),\ i \in Q_t \\ -u_{i2}^{max}(v_i) \le u_{i2}(s) \le 0 \\ 0 \le v_i(s) \le V_{max}(s) \\ t_i(s) - t_{i-1}(s) \ge T_{min} \end{cases} \tag{16}$$

the state constraints at all stations for train $i \in Q_p$:

$$\begin{cases} t_{i,m}^d(z_m) = T_{i,m}^d, m = 1,2,\dots,M-1\ \ , i \in Q_p \\ t_{i,m}^a(z_m) \in \left[T_{i,m}^a - t_a, T_{i,m}^a + t_a\right], m = 2,3,\dots,M \\ v_i(z_m) = 0, m = 1,2,\dots,M \end{cases} \tag{17}$$

the state constraints at the initial and terminal stations for train $i \in Q_f$:

$$\begin{cases} t_{i,1}^d(z_1) = T_{i,1}^d\ \ , i \in Q_f \\ t_{i,M}^a(z_M) \in \left[T_{i,M}^a - t_a, T_{i,M}^a + t_a\right] \\ v_i(z_m) = 0, m = 1, M \end{cases} \tag{18}$$

where t_a is the allowable deviation from the scheduled arrival time $T_{i,m}^a, m = 2,3,\dots,M$.

By addressing the centralised optimisation problem, we can effectively determine the reference state trajectories and control trajectories for all trains. We note that solving the centralised optimisation problem is time-consuming, making it hard to execute in real-time. Hence the centralised optimisation will be applied offline before all trains depart from the initial station. On the other hand, simply tracking the offline-determined reference trajectories may be infeasible or cause unnecessary energy consumption in the presence of input disturbances or TSR. Therefore, the next subsection centres on the online optimisation of the train control strategy and arrival times with considering the presence of input disturbances and the temporary speed restriction.

3.2 Distributed model predictive control for train trajectory optimisation

We presume that each train is independently organised and has the ability to compute and interact with other trains. We employ the predecessor-following topology, as illustrated in *Figure 2*, to simulate the communication among trains on a single-track railway that accommodates both passenger and freight trains. In greater detail, train i can only acquire information from train $i-1$, and train 1 does not receive information from any other trains. Trains with blue windows denote passenger trains, while those with red windows denote freight trains. In this subsection, we aim to design the control law $u_{i1}(s)$ and $u_{i2}(s)$ for each train i utilising the DMPC approach.

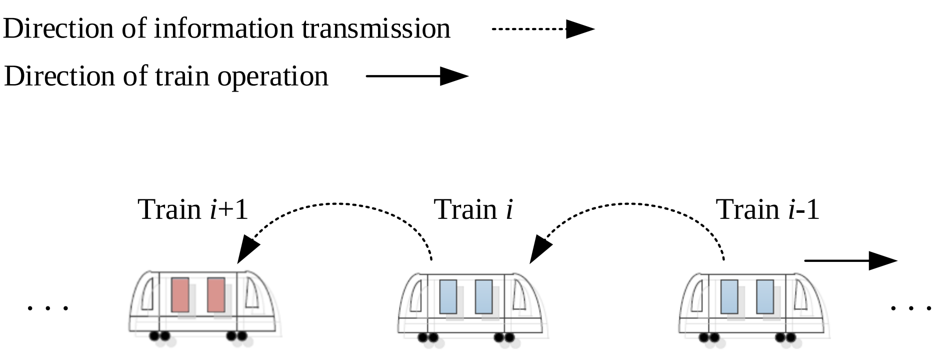


Figure 2 – Predecessor-following communication topology among the trains

The model predictive control approach iteratively applies optimal control within a shifting time horizon [24]. At every time step, the most recent system information is utilised to compute the optimal control input by minimising a predefined cost function over a set prediction horizon.

The problem of energy-efficient control can be modelled as an optimisation problem that considers train velocity and time as the state variables, and traction and braking forces as the control inputs. Let $x_i(s) = [v_i(s), t_i(s)]^T$ and $u_i(s) = [u_{i1}(s), u_{i2}(s)]^T$. Rewrite *Equation 1* into the following compact form:

$$\dot{x}_i(s) = f_i\big(x_i(s), u_i(s)\big) \tag{19}$$

For passenger train $i \in Q_p$, we address the optimal train control problem within the interval $[s_i, z_{n_i}]$, where s_i denotes the present position of train i, and z_{n_i} denotes the position of the next station along the operation direction from its current position s_i (i.e. $z_{n_{i-1}} \leq s_i < z_{n_i}$). The cost function pertaining to the energy-efficient train control problem at current time step k, spanning the prediction horizon from s_i to z_{n_i}, can be stated as follows:

$$J_{i,p} = \int_{s_i}^{z_{n_i}} u_{i1}(s)ds, i \in Q_p \tag{20}$$

The initial condition of the problem is $[v_i(s_i),\ t_i(s_i)]$, which denotes the velocity and time of train i at current position s_i. At the end of the prediction horizon, the terminal states have to satisfy the following:

$$v_i(z_{n_i}) = 0, t_i(z_{n_i}) \in [T_{i,n_i}^a - t_a, T_{i,n_i}^a + t_a] \tag{21}$$

In order to avoid conflicts, the safe constraint should be satisfied:

$$t_i(s) - t_{i-1}^p(s) \geq T_{min} \tag{22}$$

where $t_{i-1}^p(s)$ is the predicted time sequence calculated by train $i-1$ at the previous time step $k-1$, and T_{min} denotes the minimum safe spacing for train operation.

Temporary speed restrictions are a common occurrence during the train operation:

$$\begin{cases} 0 \leq v_i(s) \leq V_{max}^p(s) = V_{ssl}^p(s), \quad s \in [s_i, z_{n_i}] \\ 0 \leq v_i(s) \leq V_{max}^p(s) = min\{V_{ssl}^p(s), V_{tsr}^p(s)\}, \quad s \in [s_r, s_{r+1}] \end{cases} \tag{23}$$

where $V_{ssl}^p(s)$ is the static speed limit and $V_{tsr}^p(s)$ is the temporary speed limit for passenger trains. Therefore, the maximum speed for passenger train $V_{max}^p(s)$ is a piecewise-constant function relative to the position s. Let s_r and s_{r+1} be the starting and ending positions of the area affected by the TSR. We further assume that the ATO system receives the TSR information before the train enters this area, which is within the prediction period.

The energy-efficient control problem for passenger trains at the current time step k, within the prediction horizon from s_i to z_{n_i}, can be formulated as follows:

$$\begin{aligned} &min J_{i,p} \\ &s.t. \begin{cases} \dot{x}_i(s) = f_i(x_i(s), u_i(s)), \quad s \in [s_i, z_{n_i}] \\ v_i(z_{n_i}) = 0, t_i(z_{n_i}) \in [T_{i,n_i}^a - t_a, T_{i,n_i}^a + t_a] \\ 0 \leq u_{i1}(s) \leq u_{i1}^{max}(v_i) \\ -u_{i2}^{max}(v_i) \leq u_{i2}(s) \leq 0 \\ 0 \leq v_i(s) \leq V_{max}^p(s) = V_{ssl}^p(s), \quad s \in [s_i, z_{n_i}] \\ 0 \leq v_i(s) \leq V_{max}^p(s) = min\{V_{ssl}^p(s), V_{tsr}^p(s)\}, \quad s \in [s_r, s_{r+1}] \\ t_i(s) - t_{i-1}^p(s) \geq T_{min} \end{cases} \end{aligned} \tag{24}$$

For freight train $i \in Q_f$, we address the optimal train control problem within the interval $[s_i, z_M]$. The cost function pertaining to the energy-efficient train control problem at current time step k, spanning the prediction horizon from s_i to z_M, can be stated as follows:

$$J_{i,f} = \int_{s_i}^{z_M} u_{i1}(s) ds, i \in Q_f \tag{25}$$

At the end of the prediction horizon, the terminal states have to satisfy the following:

$$v_i(z_M) = 0, t_i(z_M) \in [T_{i,M}^a - t_a, T_{i,M}^a + t_a] \tag{26}$$

The energy-efficient control problem for freight trains at the current time step k, within the prediction horizon from s_i to z_M, can be formulated as follows:

$$\begin{aligned} &min J_{i,f} \\ &s.t. \begin{cases} \dot{x}_i(s) = f_i(x_i(s), u_i(s)), \quad s \in [s_i, z_M] \\ v_i(z_M) = 0, t_i(z_M) \in [T_{i,M}^a - t_a, T_{i,M}^a + t_a] \\ 0 \leq u_{i1}(s) \leq u_{i1}^{max}(v_i) \\ -u_{i2}^{max}(v_i) \leq u_{i2}(s) \leq 0 \\ 0 \leq v_i(s) \leq V_{max}^f(s) = V_{ssl}^f(s), \quad s \in [s_i, z_M] \\ 0 \leq v_i(s) \leq V_{max}^f(s) = min\{V_{ssl}^f(s), V_{tsr}^f(s)\}, \quad s \in [s_r, s_{r+1}] \\ t_i(s) - t_{i-1}^p(s) \geq T_{min} \end{cases} \end{aligned} \tag{27}$$

where V_{max}^f is the maximum speed for freight train, $V_{ssl}^f(s)$ is the static speed limit and $V_{tsr}^f(s)$ is the temporary speed limit for freight train.

3.3 Online train operation control algorithm

Denote $\{t_k\}_{k=0}^{+\infty}$ as the set of sampling and control updating time instants. We also let k stand for t_k. In problems (24) and (27), the real-time state $[v_i(s_i),\ t_i(s_i)]$ is measured at the current position s_i $(t_i(s_i) = t_k)$. The optimal control sequences $[u_{i1}^*(s),\ u_{i2}^*(s)]$ within prediction horizon can be obtained by solving problem (24) or (27), with only the initial control vector $[u_{i1}^*(s_i),\ u_{i2}^*(s_i)]$ implemented in the system. At the subsequent step $k+1$, the updated state is used to re-solve the optimal control problem (24) or (27), and also only the initial control vector is used for the system. By repeatedly solving a set of optimal control problems in a moving horizon approach, this DMPC-based method addresses the online train control problem based on the most up-to-date system information.

The main procedure of the DMPC algorithm for online train operation control in mixed passenger and freight lines is summarised in *Algorithm 1*.

Algorithm 1

Input: the position z_m of station m, the real-time state $[v_i(s_i),\ t_i(s_i)]$ of train i measured at the current time t_k $(t_i(s_i) = t_k)$, the scheduled arrival time $T_{i,m}^a$ of train i at station m, the scheduled departure time $T_{i,m}^d$ of train i at station m, and the temporary speed limit $V_{tsr}^p(s)$ and $V_{tsr}^f(s)$.

Output: the optimal control $[u_{i1}^*(s),\ u_{i2}^*(s)], s \in [s_i, z_{n_i}]$ for the passenger train or $[u_{i1}^*(s),\ u_{i2}^*(s)], s \in [s_i, z_M]$ for the freight train, and the predicted state trajectories $[v_i^p(s),\ t_i^p(s)], s \in [s_i, z_{n_i}]$ for the passenger train or $[v_i^p(s),\ t_i^p(s)], s \in [s_i, z_M]$ for the freight train.

1) Initialisation: Before departure, load data from the ATO database, such as speed limits and train parameters. Then, solve the centralised optimisation problem *Equations 15–18* to calculate the reference position and speed trajectories for the trip, and set the current control step $k = 0$.
2) Train i measures the real-time state $[v_i(s_i),\ t_i(s_i)]$ and solves energy-efficient control problem (24) or (27) based on the measured state. Take the optimal control $[u_{i1}^*(s),\ u_{i2}^*(s)], s \in [s_i, z_{n_i}]$ for the passenger train or $[u_{i1}^*(s),\ u_{i2}^*(s)], s \in [s_i, z_M]$ for the freight train, as the operation strategy within prediction horizon.
3) Update the predicted state trajectories $[v_i^p(s),\ t_i^p(s)], s \in [s_i, z_{n_i}]$ for the passenger train or $[v_i^p(s),\ t_i^p(s)], s \in [s_i, z_M]$ for the freight train stored previously.
4) Train i transmits its updated predicted state trajectories to the following train $i+1$.
5) Implement only the first component $[u_{i1}^*(s_i),\ u_{i2}^*(s_i)]$ of the optimal control sequences obtained from step 2) to train i.
6) Let $k = k + 1$ and return to step 2).

4. SIMULATION RESULTS

In this section, we present numerical examples that clearly validate the efficiency of our advanced algorithm. The optimisation problems mentioned in Section III can be solved by using pseudospectral methods, which transform the train control problem into a nonlinear programming (NLP) problem at the Legendre-Gauss-Radau (LGR) orthogonal collocation points. The transformed mathematical programming problem can then be solved by using the existing optimisation solvers such as GPOPS-II. And we solve the centralised optimisation problem by using GPOPS-II by transforming it into a multi-phase optimal control problem. All experiments in this section are conducted in MATLAB by using the optimisation solver GPOPS-II on a computer with 1.90GHz AMD CPU and 16G RAM.

4.1 Simulation scenario setting

Assuming there are four stations on the route, with a distance of 30 kilometres between each station. There are three passenger trains and a freight train running in this track, the first three being passenger trains and the last being freight train. For passenger trains, the original scheduled running time between an interval is 700 seconds and dwell time in a station is 120 seconds. For freight train, the original scheduled running time between the initial and terminal stations is 3170 seconds. The allowable floating value t_a of original scheduled arrival time $T_{i,m}^a$ is set to 30 seconds. The static speed limit $V_{ssl}^p(s)$ for passenger trains is set to 55 m/s, while freight trains have a static speed limit of 45 m/s. Safe margin T_{min} is set to 240 seconds. Weight coefficient w_{ip} and w_{if} are set to 10^8 and 10^7.

We select the CRH3 train and HXD1 electric locomotive for simulation in the experiment, and the freight vehicle used for simulation is the C_{80} freight vehicle. The parameter information for the CRH3 train and freight train is presented in *Tables 1 and 2,* respectively.

Table 1 – Basic parameters of CRH3 train

Property	Values
a (N/kg)	0.0141612
b (N s/m kg)	5.0400×10^{-4}
c (N s^2/m^2 kg)	1.9912×10^{-5}
Train mass (t)	536
Maximum operating speed (m/s)	55

Table 2 – Basic parameters of freight train

Property	Values
a (N/kg)	100.76
b (N s/m kg)	0.5222
c (N s^2/m^2 kg)	0.0138
Train mass (t)	10676
Maximum operating speed (m/s)	45

The CRH3 train's maximum traction and braking forces, each expressed in kN, are stated as follows:

$$u_{i1}^{max}(v_i)=\begin{cases}300-0.285v_i, & v_i<\dfrac{119km}{h}\\ \dfrac{31500}{v_i}, & v_i\geq\dfrac{119km}{h}\end{cases} \tag{28}$$

$$u_{i2}^{max}(v_i)=\begin{cases}300-0.285v_i, & v_i<\dfrac{106.7km}{h}\\ \dfrac{28800}{v_i}, & v_i\geq\dfrac{106.7km}{h}\end{cases} \tag{29}$$

The HXD1 electric locomotive's maximum traction and braking forces, each expressed in kN, are stated as follows:

$$u_{i1}^{max}(v_i)=\begin{cases}760, & 0<v_i\leq\dfrac{5km}{h}\\ 779-3.8v_i, & \dfrac{5km}{h}<v_i\leq\dfrac{65km}{h}\\ \dfrac{34560}{v_i}, & v_i>\dfrac{65km}{h}\end{cases} \tag{30}$$

$$u_{i2}^{max}(v_i)=\begin{cases}\dfrac{461v_i}{5}, & 0<v_i\leq\dfrac{5km}{h}\\ 461, & \dfrac{5km}{h}<v_i\leq\dfrac{75km}{h}\\ \dfrac{34560}{v_i}, & v_i>\dfrac{75km}{h}\end{cases} \tag{31}$$

4.2 Simulation results and analysis

We present two numerical examples to demonstrate the efficiency and energy efficiency of the proposed DMPC-based online train control algorithm for passenger and freight trains. The first example solves the online train control problem in the presence of input disturbances. In the second example, we take TSR into account to verify the efficiency of the proposed algorithm. In both examples, we will compare the train operation result under the online control of *Algorithm 1* with that under offline control. Offline control represents executing the centralised optimised control input sequence into the system.

Example 1 (Scenarios with input disturbances): We generate random numbers from -9000 to 9000 to simulate the real-time input disturbances in *Equation 4* by using the rand function in MATLAB. The results of these experiments are depicted in *Figures 3–4* and presented in *Table 3*.

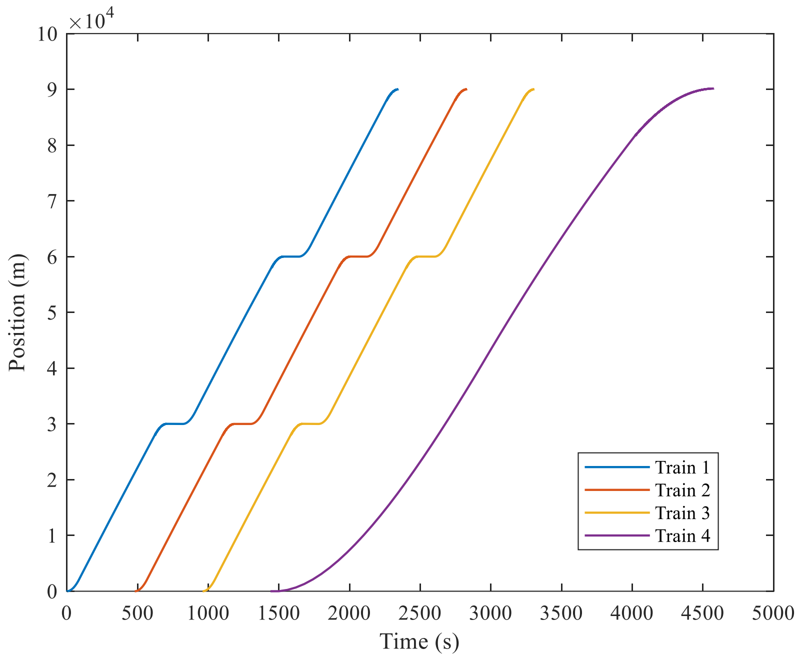


Figure 3 – The position trajectories of all trains under Algorithm 1 in Example 1

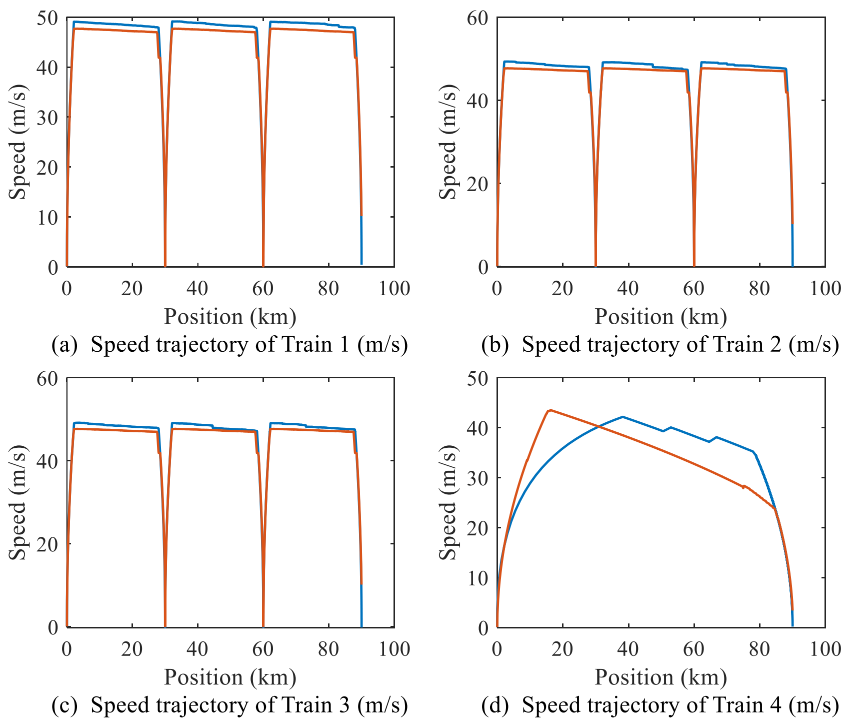


Figure 4 – Speed trajectories of all trains in Example 1 under Algorithm 1 (blue curves) and offline control (red curves)

Table 3 – The arrival states and energy consumption in Example 1

	Train i	$(v_i(z_2), t^a_{i,2})$	$(v_i(z_3), t^a_{i,3})$	$(v_i(z_4), t^a_{i,4})$	**Energy**
Offline control	$i=1$	(10.1 m/s, 703.5 s)	(10.5 m/s, 1522.6 s)	(10.2 m/s, 2342.9 s)	2.15127×10^{10} kJ
	$i=2$	(9.9 m/s, 1184.2 s)	(9.8 m/s, 2004.6 s)	(9.9 m/s, 2824.2 s)	
	$i=3$	(10.2 m/s, 1663.3 s)	(10.6 m/s, 2482.5 s)	(9.6 m/s, 3304.9 s)	
	$i=4$	-	-	(3.5 m/s, 4564.1 s)	
Algorithm 1	$i=1$	(0.0 m/s, 704.4 s)	(0.0 m/s, 1525.2 s)	(0.0 m/s, 2343.6 s)	1.98522×10^{10} kJ
	$i=2$	(0.0 m/s, 1185.6 s)	(0.0 m/s, 2006.4 s)	(0.0 m/s, 2827.2 s)	
	$i=3$	(0.0 m/s, 1666.8 s)	(0.0 m/s, 2490.0 s)	(0.0 m/s, 3308.4 s)	
	$i=4$	-	-	(0.0 m/s, 4576.8 s)	

Figure 3 shows the position trajectories of all trains under *Algorithm 1*. In *Figure 4*, the red curves represent speed trajectories of all trains under the offline control, while the blue curves represent speed trajectories of all trains under *Algorithm 1*. *Figure 4* shows that input disturbances cause fluctuations in speed profiles of all trains under *Algorithm 1*. And the fluctuations of the freight train are larger than that of the passenger trains. This may be because passenger trains have fixed stopping time at each station. Conversely, the DMPC optimisation of freight train is done for the terminal position every time, with a larger optimisation range than passenger trains. Moreover, a single centralised optimisation takes 288 seconds and is not applicable to online train control. However, the average solution time under *Algorithm 1* for each step is 0.39 seconds, which is less than the sampling time period of 1.2 seconds, allowing for online train control to be achievable.

As shown in *Table 3*, the offline control makes the train arrive at the station at an unsafe speed when there is input disturbance. For example, the speed of passenger trains at the second, third and fourth stations is close to 10 m/s. However, the online control algorithm based on the DMPC can successfully make the trains stop at each station. Furthermore, the traction energy consumption of the four trains calculated by offline control and *Algorithm 1* are 2. 15127×10^{10} kJ, and 1.98522×10^{10} kJ, respectively. It reveals that trains controlled by *Algorithm 1* consume less traction energy under the input disturbances.

To evaluate the performance of *Algorithm 1* with a larger number of trains, we conduct an experiment that involved five passenger trains and two freight trains, the results of which are depicted in *Figures 5–6*. *Figure 5* shows the position trajectories of five passenger trains and two freight trains under *Algorithm 1*. In *Figure 6*, the red curves represent speed trajectories of all trains under the offline control, while the blue curves represent speed trajectories of all trains under *Algorithm 1*. Solving the single centralised optimisation problem for the offline control takes 36.85 minutes, which is more than seven times of the solution time for the case with four trains. However, the average solution time under *Algorithm 1* at each step is 0.38 seconds, which is similar as that for the case with four trains. It indicates that the solution time of centralised optimisation severely increases with the number of trains, while the solution time of *Algorithm 1* based on the DMPC would not increase with the number of trains. As shown in *Figure 6*, the offline control makes the train arrive at the station at an unsafe speed when there is input disturbance. However, the online control algorithm based on the DMPC can successfully make the trains stop at each station. Furthermore, the traction energy consumption of the seven trains calculated by offline control and *Algorithm 1* are 3.91403×10^{10} kJ and 3.61587×10^{10} kJ, respectively, revealing that *Algorithm 1* proposed in this paper can also decrease the total traction energy consumption.

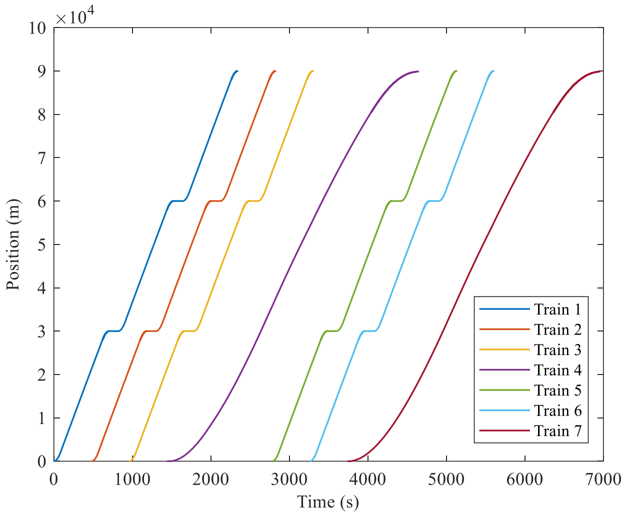
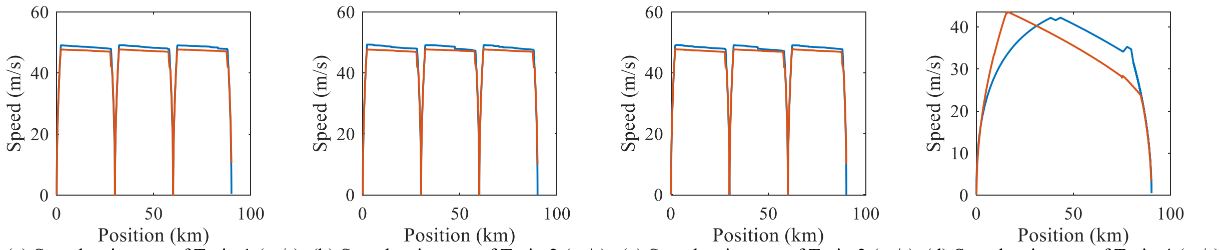


Figure 5 – The position trajectories of seven trains under Algorithm 1 in Example 1



(a) Speed trajectory of Train 1 (m/s) (b) Speed trajectory of Train 2 (m/s) (c) Speed trajectory of Train 3 (m/s) (d) Speed trajectory of Train 4 (m/s)

(e) Speed trajectory of Train 5 (m/s) (f) Speed trajectory of Train 6 (m/s) (g) Speed trajectory of Train 7 (m/s)

Figure 6 – Speed trajectories of seven trains in Example 1 under Algorithm 1 (blue curves) and offline control (red curves)

Example 2 (Scenarios with TSR): A TSR of 40 m/s for passenger trains and 35 m/s for freight train occurs at the interval [45, 50] km, and the four trains will receive this TSR information at 961.2 seconds. The simulation results obtained are depicted in *Figures 7–8* and shown in *Table 4*.

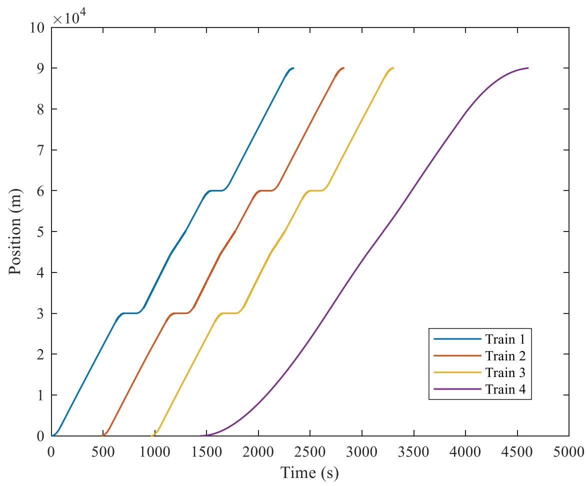


Figure 7 – The position trajectories of all trains under Algorithm 1 in Example 2

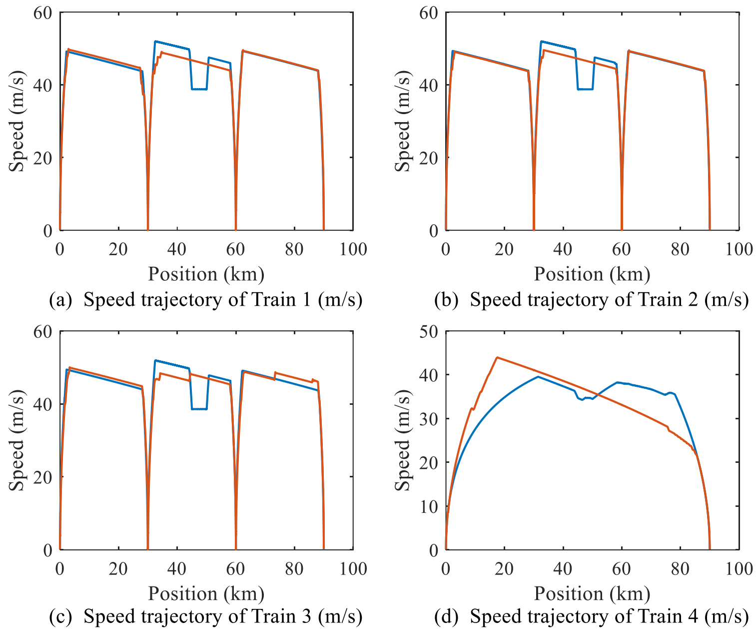


Figure 8 – Speed trajectories of all trains in Example 2 under Algorithm 1 (blue curves) and offline control (red curves)

Table 4 – The arrival states and energy consumption in Example 2

	Train i	$(v_i(z_2), t_{i,2}^a)$	$(v_i(z_3), t_{i,3}^a)$	$(v_i(z_4), t_{i,4}^a)$	**Energy**
Offline control	$i=1$	(0.0 m/s, 700.0 s)	(0.0 m/s, 1520.0 s)	(0.0 m/s, 2340.0 s)	2.15127×10^{10} kJ
	$i=2$	(0.0 m/s, 1180.0 s)	(0.0 m/s, 2000.0 s)	(0.0 m/s, 2820.0 s)	
	$i=3$	(0.0 m/s, 1660.0 s)	(0.0 m/s, 2480.0 s)	(0.0 m/s, 3300.0 s)	
	$i=4$	-	-	(0.0 m/s, 4580.0 s)	
Algorithm 1	$i=1$	(0.0 m/s, 706.8 s)	(0.0 m/s, 1548.0 s)	(0.0 m/s, 2350.8 s)	2.21945×10^{10} kJ
	$i=2$	(0.0 m/s, 1190.4 s)	(0.0 m/s, 2029.2 s)	(0.0 m/s, 2826.5 s)	
	$i=3$	(0.0 m/s, 1671.6 s)	(0.0 m/s, 2506.8 s)	(0.0 m/s, 3309.6 s)	
	$i=4$	-	-	(0.0 m/s, 4609.2 s)	

As shown in *Figure 7*, the position trajectories of all trains bend slightly in the second section due to TSR. In *Figure 8*, the red curves represent speed trajectories of all trains under the offline control, while the blue curves represent speed trajectories of all trains under *Algorithm 1*. As illustrated in *Figure 8*, offline control fails to meet the temporary speed restrictions. However, all trains satisfy the temporary speed restrictions within the interval [45, 50] km under *Algorithm 1*. And because the trains are informed of the temporary speed restrictions in advance, the trains strategically accelerate prior to reaching the section to arrive at the next station timely. And it can be seen from Table 4 that trains consume more traction energy due to TSR during operation.

5. CONCLUSION

In this paper, we propose an online control algorithm for energy-saving train operation in mixed passenger and freight lines. Firstly, we design a centralised optimisation method to generate energy-saving reference profiles for both passenger and freight trains, considering their respective operational objectives and constraints. The method aims to improve the punctuality of passenger trains and to reduce the total running time of freight trains. Secondly, we propose the DMPC based online trajectory optimisation method for both types of trains, subject to operational constraints such as safety, punctuality, static speed limits and temporary speed restrictions. Then we formulate an online train operation control algorithm based on the centralised optimisation method for the initialisation of train trajectories and the DMPC method for the online trajectory planning. Finally, we conduct two numerical experiments, the first one of which involves four trains subject to input disturbances in operation, and the second one examines the impact of TSR on the train operation. The simulation results clearly demonstrate the effectiveness and energy efficiency of our proposed algorithm.

ACKNOWLEDGEMENTS

This work is partially supported by the National Natural Science Foundation of China (Nos. 62273014, U2233211, 62203023), by the Beijing Nova Program (No. 20220484133), by R&D Program of Beijing Municipal Education Commission (Nos. KZ20231000523, KM202310005032), and by the Beijing Municipal College Faculty Construction Plan for Outstanding Young Talents (No. BPHR202203011).

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王丹丹，邓恒，詹璟原，张利国

基于模型预测控制的客货共线列车节能运行在线控制

摘要

论文研究单线铁路上客货共线列车的在线节能运行控制问题。首先，设计了一个集中式优化方法，为客运和货运列车生成节能参考速度曲线，以集中式的方法提高客运列车的准时性和减少货运列车的总运行时间。然后，提出了基于分布式模型预测控制（DMPC）的在线轨迹优化问题，该问题适用于两种类型的列车，并且它们受到各自的运营约束，包括安全性、准时性、静态限速和临时限速。然后，设计了一种客货共线列车在线节能运行控制算法，算法使用集中式优化方法初始化列车运行轨迹，并采用 DMPC 方法对列车运行轨迹进行在线规划。最后，将提出的算法应用于单线铁路上客货共线列车的案例研究，数值模拟结果表明，所提算法能够在存在输入干扰和临时限速的情况下，实现列车运行的在线节能控制。

关键词

列车运行最优控制；节能运行；分布式模型预测控制；客货共线铁路