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Two-Stage Greener Four-Dimensional Trajectory Optimisation with Combined Holding Strategies

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ABSTRACT

The rapid growth of air transport demand poses challenges to the development of the global air traffic system, including increased delays and environmental pollution. To alleviate these problems, a two-stage greener 4D trajectory optimisation framework is proposed by integrating air traffic flow management (ATFM) with the trajectory-based operations (TBO). In the first stage, a multi-objective optimisation model is developed for network-wide delay management to minimise delay costs and CO_2 emissions. The combination of ground holding, standard airborne holding and economic airborne holding is adopted to control the controlled time of arrival (CTA) at each critical node along the trajectory. In the second stage, the 4D trajectory optimisation method is conducted based on the CTA constraints of the negotiated solution from the first stage to generate specific "runway-to-runway" 4D trajectory for the individual flight. The method is tested on the Shanghai-Beijing-Guangzhou air traffic network. Results indicate that including the economic airborne holding provides a more flexible trade-off between delay costs and CO_2 emissions, facilitating stakeholder negotiations. Specifically, a 1% increase in delay cost can lead to a reduction in CO_2 emissions by approximately 0.51%. The proposed method presents a promising solution for achieving greener and more precise ATFM within the TBO paradigm.

KEYWORDS

air traffic flow management; trajectory-based operation; two-stage optimisation; greener 4D trajectory.

1. INTRODUCTION

The rapid growth in air transport demand has led to increased airspace congestion, flight delays and environmental pollution. In 2019, delayed flights accounted for 20.8% and 25% of all flights in the United States and China, respectively [1]. In addition, according to the International Civil Aviation Organisation (ICAO) Environmental Report, one of the aviation industry's targets is to reduce net emissions of carbon dioxide (CO_2) or other greenhouse gases by 50% by 2050 compared to 2005 [2]. Air traffic management (ATM) is considered the most effective means of reducing airspace congestion, flight delays and emissions [2], and trajectory-based operation (TBO) is one of the key characteristics of the new generation air transportation systems. TBO aims to achieve four-dimensional (4D) trajectory planning and control before take-off and during flight through continuous information exchange between various stakeholders, which is recognised as a promising solution to improve the precision, collaboration and environmental impact of air traffic operation. Therefore, there is a growing interest in integrating TBO into the ATM paradigm, although there are still open questions to address. There are two main challenges in ATM under TBO environment: (1) managing air traffic flow at the macro level to maintain balance between flight demand and airspace capacity; (2) precisely controlling the flight trajectory at the micro level. Two technologies called air traffic flow management (ATFM) and 4D trajectory planning are specifically developed for the corresponding challenges, respectively.

ATFM was initially introduced by the ICAO to smooth the air traffic flow and alleviate the demand-capacity imbalances, thus improving the safety and operational efficiency of air traffic systems. In 1987, Odoni first expounded the ATFM problem systematically, and proposed a stochastic and static model based on the ground delay programs (GDPs) [3]. Following this groundbreaking work, ATFM methods in different dimensions (i.e. horizontal, vertical and temporal) and phases (i.e. strategic, pre-tactical and tactical) have been proposed, including airborne holding [4], rerouting [5], speed adjustment [6], miles-in-trail (MIT) [7] and minutes-in-trail [8]. However, with the increasing air traffic demand, it is difficult to achieve demand-capacity balancing (DCB) only by using a single ATFM method alone. Consequently, there has been a growing research focus on the combination of multiple ATFM strategies [9, 10]. Scholars have integrated ATFM technologies and comprehensively analysed the benefits and defects of various ATFM technologies under different operation scenarios [11]. Moreover, some scholars have studied the changes of air traffic demand and sector capacity under the influence of uncertain factors, as well as the real-time adjustment of ATFM strategies based on the updated sector capacity [12]. The NP-hard nature of the ATFM problem has been demonstrated by Bertsimas [9]. To enhance computational efficiency, heuristic algorithms and machine learning techniques have been employed by scholars to solve ATFM problems [13, 14]. In Europe, the collaborative decision-making (CDM) concept developed by EUROCONTROL has been widely implemented to enhance the efficiency and fairness of ATFM [15]. Research has demonstrated that CDM facilitates effective stakeholder collaboration by integrating real-time information exchange among air traffic controllers, airlines and airports [16]. This has enabled significant advancements in strategic and pre-tactical planning for DCB across the European airspace [17].

With the introduction of TBO, the ATM paradigm has shifted from clearance-based tactical control to trajectory-based strategic management, which defines a 4D trajectory for each flight, consisting of a series of waypoints and corresponding controlled time of arrival (CTA). Scholars studied the 4D trajectory optimisation with CTA constraint at different flight stages such as climbing, cruising and descending, and explored the influence of CTA on trajectory optimisation under uncertain factors such as wind [18]. In addition, coordinated optimisation of air traffic flow and 4D trajectory has gradually become popular in recent years. Diao proposed integer ATFM optimisation model which integrates 4D trajectory-based operations under the sector-less airspace configuration, and the heuristic approach via column generation is developed [19]. Based on GDPs, Xu proposed a pre-tactical-and-tactical integrated 4D trajectory planning framework [20]. Chen proposed a pre-tactical-and-tactical integrated 4D trajectory planning framework for the demand and capacity balancing problem, and a hybrid optimisation strategy is designed to solve the general conflict [21]. Zhou proposed a multiobjective 4D trajectory synergetic optimisation method to match the flight traffic with airspace capacity, reduce potential conflicts and fuel consumption [22]. The characteristics of the ATFM methods in related studies are listed in *Table 1*.

Extensive research has been conducted on ATFM and 4D trajectory planning with CTA at both strategic and pre-tactical levels. However, certain limitations persist in the existing literature. Firstly, the studies on ATFM problem primarily focus on achieving a balance between flight demand and airspace capacity, often neglecting realistic constraints such as flight conflicts. Secondly, the primary optimisation objective of ATFM problem is to minimise operational costs, including delay costs, rerouting costs and fuel consumption costs. However, there is a lack of research on reducing CO_2 emissions at the network level. Particularly, the trade-off between CO_2 emissions and network delay costs remains unexplored. Lastly, limited studies have been conducted on the integration of ATFM and 4D trajectory optimisation. To address this gap, a two-stage green 4D trajectory optimisation method is proposed with the integration of network-level ATFM and individual-level trajectory optimisation. The main contributions of this paper are listed as follows:

1) A two-stage greener 4D trajectory optimisation framework is proposed to integrate ATFM and 4D trajectory optimisation. In the first stage, the delay management process is implemented to provide a set of optimised solutions for delay assignment from the network-wide perspective. In the second stage, the individual trajectory for each flight is optimised based on the CTA constraints applied by the negotiated solution from the first stage.
2) The economic airborne holding strategy is introduced to overcome the limitations of conventional holding strategies in ATFM, such as GDPs and airborne holding. The economic airborne holding is performed based on a comprehensive analysis of aircraft performance during different flight phases, enabling the assignment of airborne delay without the extra fuel consumption.

3) A multi-objective mixed integer linear programming (MMILP) model with the objectives of minimising delay cost and CO_2 emissions is established for delay management. The fairness of delayed distribution and the conflict-free constraint are considered in the model. The combination of ground holding, standard airborne holding and economic airborne holding is employed for delay assignment, and the trade-off relationship between two objectives under different strategies is explored.
4) The proposed method is tested on the scenario constructed based on the Shanghai-Beijing-Guangzhou network. The experimental results show that including the economic airborne holding strategy provides a more flexible trade-off between delay costs and CO_2 emissions.

The rest of this paper is organised as follows: Section 2 introduces the holding strategies in delay management of ATFM; Section 3 introduces the overall framework of the two-stage greener 4D trajectory optimisation method; Section 4 presents the multi-objective optimisation model for the delay management in the first stage; Section 5 presents the 4D trajectory optimisation model for the second stage; Section 6 conducts numerical experiments and analyses the results. Section 7 discusses the conclusions and the future research.

Table 1 – Characteristics of the ATFM methods in related studies

Related work	**Strategy (G/A/S/R)[a]**	**Multi-objective**	**Equity**	**Environment (Fuel/CO_2)**	**Conflict**	**Scope (MN/MT)[b]**
Odoni [3]	G/A	×	✓	×	×	MN
Agust [5]	G/A/S/R	×	×	×	×	MN
Bertsimas [9]	G/A/R	×	×	×	×	MN
Hamdan [10]	G/A/S/R	×	×	CO_2	×	MN
Diao [19]	G/A/R	✓	×	Fuel	×	MN+MT
Xu [20]	G/A/S	×	×	Fuel	×	MN+MT
Chen [21]	G	×	×	×	✓	MN
Zhou [22]	S/R	✓	×	Fuel	✓	MN+MT
This paper	G/A/S	✓	✓	Fuel+ CO_2	✓	MN+MT

[a] G: ground holding, A: airborne holding, S: speed control, R: rerouting; [b] MN: macroscopic network, MT: microscopic trajectory.

2. HOLDING STRATEGIES IN DELAY MANAGEMENT

In the field of ATFM, the delay management plays a vital role in smoothing air traffic operation and reducing the workload of air traffic controllers. This section provides an overview of two conventional strategies and introduces a new holding strategy called economic holding.

2.1 GDPs and airborne holding

GDPs delay the flight departure to manage predicted congestion at the destination airport or along the route. Holding on the apron reduces fuel consumption and CO_2 emissions, and eases air traffic controllers' workload, enhancing safety. However, GDPs have limitations. Sector capacity can fluctuate due to limited scheduling flexibility and air traffic uncertainties, causing planned entry slots to become invalid. Typically, this strategy is conservative in dealing with uncertainties, resulting in longer delays than necessary.

Airborne holding is used for short-term imbalances between airspace capacity and demand. Controllers direct aircraft to follow holding patterns at designated points, effectively utilising airspace and airport capacity during these imbalances. However, airborne holding has higher operational costs, including safety concerns, and requires specific procedures or airspace that may be impractical. It also leads to increased fuel consumption and CO_2 emissions.

2.2 Economic airborne holding

Considering the higher operating and environmental costs of airborne holding compared to GDPs, improving the standard airborne holding strategy is essential. Recent research has identified an economic airborne holding strategy, which uses an equivalent speed lower than the planned cruising speed to absorb delays without extra fuel consumption by adjusting the CTA through precise speed and altitude control.

During the cruise phase, the relationship between fuel efficiency and cruise speed is shown in *Figure 1(a)*. The maximum range cruise (MRC) speed represents the speed at which the aircraft achieves the maximum range. The planned cruise speed, denoted as v_{om}^{cruise}, typically exceeds the MRC speed. Additionally, there exists a minimum cruising speed, v_{eq}^{cruise}, referred to as the equivalent speed, which exhibits the same fuel efficiency as the planned cruise speed. Notably, v_{nom}^{cruise} is generally greater than v_{eq}^{cruise}. For cruise speeds ranging between v_{nom}^{cruise} and v_{eq}^{cruise}, the fuel consumption is either lower or equal to the fuel consumption associated with the planned trajectory.

According to the Performance Engineering Program (PEP) of Airbus, the relationship between flight speed and fuel consumption during the typical full-weight climbing and descending stages of A320 aircraft is shown in *Figure 1(b)*. Similar to the cruising stage, there is also an equivalent speed in the climbing and descending stages. Therefore, it is feasible to extend economic airborne holding to the whole flight stage.

The relationship between fuel mileage per unit and flight Mach number at different cruising altitudes for a typical full-weight A320 aircraft is derived as shown in *Figure 1(c)*. The results show that the planned flight altitude may not be the optimal cruising altitude as the flight speed decreases, which indicates that changing cruising altitude with speed can be beneficial for implementing the economic airborne holding strategy.

The economic airborne holding strategy offers several advantages over GDPs and standard airborne holding. Unlike GDPs, which are pre-take-off measures, this strategy allows dynamic adjustment of the CTA during flight. Compared to standard airborne holding, the economic airborne holding strategy can manage delays efficiently without extra fuel consumption. Furthermore, the economic airborne holding strategy enables achieving the system-wide trade-off between the environment and efficiency, due to its more flexible delay assignment mechanism. *Figure 1(d)* shows the feature of these three holding strategies in the ATFM model.

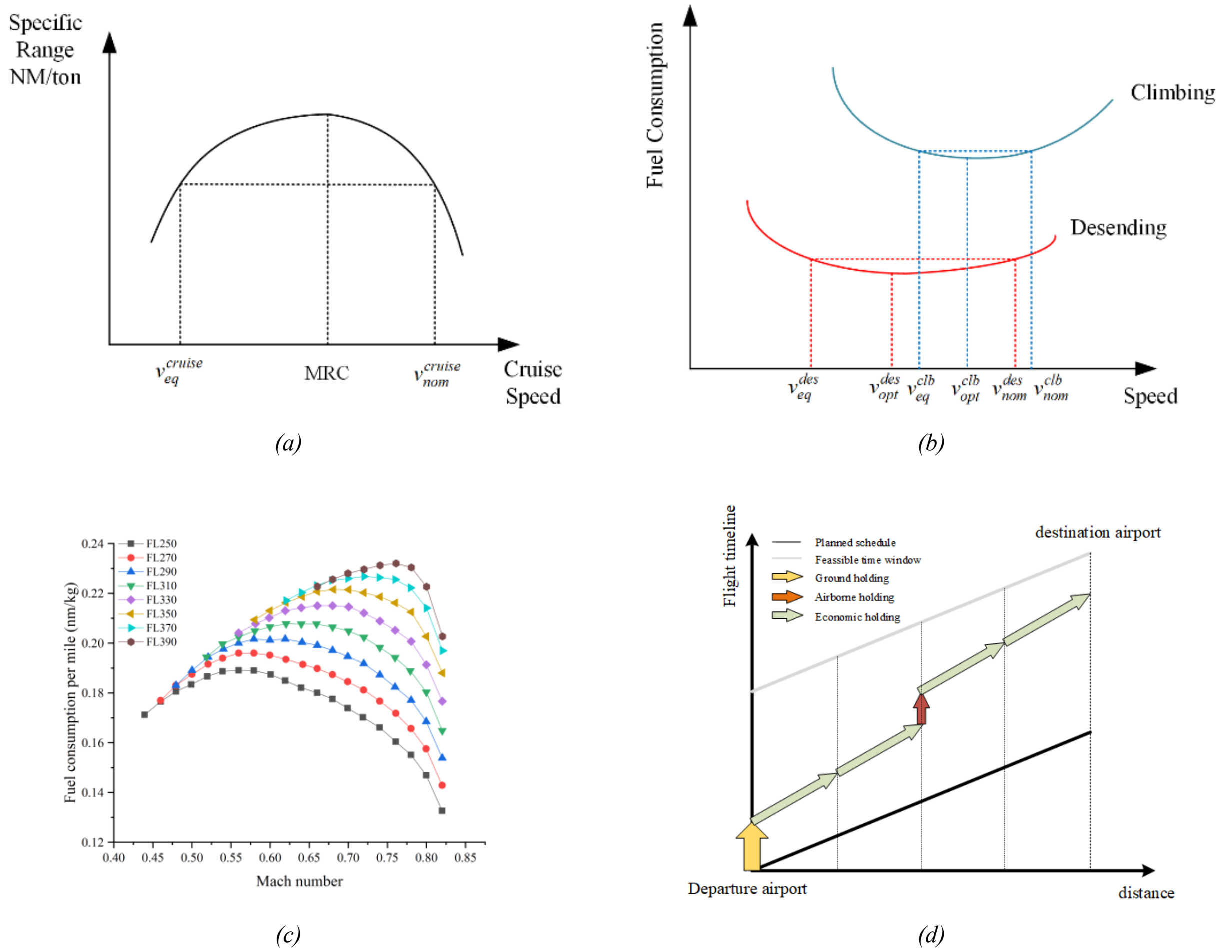


Figure 1 – Illustration of economic airborne holding mechanism: a) function of cruise speed and fuel efficiency in cruise phase; b) function of flight speed and fuel consumption in climbing and descending phases; c) optimal unit fuel mileage vs. Mach number at different cruising altitude; d) flight time vs. distance of three holding strategies

3. TWO-STAGE OPTIMISATION FRAMEWORK

To generate efficient and green 4D trajectories under constrained airspace capacity and facilitate CDM in the strategic trajectory planning process, a two-stage greener 4D trajectory optimisation framework is proposed in this paper. The workflow of the proposed framework is presented in *Figure 2*. This framework integrates ATFM and 4D trajectory optimisation based on the CDM process between network manager (NM) and airspace users (AUs).

At the level of large-scale air traffic networks, airspace congestion and greenhouse effect have become increasingly prominent. Hence, in the first stage, the delay management is conducted by NM to assign flight delays in a manner that reduces delay costs and CO_2 emissions under capacity constraints. To enhance the flexibility in delay assignment, the combination of three holding strategies in ATFM is used, including ground holding, airborne holding and economic airborne holding. The trade-off relationship between delay costs and CO_2 emissions is explored for different air traffic flow management strategies.

The CDM process is performed based on the set of non-dominated solutions optimised in stage 1, which provides the information about the trade-off existing between two objectives. The information is synchronised among all the stakeholders in the ATM system based on the information sharing systems. The negotiation process is conducted between the NM and AUs to select a commonly-agreed solution according to their preferences.

In the second stage, the 4D trajectory optimisation method is conducted by AUs to generate a "runway-to-runway" 4D trajectory for each flight, considering the wind field data and aircraft performance. The CTAs at the waypoints along the trajectory determined by the agreed solution serve as the constraints of the model. Both speed control and altitude adjustment are performed to improve the flight efficiency. Based on the optimised trajectories submitted by AUs, air traffic controllers (ATCos) will make further adjustments during the pre-tactical or tactical phases to avoid potential conflicts and respond to unexpected disturbances. Tactical conflict resolution and real-time adjustments by ATCos falls outside the scope of the proposed framework.

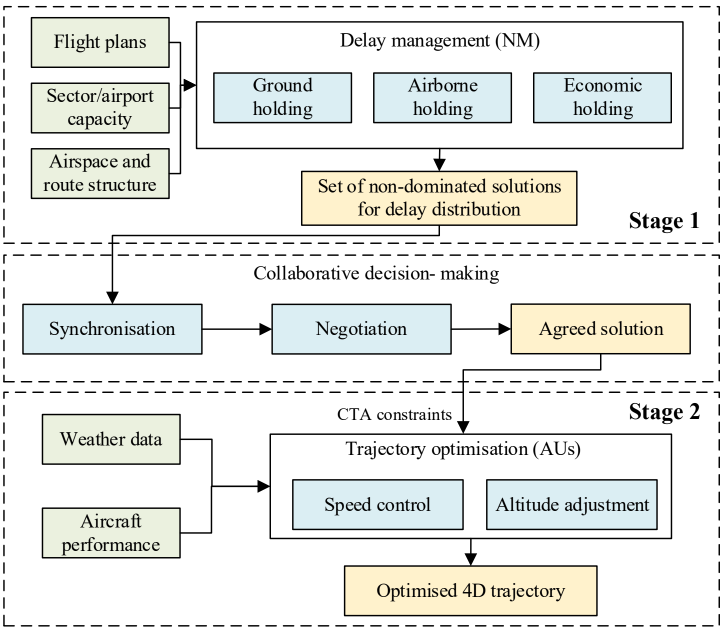


Figure 2 – The workflow of the two-stage greener 4D trajectory optimisation framework

4. GREEN DELAY MANAGEMENT MODEL (STAGE 1)

In the current sector-based ATM system, a flight crosses a sequence of sectors through the waypoints at the sector boundaries before arriving at its destination airport. Hence, the airspace can be represented as a route network $G_{net} = (V, E)$, where $V = \{j_1, j_2, \ldots, j_n\}$ is the set of nodes, representing waypoints at sector boundaries, departure and arrival airports, and E is the set of edges connecting pairs of nodes. An edge between two waypoints at boundaries or between the waypoint and the airport within the same sector. The delay management aims to achieve DCB by assigning delays under various operational constraints (e.g. capacity constraint, conflict-free constraint) to control the CTAs at the waypoints along the scheduled trajectory and controlled time of departure (CTD) for each flight within the airspace. It should be noted that, conflict-free constraint considered in this model refers to the requirement that the entry time of any two consecutive flights passing through the same sector meets the predetermined time interval, regardless of the flight trajectory geometry. To simplify this problem, the following basic assumptions are made:

1) The sector capacity is assumed to be known and fixed;
2) Controllers' intervention in modifying flight trajectory is not taken into account;
3) The departure time of the flight can only be delayed and cannot be advanced.

4.1 Decision variables

To effectively assign ground and airborne delay under various operational constraints, the proposed model combines three ways to handle delays, including ground holding, airborne holding and economic airborne holding. Accordingly, the decision variables of the model are defined as follows:

$$x_{f,t}^{j} = \begin{cases} 1 & \text{if flight } f \text{ departs from node } j \text{ by time } t \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$y_{f,t}^{j} = \begin{cases} 1 & \text{if flight } f \text{ arrives at node } j \text{ by time } t \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

It should be noted that, according to the reference [9], "by" time is used, rather than "at" time, which would enable to improve computing efficiency. The "at" time can be derived by $t\left(x_{f,t}^{j} - x_{f,t-1}^{j}\right)$ and $t\left(y_{f,t}^{j} - y_{f,t-1}^{j}\right)$.

4.2 Objective function

The first objective considered in this model is to minimise the total delay cost. In addition, in line with the growing emphasis on green aviation, the model also considers the CO_2 emissions of flights as a significant factor that must be taken into account by NM. As a result, a dual objective optimisation model is established, which simultaneously considers the total delay cost and CO_2 emissions. The total delay cost is defined as the weighted sum of the delays caused by three holding strategies:

$$\min cost_{TD} = \min(GH + \alpha AH + \beta EH) \tag{3}$$

where the $cost_{TD}$ is total delay cost; the GH is ground holding time; the AH is airborne holding time; the EH is economic airborne holding time; the α and β are the cost weights for airborne holding and economic airborne holding with regard to the ground holding, respectively. Typically, airborne holding incurs higher costs due to additional fuel consumption, and is therefore assigned a weight $\alpha > 1$. In contrast, economic airborne holding, which avoids extra fuel consumption by utilising speed controls, is more cost-efficient and is assigned a weight $\beta < 1$. Let the total delay time be the sum of the delays caused by three holding strategies, $TD = GH + AH + EH$. Then, the objective function can be derived as follows:

$$\min\,[\beta TD + (\alpha - \beta)AH + (1 - \beta)GH] \tag{4}$$

where TD, AH and GH can be calculated as follows:

$$TD = \sum_{f \in F} \sum_{t \in T_f^a} \sum_{P(f,n_f)=a} \left(t - r_f^a\right)\left(y_{f,t}^a - y_{f,t-1}^a\right) \tag{5}$$

$$AH = \sum_{f \in F} \sum_{t \in T_f^w} \sum_{w \in P(f,i):1<i<n_f} t\left(x_{f,t}^w - x_{f,t-1}^w - y_{f,t}^w + y_{f,t-1}^w\right) \tag{6}$$

$$GH = \sum_{f \in F} \sum_{t \in T_f^a} \sum_{P(f,1)=a} (t - r_f^a)(x_{f,t}^a - x_{f,t-1}^a) \tag{7}$$

where $a \in A$ is the set of airports; T_f^j is the available time window of flight f at node j; n_f^j is the predefined number of nodes of flight f according to the scheduled trajectory; r_f^a is the scheduled arrival time at the node j of flight f; $w \in W$ is the set of sector; $P(f,i)$ is the properties of the node j passed by flight f, which represents the departure airport when $j = 1$, arrival airport when $j = n_f$, and the entrance node of the sector when $1 < j < n_f$.

Taking into account the fairness of delay assignment, the total delay time can be multiplied by a coefficient $\gamma = \left(t - r_f^a\right)^{1+\delta} (\delta > 0)$. In this way, delays would be distributed moderately among all flights, rather than a large number of delays being assigned to any one flight. Therefore, the TD can be derived as follows, and the AH and GH is unchanged.

$$TD = \sum_{f \in F} \sum_{t \in T_f^a} \sum_{P(f,n_f)=a} \left(t - r_f^a\right)^{1+\delta} \left(y_{f,t}^a - y_{f,t-1}^a\right) \tag{8}$$

For the network CO_2 emissions, the objective function is shown as follows [23]:

$$\min emi = \sum_{f \in F} \frac{TF_f}{4} \times E_f \times R_A \times N_f \tag{9}$$

where E_f is the average CO_2 emissions per passenger per hour of flight f; R_A is the radiative forcing factor; N_f is the number of passengers of flight f; TF_f is the total flight time of flight f, which is calculated as follows:

$$TF_f = ST_f + TD_f - GH_f \tag{10}$$

where ST_f is the scheduled flight time of flight f.

4.3 Constraints

$$\sum_{t \in T_f^j} \left(x_{f,t}^j - x_{f,t-1}^j\right) = 1 \forall f \in F, \forall j \in P_f \tag{11}$$

$$\sum_{t \in T_f^j} \left(y_{f,t}^j - y_{f,t-1}^j\right) = 1 \forall f \in F, \forall j \in P_f \tag{12}$$

$$\sum_{f \in F} \left(x_{f,t}^j - x_{f,t-1}^j\right) \leq 1 \forall t \in T_f^j, \forall j \in P_f \tag{13}$$

$$\sum_{f \in F} \left(y_{f,t}^j - y_{f,t-1}^j\right) \leq 1 \forall t \in T_f^j, \forall j \in P_f \tag{14}$$

$$x_{f,t}^j - x_{f,t-1}^j \geq 0 \quad \forall f \in F, \forall j \in P_f, \forall t \in T_f^j \tag{15}$$

$$y_{f,t}^{j} - y_{f,t-1}^{j} \geq 0 \forall f \in F, \forall j \in P_f, \forall t \in T_f^j \tag{16}$$

$$x_{f,t}^{w} - y_{f,t}^{w} \leq 0 \quad \forall f \in F, \forall P(f,i): 1 < i < n_f, \forall t \in T_f^w \tag{17}$$

$$\sum_{t \in T_f^w} \sum_{w \in P(f,i): 1<i<n_f} t\left(x_{f,t}^{w} - x_{f,t-1}^{w} - y_{f,t}^{w} + y_{f,t-1}^{w}\right) \leq u_f \quad \forall f \in F, \forall P(f,i): 1 < i < n_f, \forall t \in T_f^w \tag{18}$$

$$y_{f,t'}^{j'} - x_{f,t}^{j} \leq 0 \forall f \in F, \forall i[1, n_f - 1], P(f,i) = j, P(f,i+1) = j', \forall t \in T_f^j, t' = t + z_f^{j,j'} \tag{19}$$

$$y_{f,t'}^{j'} - x_{f,t}^{j} \geq 0 \forall f \in F, \forall i \in [1, n_f - 1], P(f,i) = j, P(f,i+1) = j', \forall t \in T_f^j, \forall t' \in T_f^{j'}, t' = t + z_f^{j,j'} + v_f^{j,j'} \tag{20}$$

$$y_{j,t}^{f} - y_{j,t-1}^{f} = I_{j,t} \tag{21}$$

$$\sum_{t=t-\frac{1}{d_j}+1}^{t} I_{j,t} \leq 1 \tag{22}$$

$$I_{j,t} = \begin{cases} 1 & \text{node } j \text{ is passed at time } t \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

$$\sum_{f \in F: P(f,1)=a} \sum_{t \in T_f^a \cap T(\tau)} \left(x_{f,t}^{a} - x_{f,t-1}^{a}\right) \leq C_{dep}^{a}(\tau) \quad \forall a \in A, \forall \tau \in \Upsilon \tag{24}$$

$$\sum_{f \in F: P(f,n_f)=a} \sum_{t \in T_f^a \cap T(\tau)} (y_{f,t}^{a} - y_{f,t-1}^{a}) \leq C_{arr}^{a}(\tau) \quad \forall a \in A, \forall \tau \in \Upsilon \tag{25}$$

$$\sum_{f \in F: P(f,i)=w, i \in [1.n_f-1]} \sum_{t \in T_f^w \cap T(\tau)} (x_{f,t}^{w} - x_{f,t-1}^{w}) \leq C_{\sec}(\tau) \quad \forall w \in s \subset S, \forall \tau \in \Upsilon \tag{26}$$

Equations 11-14 represent the uniqueness constraints, ensuring each flight f must pass through its predefined node P_f according to the scheduled trajectory with only one slot allocated per flight at each node, and each slot at any node being assigned to only one flight. *Equations 15-16* are continuity constraints, indicating that the decision variables are continuous in time. *Equations 17-18* are the airborne holding time constraints to ensure that airborne holding time should not exceed the threshold value. *Equations 19-20* are the economic airborne holding feasibility constraints, where $z_f^{j,j'}$ is scheduled flight time for flight f from j to j'; $v_f^{j,j'}$ is the maximum delay generated from j to j' by economic airborne holding strategy. The model assumes flight speed does not exceed the planned speed, and the delay generated by economic airborne holding cannot exceed its threshold. *Equations 21-23* are the horizontal separation constraints, where $I_{j,t}$ is the time interval that two consecutive aircraft pass the node j (sortie/min), which is calculated by dividing the protocol separation by the predefined flight speed. In this paper, the predefined flight speed is 850 km/h. *Equations 24-26* are capacity constraints, where $s \in S$ is the set of capacity restricted sector; $\tau \in \Upsilon$ is the granularity of air traffic demand and airspace capacity; $C_{dep}^{a}(\tau)$, $C_{arr}^{a}(\tau)$ and $C_{\sec}(\tau)$ are the capacity of departure airport, arrival airport and sector capacity, respectively.

5. 4D TRAJECTORY OPTIMISATION MODEL (STAGE 2)

Based on the agreed solution selected from the non-dominated solutions of delay management, the 4D trajectory optimisation method is employed by AUs to precisely control the individual trajectory for each flight. The 4D trajectory optimisation problem was simplified by dividing the vertical profile into three stages: climbing, cruising and descending. The horizontal trajectory of the aircraft was discretised in 1 km increments. By transforming the time-continuous trajectory optimisation problem into a mixed-integer nonlinear program (MINLP), a limited set of decision variables was obtained. Both the speed and vertical profiles of the trajectory are optimised by speed control and altitude adjustment to minimise the fuel consumption and flight time. Various operational constraints are considered in the model, such as aircraft dynamics, ATFM regulations and passenger comforts. To simplify this problem, the following basic assumptions are made:

1) The clean configuration of the aircraft is adopted for optimisation;
2) The human interventions to the flight trajectories are not considered.

5.1 Decision variables

In this model, the horizontal trajectory was discretised, and the continuous 4D trajectory optimisation is transformed into a problem involving determining the flight speed and altitude at the nodes. Therefore, decision variables are the flight speed v_i^f and flight altitude h_i^f of the flight f at node i. It should be noted that, different from the meaning of node j, node i is the node separated by 1 km after the horizontal trajectory is discretised.

5.2 Objective function

The objective of the 4D trajectory optimisation model is the total fuel consumption and flight time of the aircraft:

$$min\left\{\sum_{i} F_{i_n,i_{n+1}}^{f}, \sum_{i} t_{i_n,i_{n+1}}^{f}\right\} \tag{27}$$

where $F_{i_n,i_{n+1}}^{f}$ and $t_{i_n,i_{n+1}}^{f}$ respectively represent the fuel consumption calculated based on the Base of Aircraft Data (BADA) model and flight time of flight f on the flight segment (i_n, i_{n+1}).

5.3 Constraints

$$h_{i,controlmin}^{f} \leq h_i^f \leq min\{h_{i,controlmax}^{f}, h_{max}^{f}\} \tag{28}$$

$$\frac{h_i^f}{100} = \begin{cases} 2N & \text{westbound} \\ 2N+1 & \text{eastbound} \end{cases} \quad \forall N \in \{0,1,2,\dots\dots\} \tag{29}$$

$$max\{v_{i,controlmin}^{f}, v_{i,min}^{f}\} \leq v_i^f \leq min\{v_{i,controlmax,}^{f} v_{i,max}^{f}\} \tag{30}$$

$$\frac{\left(v_{i_{n+1}}^{f}\right)^2 - \left(v_{i_n}^{f}\right)^2}{2 \cdot d} \leq a_{max} \tag{31}$$

$$\frac{\left|h^f_{i_{n+1}} - h^f_{i_n}\right|}{t^f_{i_n,i_{n+1}}} \leq x^f_{i_n,i_{n+1}} C^f_{Lmax} + y^f_{i_n,i_{n+1}} D^f_{Emax} \tag{32}$$

$$\omega^f_{i'} \in T_s \tag{33}$$

$$x^f_{i_n,i_{n+1}} = \begin{cases} 1 & h^f_{i_{n+1}} - h^f_{i_n} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{34}$$

$$y^f_{i_n,i_{n+1}} = \begin{cases} 1 & h^f_{i_{n+1}} - h^f_{i_n} < 0 \\ 0 & \text{otherwise} \end{cases} \tag{35}$$

Equation (28) represents flight altitude constraint, where h^f_{max} is the maximum altitude of the aircraft under the constraint of its properties; $h^f_{i,controlmin}$ and $h^f_{i,controlmax}$ are the minimum and maximum altitude thresholds of aircraft at node i according to air traffic control rules. *Equation (29)* states that the altitude for each flight should comply with the "odd for eastbound and even for westbound" regulations. *Equation (30)* represents the flight speed constraints, where $v^f_{i,min}$ and $v^f_{i,max}$ are the minimum and maximum speed of flight f at node i under the constraint of its performance; $v^f_{i,controlmin}$ and $v^f_{i,controlmax,}$ are the minimum and maximum speed thresholds of aircraft at node i according to air traffic control rules. *Equation (31)* ensures that flight acceleration should not be excessive in consideration of flight safety and passenger comfort, where $a_{max} = 0.6096\ \mathrm{m/s^2}$ is the maximum longitudinal acceleration recommended in the BADA 3.11 GPF file. *Equation (32)* ensures that the rates of climb and descent of the aircraft cannot exceed the maximum rate of climb or the maximum rate of descent, where C^f_{Lmax} and D^f_{Emax} are the aircraft's maximum rate of climb and maximum rate of descent, respectively; $x^f_{i_n,i_{n+1}}$ and $y^f_{i_n,i_{n+1}}$ are state variables. In this paper, the rate of climb and the rate of descent introduced from BADA 3.11 PTF file are used as the thresholds for the climb and descent operations. *Equation (33)* represents the sector capacity constraints, where i' is the node that enters sector s; $\omega^f_{i'}$ is the time aircraft flight f passes node i'; T_s is the set of available time slots of sector s. *Equation (34)* and *Equation (35)* define the states of the aircraft being in a climbing and descending phase, respectively.

6. EXPERIMENTAL ANALYSIS

In this section, we designed a simulation scenario based on the route network among three busiest airports in China to verify the effectiveness and efficiency of the proposed method. The experiment is implemented using software GAMS 25.1.3 and CPLEX on the laptop computer, with Intel Core i5-6300HQ CPU 2.30GHz quad-core processor, 8 GB memory, Windows 10 operating system.

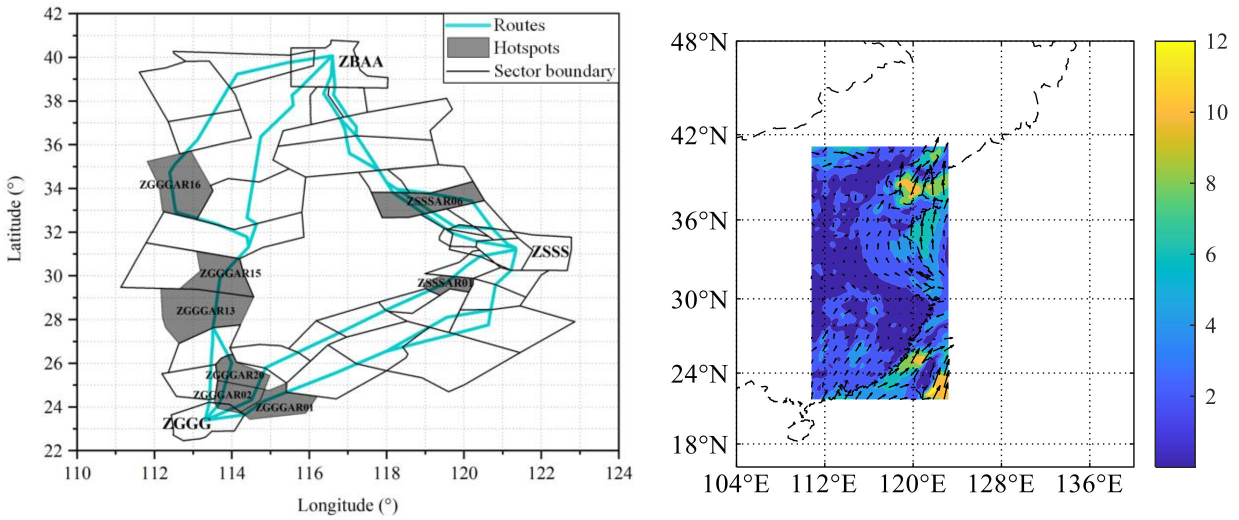
6.1 Scenario setup

As shown in *Figure 3(a)*, a scenario is selected for the analysis based on the route network which consists of the three busiest airports in China, ZBAA, ZSSS and ZGGG. A total of 64 flights of ZBAA, ZSSS and ZGGG route network from 6:00 to 12:00 on 1 June 2019, are selected. By analysing the historical air traffic data, 8 sectors are identified as "hotspots", which represent the areas where the imbalance between flow and capacity occurs. The published capacity of each sector is obtained from the data of Civil Aviation Administration of China, which specifies the maximum number of aircraft that can operate within each sector per hour. The available sector capacity is calculated by subtracting the traffic flows that do not belong to the route network of ZBAA, ZSSS and ZGGG from the published sector capacity. Based on the 4D trajectory, a flight is considered to belong to the route network if its position and time indicate that it enters any sector within the studied airspace or arrives at/departs from any airport within the studied airspace during the given time domain.

6.2 Parameter settings

The parameters of the proposed method are set as following: according to the reference [24], the airborne holding cost factor $\alpha = 1.2$; the economic airborne holding cost factor $\beta = 0.8$; $\delta = 0.05$ is set as fairness factor; the maximum airborne holding time per flight $u_f = 60$ min. The maximum delay time of economic airborne holding without extra fuel cost $v_f^{j,j'}$ is 20% of scheduled flight time [24]. The CO_2 emission per passenger per hour of flight E_f is 90 kg/hr/passenger, and the radiative forcing factor R_A is 2 [23]. The number of passengers N_f is 81% of the maximum number of passengers [25]; the time granularity of air traffic demand statistics is 1 hour; to simplify the problem, the economic airborne holding strategy does not change the cruising altitude; the discrete time interval is set to 1 min; the minimum interval between consecutive flights passing through the same route point is 2 min. It should be noted that the planned trajectory data come from the airline release form.

Taking into account that the wind has a significant effect on the flight path optimisation results, in this paper, meteorological data are introduced in the 4D trajectory optimisation method. The data of weather in this paper are mainly derived from the GRIB2 files published by Global Forecast System (GFS) of National Oceanic and Atmospheric Administration (NOAA), and the GFS data are further divided into GFS analysis data and GFS forecast data. In this paper, the weather data of the $0.5° \times 0.5°$ latitude and longitude grid are selected in the GFS analysis data. *Figure 3(b)* illustrates the distribution of the GFS wind data, where the direction of the arrows indicates the wind direction, and their length represents the wind speed.



(a) *(b)*

Figure 3 – Illustration of the simulation scenario: a) ZBAA, ZSSS and ZGGG route network structure; b) GFS wind data

6.3 Solution approaches

For the first stage model, to solve the bi-objective optimisation model, the weighted comprehensive criterion method (WCCM) is used. The core idea of WCCM is to convert the multi-objective optimisation problem into a single-objective optimisation problem by assigning corresponding weights to different sub-objective functions, and weighting and summing all objective functions:

$$\min E = \gamma \left(\frac{Cost_{TD} - Cost_{TD}^{opt}}{Cost_{TD}^{opt}} \right) + (1-\gamma) \left(\frac{Emi - Emi^{opt}}{Emi^{opt}} \right)$$

where $Cost_{TD}$ and Emi^{opt} are optimal solutions of total delay cost and CO_2 emission respectively; $\gamma \in [0,1]$ is the weighting factor. The Pareto optimal front can be obtained by changing γ, and the step of γ is set as 0.05.

For the second stage model, the non-dominated sorting genetic algorithm (NSGA-II) was used to solve multi-objective trajectory optimisation problem. The details of the algorithm design can be referred to our previously published paper [26].

6.4 Result analysis

To verify the efficiency of employing economic airborne holding in delay management, the conventional ATFM method with ground holding and airborne holding is conducted with the same parameter settings for comparison. The Pareto optimal fronts of the proposed ATFM method (Case 1) and the conventional ATFM method (Case 2) are shown in *Figure 4*. There is only one feasible solution in Case 2, while the proposed ATFM method with more flexible economic airborne holding strategy is able to provide information about the trade-off existing between the delay cost and CO_2 emission for the AUs and NM. Meanwhile, for Case 1, the total delay cost has a significant impact on CO_2 emissions and these two objectives present a nearly linear relationship within a specific delay interval. Specifically, each 1% increase in delay cost reduces CO_2 emissions by 0.51%.

In order to further analyse the delay distribution of the optimisation results, the optimal solutions with minimum delay cost and CO_2 emission are selected from the set of non-dominated solutions as the illustrative examples. *Figure 5(a)* and *Figure 5(b)* show the delay distribution of each flight in the solution with optimal CO_2 emission and the solution with optimal delay cost, respectively. *Table 2* shows the metrics of flight delays for two illustrative solutions. Since the CO_2 emission has a proportional relationship with flight time according to *Equation 9*, the ground holding strategy is adopted for all flights in the emission-optimal solution in order not to increase airborne flight time, which can also explain that there is only one feasible solution to the traditional ATFM model. As for the solution with optimal delay cost, it can be seen that flight delays are primarily caused by ground holding and economic holding. The total delay time is reduced, since the economic airborne holding strategy can make more efficient use of the spatial resources to handle the imbalance between flow and capacity within multiple restricted sectors and the destination airport. In addition, more flights are assigned with delays, making delay distribution more fair.

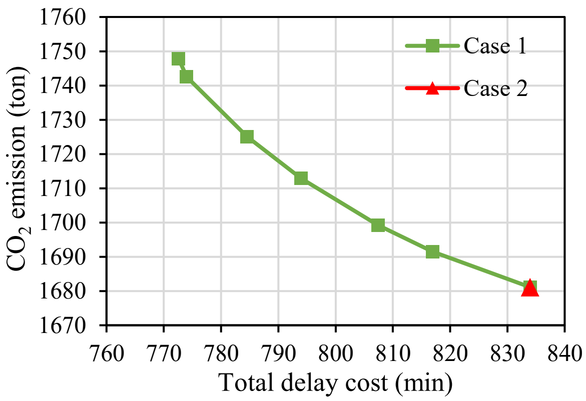
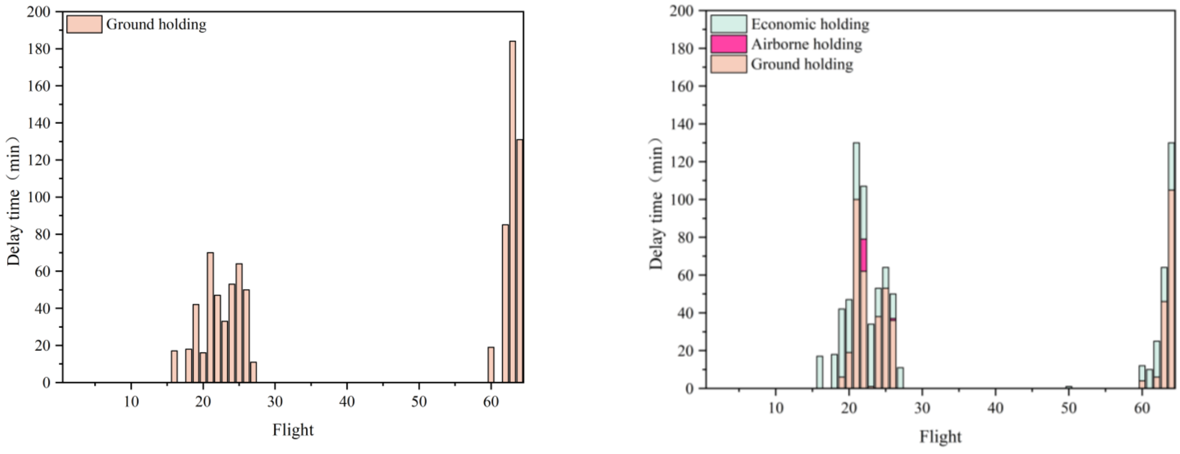


Figure 4 – Pareto optimal front for two compared ATFM methods

Table 2 – Flight delays of two illustrative solutions

Metrics	Cost-optimal solution	Emission-optimal solution
Total delay time (min)	815	834
Number of delayed flights	17	15
Average delay time (min)	47.9	56
Ground holding (min)	501	834
Airborne holding (min)	13	0
Economic airborne holding (min)	320	0

In order to explore the influence of the weight coefficient γ change on the parameters such as ground holding, airborne holding and economic airborne holding delay time and CO_2 emission, a sensitivity analysis was carried out in this paper, and the results are shown in *Figure 6*. When $\gamma \in [0,0.2]$, the objective of reducing CO_2 emissions has a higher priority, and ground holding is widely used for ATFM that does not change the airborne flight time, and is always maintained at a maximum value of 840 min; when $\gamma \in (0.2,0.5]$, with the continuous increase of parameter γ, the total delay cost gradually becomes the main optimisation objective, the proportion of economic airborne holding gradually increases and reaches 302 min, the ground holding delay time is reduced to 501 min, and the airborne delay time is increased to 14 min. When $\gamma \in (0.5,0.6]$, the total delay cost of network is still the main optimisation objective, and the optimisation result remains unchanged. When $\gamma = 0.65$, the economic airborne holding delay reaches the maximum value of 321 min, the ground holding delay time is reduced to 476 min, and the airborne delay time is increased to 18 min. When $\gamma \in [0.65,1]$, since the economic airborne holding has reached the upper bound of feasible delay, and due to the high cost of airborne holding, it is impossible to balance the delay cost and CO_2 emissions by continuing to increase the airborne delay, so the above parameters remain unchanged.



(a) *(b)*

Figure 5 – Delay of flights for two illustrative solutions: a) solution with optimal CO_2 emission; b) solution with optimal delay cost

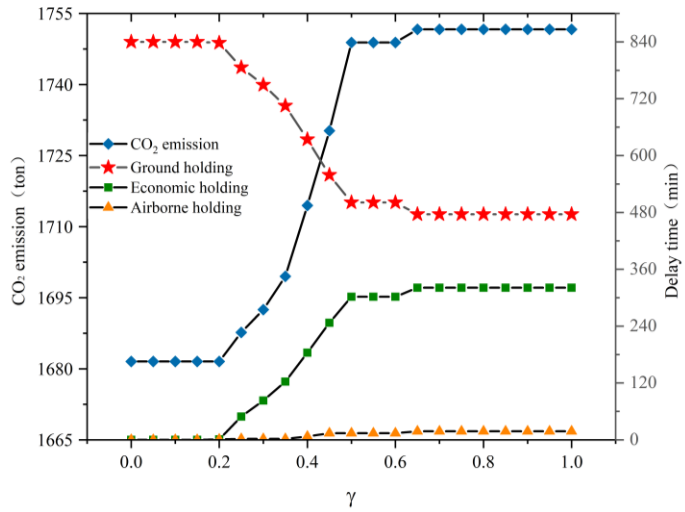
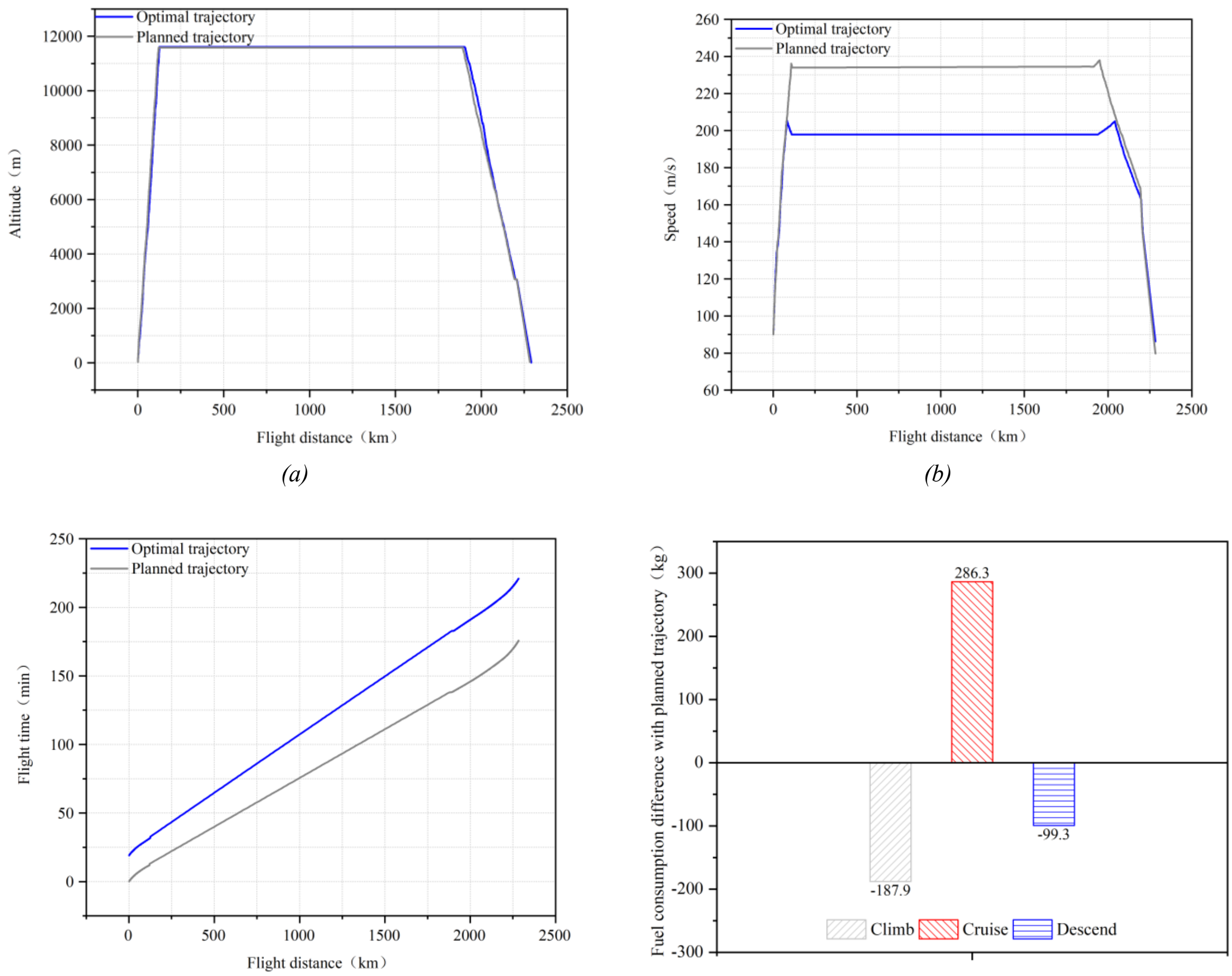


Figure 6 – Sensitivity analysis of coefficient γ

The solution with optimal delay cost from delay management in stage 1 is selected as agreed solution to serve as an illustrative example. Based on the selected solution, the aircraft flying over the restricted sector and the take-off and landing time are used as CTA and CTD constraints of the 4D trajectory optimisation model in stage 2. In addition, the performance constraints are set to ensure that the fuel consumption of the optimised trajectory is not greater than the planned trajectory. As shown in *Figure 7*, the trajectory parameters of flight CCA1335 (B744) are selected for analysis, including the altitude, speed, time and fuel consumption. Compared with the planned trajectory, the optimised trajectory maintains the same cruise altitude but reduces the cruise speed to perform economic airborne holding. In terms of flight time, the flight is assigned a total delay of 47 minutes, consisting of 19 minutes of ground delay and 28 minutes of airborne delay, with all airborne delay managed through economic airborne holding. As shown in *Figure 7(d)*, for the optimised trajectory, the fuel consumption decreases in the climb and descent phases due to reduced speeds, which lower the fuel consumption rate. For the cruise phase, the reduced cruise speed causes the original cruise altitude to no longer align with the optimal altitude for fuel efficiency, leading to an increased fuel consumption rate. Additionally, the extended flight time from reduced speed further contributes to higher total fuel consumption during the cruise phase. For the total fuel consumption, the optimised trajectory achieves a fuel consumption reduction of 0.9 kg compared with the planned trajectory, demonstrating the effectiveness of the proposed 4D trajectory optimisation method in planning conflict-free and green 4D trajectories without incurring additional fuel costs.



(a) *(b)* *(c)* *(d)*

Figure 7 – Optimised trajectory: a) altitude profile; b) speed profile; c) time profile; d) fuel consumption difference

7. CONCLUSION

In this paper, we propose a two-stage greener 4D trajectory optimisation method that integrates network-wide ATFM with individual 4D trajectory optimisation to reduce delay and environmental costs. In the first stage, a multi-objective optimisation model for delay management is formulated, considering sector capacity constraints, conflict-free requirements and fair delay allocation. To improve on conventional holding strategies, an economic airborne holding strategy is introduced to assign airborne delay without extra fuel consumption. Through comprehensive analysis of a large-scale air traffic network scenario, the trade-off between delay costs and CO_2 emissions under different ATFM strategies is investigated. The results highlight the significant impact of chosen ATFM strategies on this trade-off. Specifically, the economic airborne holding strategy offers a broader set of non-dominated solutions in the objective space, aiding stakeholder negotiations. The findings show that a 1% increase in delay costs can lead to a 0.51% reduction in CO_2 emissions. In the second stage, the 4D trajectory optimisation method generates specific “runway-to-runway” 4D trajectories considering wind fields, resulting in a 0.9 kg reduction in fuel consumption compared to the initial trajectories.

Future research will explore the impact of factors such as aircraft type and flight range on the economic airborne holding strategy through simulations. Additionally, rerouting strategies will be incorporated to enhance spatial resource utilisation in ATFM within the TBO context. Future research will extend the scope to include other significant non-CO_2 emissions, such as nitrogen oxides and contrail formation to provide a more comprehensive assessment of environmental impacts. Considering the significant impact of weather uncertainty on sector and runway capacity, weather forecast data will be integrated into the two-stage 4D trajectory optimisation method to improve robustness. These extensions aim to provide a more comprehensive understanding of the benefits and implications of ATFM strategies for achieving greener and more efficient network traffic flow management within the TBO paradigm.

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