A Pareto-Optimal Carbon Allowance Policy – Balancing Emission Reduction and Commuter Benefits in Urban Transportation Systems

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ABSTRACT

This paper explores the impact of the personal carbon allowance (PCA) policy on a bi-modal transportation system, examining travel mode shifts, congestion patterns and policy effectiveness. The optimal carbon allowance quantity and the policy's social acceptability are assessed through numerical simulations and Pareto optimisation. Results indicate that, in this parametric simulation, a strict allocation of 8,650 units achieves a 64% emission reduction while meeting Pareto efficiency, whereas a higher allowance of 14,300 units prioritises public acceptance with a 41% emissions reduction. However, the effectiveness of the PCA policy is influenced by factors such as road capacity stability and the level of public transportation development. First, low or unstable road capacity diminishes behavioural shifts and carbon emission reductions. However, when accidents are promptly addressed, the policy's effectiveness is less impacted. Second, if public transit capacity is insufficient and passengers experience overcrowding, reducing car commuters without undermining passenger benefits proves challenging. The study quantifies the trade-offs between emission reduction goals and societal welfare, identifies infrastructure limitations and demonstrates the policy's adaptability through dynamic adjustments. Although assuming homogeneous traveller behaviour and static demand, this work establishes a framework for implementing behaviourally-informed carbon allowances policy in real-world transport systems.

KEYWORDS

personal carbon allowances; Vickrey model; traffic congestion; transit priority; travel mode choice.

1. INTRODUCTION

According to the sixth report of the United Nations Intergovernmental Panel on Climate Change, increasing global warming poses a significant threat to human survival and development [1-3]. Research indicates that greenhouse gas emissions from human activities, such as fossil fuel combustion and land-use changes, are major contributors to global warming [4-6]. The transportation sector alone accounts for approximately one-fifth of global carbon dioxide emissions, with road transport responsible for three-quarters of those emissions, equivalent to 15% of total CO_2 emissions [7]. The International Energy Agency (IEA) projects that without intervention, global transport demand will double by 2070, with car ownership rates increasing by 60% [8]. Therefore, it is imperative to implement measures to reduce greenhouse gas emissions in the transportation system.

Vehicles, predominantly powered by fossil fuels, continue to produce greenhouse gases and other harmful pollutants during operation. Furthermore, under congested conditions, when speeds fall below 30 mph, greenhouse gas emissions per mile increase sharply due to frequent braking, acceleration and gear changes [9].

The “2018 China Motor Vehicle Environmental Management Annual Report” found that pollutant emissions from vehicles under traffic congestion are 5-10 times higher than under free-flow conditions, although this figure is on the higher end of estimates in the literature.

Carbon pricing policies, such as carbon taxes and carbon trading, have become crucial tools for managing greenhouse gas emissions [10]. However, tax policies lack control over the total reduction in carbon emissions and may face public resistance [11]. Unlike the relatively stable prices of carbon taxes, the price of carbon allowances under a trading system can fluctuate with the market, offering greater policy flexibility [12-13].

Hillman [14] introduced the concept of personal carbon allowances (PCA), also known as tradable energy quotas (TEQs), personal carbon rations and carbon credits. In 2004, the UK began experimenting with and studying PCA policies [15]. Scholars have since explored various ways to introduce and implement PCAs, conducting sociological studies. Much of the debate around PCAs has centred on how to allocate allowances and determine appropriate trading mechanisms. Proponents of PCAs have suggested an equal distribution scheme [14-19]. Hyams [16] proposed an equal distribution with a reserve for special cases. Regarding trading, Hillman [17] likened carbon allowances to currency, with prices fluctuating based on government-set totals. Hyams [16] outlined several PCA trading modes, including non-tradable carbon allowances, partially tradable allowances and fully tradable allowances.

Various analytical methods have been applied to study PCAs within transportation systems. Since 2010, research has focused on different aspects of transport. For example, McNamara and Caulfield (2013) evaluated a personal carbon trading scheme (PCTS) using consumer surplus analysis and data from Dublin and the Western Border Region (WBR) of Ireland [20]. Wadud [21] qualitatively assessed the tradable carbon permits policy in the context of carbon taxes and upstream tradable permits using a 2002 US micro dataset. Stated preference (SP) methods have also been employed, such as in a study comparing transport choices under personal carbon trading (PCT) and carbon tax (CT) policies, which involved interviews with 268 individuals [22]. Other studies have used questionnaires to investigate PCA policies. Wadud and Chintakayala [23] surveyed the impact of tradable carbon permits on both transport emissions and in-home energy use, conducting 1,000 interviews in August 2016. Li et al. [24] compared electric vehicle purchase choices under different policies, including personal carbon trading, road tolls, vehicle and vessel taxes, purchase taxes and government subsidies. This study, conducted in 2016 across six cities in Jiangsu province, involved 200 questionnaires per city. A subsequent study by Li et al. [25] compared electric vehicle purchase choices between PCT and CT policies, distributing 263 questionnaires for CT and 254 for PCT. Nonetheless, more detailed analyses combining specific data and models are needed to fully understand the impacts of such schemes.

For the research gap and focus, while PCAs for transport are actively discussed in research, practical implications, such as their impact on travel mode choice, are only beginning to be explored. Most existing studies on PCAs have focused on policy promotion and qualitative analysis, with limited modelling work. Some studies use a questionnaire to get data. For instance, Gong et al. [26] developed a data-driven framework to analyse mode choice behaviour, highlighting the role of socioeconomic factors and accessibility in shaping travel decisions. Additionally, they often overlook the role of road traffic conditions when considering carbon allowances. Only a few works consider the congestion situation. For instance, Aziz et al. [27] proposed dynamic traffic assignment models to evaluate congestion mitigation strategies under varying demand scenarios. Given that driving in congested conditions significantly elevates carbon emissions, avoiding peak-hour driving could be a crucial behavioural response. However, few studies have integrated the mode choice and congestion situation into carbon allowances analysis, creating a gap that this paper aims to address.

Besides, the experience of carbon trading in Europe underscores the importance of correctly determining the total amount of carbon allowances. The EU Emissions Trading System, launched on 1 January 2005, was the world’s first major initiative in carbon trading. While it has effectively reduced carbon emissions, the initial phases faced challenges due to the over-allocation of carbon allowances, which weakened the policy’s effectiveness. During the first phase (2005-2007), allowances were over-allocated by almost 8%, and by 7% in the second phase (2008-2012). This over-allocation led to sharp fluctuations in carbon prices, with prices nearly reaching zero at the end of the first phase, significantly diminishing the impact on emission reduction. Given this, determining the optimal total amount of carbon allowances is crucial for the successful implementation of such policies.

This work aims to fill a gap in the existing literature on carbon allowances. We focus on the use of fossil fuel vehicles in the transportation system, particularly under congestion conditions. We develop a micro-perspective model focusing on travellers’ mode choices and benefits to study the effects of carbon allowance policies and explore the optimal total amount of personal carbon allowances to be distributed, along with the corresponding price.

The paper is organised as follows: Section 1 reviews and justifies the choice of the Vickery bottleneck model as the foundation for the case study. Section 2 develops a bi-modal transportation system. The network includes a highway represented by a Vickrey bottleneck and a transit line with an affine crowding cost. We then propose a personal carbon allowance (PCAs) scheme aimed at reducing traffic-related carbon emissions and analyse transportation mode choices before and after policy implementation. We also assess the policy's impact on congestion and transit priority. Given the significant effect of the total carbon allowances on policy effectiveness, we seek to determine the optimal total amount. Additionally, we consider the fairness of the policy through a Pareto optimisation approach, where a Pareto improvement benefits at least one person without harming others. Section 3 presents two numerical examples, while Section 4 concludes and suggests future research directions. Our results demonstrate that distributing an optimal amount of carbon allowances can effectively encourage public transportation use and reduce car dependency. By applying Pareto improvement constraints, we identify the optimal total carbon allowances, providing valuable insights for government policy decisions.

2. TRAFFIC PATTERN AND MANAGEMENT IN A SINGLE OD PAIR

In this section, firstly, we review the bottleneck model [28], then we build the bi-modal equilibrium between a parallel transit line and a bottleneck-constrained road, without considering carbon allowance restraints. Then we derive the traffic pattern and travel cost in the bi-modal network, considering carbon allowance restrictions. A uniformly distributed personal carbon allowance policy is proposed, and the implementation of personal carbon allowances under free trading is investigated.

2.1 Bi-modal equilibrium without carbon allowance restrictions

Consider two travel modes of a road with a bottleneck, and a parallel transit line is available for commuting between a residential area and workplaces. A bi-modal transportation system is established, consisting of a bottleneck-constrained highway and a public transportation line (metro and buses with a bus lane) that accounts for in-cabin congestion costs. N_{ab} and N_{tb} denote the number of auto and transit commuters, respectively. The total number of commuters, $N = N_{ab} + N_{tb}$, is assumed to be given and fixed. The external demand elasticity is ignored as assumed in Tabuchi's model [29], however, by allowing inter-modal competition, internal elasticity is considered. Consider homogeneous commuters with a common preferred arrival time t^* at destination, and early or late arrival will be penalised. α denotes the value of a unit time for travelling, β denotes the schedule early penalty for a unit time, and γ denotes the schedule late penalty for a unit time. According to the empirical evidence [30], it is often assumed that $\gamma > \alpha > \beta > 0$. s denotes the capacity of the bottleneck.

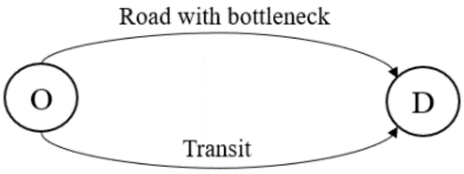


Figure 1 – A bi-modal transportation network

To be more relevant to reality, we consider the traffic free flow time t_{f} not being zero, the travel cost in user equilibrium is consists of fixed cost (car purchase and insurance), marginal cost (fuel and maintenance) and travel time cost (queue time cost and free flow time cost). We use P_C to denote the fixed cost. We use P_{AC} to denote the marginal cost for driving a car per trip. In the real transportation system, there may be many unexpected accidents. When considering the capacity of the road with a bottleneck, we may also consider the impact of random accidents. Assume the accident starts at time t_s, and we need a time period T_s to deal with this accident and make the capacity of the road fully available. Δs is the capacity change due to the accident. $\Delta s = s - s_a$.

Assume s_a is the capacity of the bottleneck after the accident, which is determined by the severity of the accident. Before the carbon allowances policy, we reviewed the model with variable road capacity [31], and then we calculated the travel cost for each car traveller $P_A\ (N_{ab})$ in user equilibrium, as shown in *Equation 1*.

$$P_A\ (N_{ab}) = P_{AC} + \frac{\beta\gamma(N_{ab} + \Delta s * T_s)}{(\beta + \gamma)s} + \alpha t_f \tag{1}$$

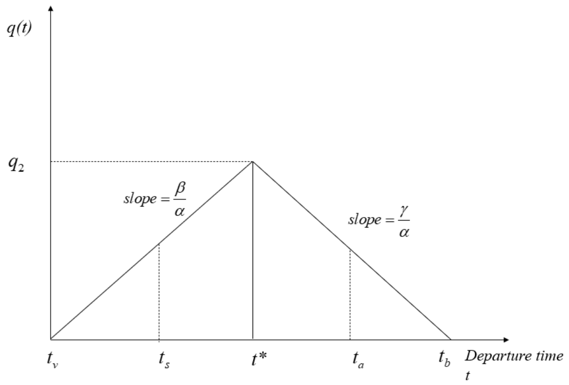


Figure 2 – Queuing time q(t) for the commuter who leaves the bottleneck at time t

Due to the fixed timetable of transit, the travel time by transit T_l is assumed to be fixed. The subway operates on dedicated rail tracks, and buses utilise exclusive bus lanes, ensuring that transit services remain unaffected by road congestion. The transit capacity is assumed to be sufficiently large, however, people may feel uncomfortable when there are too many people in the transit carriage, which is assumed as $P_{crowd}(N_t)$, a strictly increasing function of the number of transit commuters. The model assumes a homogeneous population of travellers, each owning a private car and selecting their commuting mode based on a utility-maximising behaviour that prioritises the lowest generalised travel cost (including time and monetary expenses) for every trip. Travel cost of riding transit P_T is given as a function composed of the cost of transit ticket P_{TP}, the cost of travel time, the discomfort cost of crowding for passengers P_{crowd}, and car purchase and insurance cost P_C.

Considering congestion discomfort, the transit crowding cost can be denoted by a linear function that increases with the number of transit passengers [32]. a, b are constants, we assume $P_{crowd}\ (N_{tb}) = aN_{tb} + b = a(N - N_{ab}) + b$. Then the transit cost is shown in *Equation 2*.

$$P_T\ (N_{tb}) = P_{TP} + \alpha T_l + P_{crowd}\ (N_{tb}) = P_{TP} + \alpha T_l + a(N - N_{ab}) + b \tag{2}$$

In user equilibrium, the travel cost of all the commuters is the same, thus, we can get *Equation 3*.

$$P_A\ (N_{ab}) = P_T(N_{tb}) \tag{3}$$

Substitute *Equations 1 and 2* into *Equation 3*, we can obtain *Equation 4* to calculate the total number of car commuters N_{ab}.

$$N_{ab} = \frac{(P_{TP} - P_{AC} + \alpha(T_l - t_f) + b + aN)(\beta + \gamma)s - \beta\gamma\Delta s * T_s}{(\beta + \gamma)sa + \beta\gamma} \tag{4}$$

The number of commuters should not be negative; thus, we can get *Equations 5–7*.

$$N_{ab} \geq 0 \tag{5}$$

$$N_{tb} \geq 0 \tag{6}$$

$$N_{ab} + N_{tb} = N \tag{7}$$

Let Q_b denote the total queue time of all the commuters travelling by car. From the above equations, we can obtain *Equation 8*.

$$Q_b = \int_{t_v}^{t_N} q(t)s(t)dt = \frac{\beta}{2\alpha}[s(t_s - t_v)^2 + s_a(t^* - t_s)(t^* + t_s - 2t_v)] + \frac{\gamma}{2\alpha}[s_a(t_a - t^*)(2t_b - t^* - t_a) + s(t_b - t_a)^2] \quad (8)$$

For the vehicles on the highway, the whole travel can be viewed as composed of two aggregate modes: cruising on the freeway and queuing at the bottleneck. Use λ_f and λ_q to denote the carbon allowance consumption quantity per unit time on cruising and queuing, respectively. For private cars, since the fuel consumption and carbon emission in queuing state is greater than that in free flow state [9], it would be reasonable to assume that $\lambda_q > \lambda_f$, let G_b denote the total amount of fuel consumed in this OD, as shown in *Equation 9*.

$$G_b = \lambda_f T_f + \lambda_q Q_b = \lambda_f t_f N_{ab} + \frac{\beta\lambda_q}{2\alpha}[s(t_s - t_v)^2 + s_a(t^* - t_s)(t^* + t_s - 2t_v)] + \frac{\gamma\lambda_q}{2\alpha}[s_a(t_a - t^*)(2t_b - t^* - t_a) + s(t_b - t_a)^2] \quad (9)$$

$T(t) = Q(t)/s$ is the queuing time for a driver arriving at the bottleneck at time t, $Q(t)$ is the queue length at time t. $N_{ab}(G_\mathrm{b})$ is the inverse function of $G_\mathrm{b}(N_{ab})$.

$$N_{ab} = f^{-1}(G_b) \quad (10)$$

The longest queue time for one commuter q_2 is shown in *Equation 11*

$$q_2 = \frac{\beta\gamma(N_{ab} + \Delta s * T_s)}{\alpha(\beta + \gamma)s}. \quad (11)$$

The queue start time t_v and end time t_b are calculated as follows:

$$t_v = t^* + \frac{\gamma(st_s + s_a T_s - st_a - N_{ab})}{(\beta + \gamma)s} \quad (12)$$

$$t_b = t^* + \frac{\beta(N_{ab} + st_a - st_s - s_a T_s)}{(\beta + \gamma)s} \quad (13)$$

2.2 Bi-modal equilibrium with personal carbon allowance restrictions

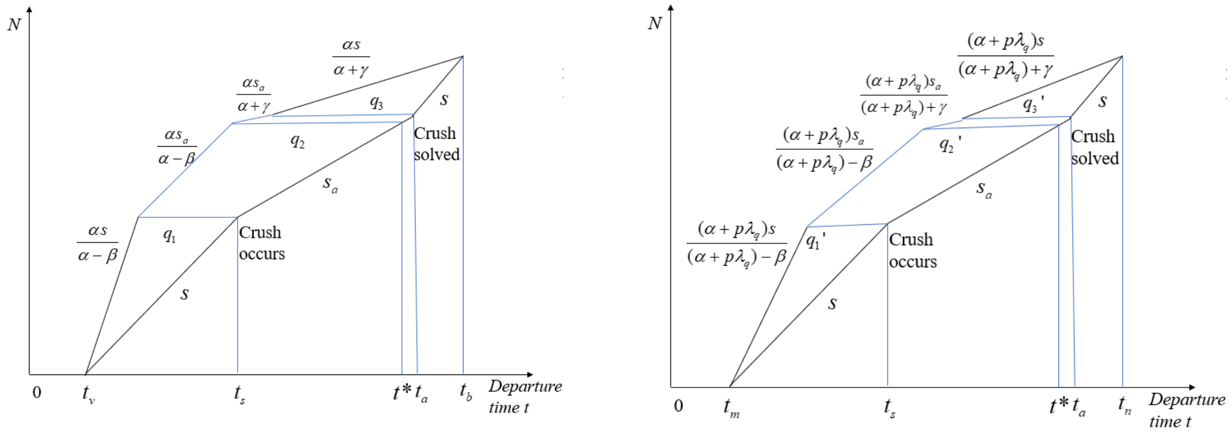
We now consider the bi-modal equilibrium when the amount of personal carbon allowances is limited. Let G_s denote the total amount of carbon allowances to be distributed for one trip in this traffic system with one OD pair. For fairness, we assume the carbon allowances are uniformly distributed between all travellers and are freely traded in the market. Let ω denote the carbon allowances for each commuter.

$$\omega = \frac{G_s}{N} \quad (14)$$

Let p denote the price of a unit carbon allowance, N_{as} denote the number of car travellers after implementing the carbon allowance policy, N_{ts}denote the number of transit travellers after implementing the carbon allowance policy. After the carbon allowances policy, in user equilibrium, travellers need to consider the prices of the carbon allowances, thus, the departure rate is changed compared to that before the carbon allowances policy.

For homogeneous commuters, the value of time α and the preferred arrival time $t^*$are identical. Although travellers experience different queue times depending on when they travel during the day, from a long-term perspective, since everyone has an equal chance of driving early or late, the average queue time for each driver in one trip can be viewed as the same. Thus, we use the average queue time to calculate the personal carbon allowance consumption quantity for each trip (c_{rd}). When the policy is implemented, travellers will not only consider their time value but also the value of carbon allowances consumed while queuing. This leads to a change in the departure rate compared to the rate before the carbon allowances policy, as well as a change in the average queue time. In user equilibrium, the travel cost for each commuter is identical, the first commuter and the last commuter do not need to queue, and the cost for free flow time is the same, t' is the departure time of the one who arrives at t^*. Then we have *Equation 16*, substituting *Equations 2 and 3*, we can get the departure rate as shown in *Figure 3*.

$$\beta(t^* - t_0) = \gamma(t_n - t^*) = \alpha(t^* - t') + p\lambda_q(t^* - t') \tag{15}$$



(a) *(b)*

Figure 3 – Traffic pattern on the bottleneck without free flow time: a) before implementing the carbon allowances policy; b) after implementing the carbon allowances policy

After the policy, in user equilibrium, we obtain the longest queuing time q_2' and the average queuing time c_{rd} (not considering free flow time) for travellers are:

$$q_2' = \frac{\beta\gamma(N_e + \Delta s * T_s)}{(\alpha + pe_e e} \tag{16}$$

$$c_{rd} = \frac{Q_s}{N_{as}} \tag{17}$$

The travel costs for car and transit travellers after the implementation of the carbon allowances policy are expressed in *Equations 18 and 19*. P_C is the fixed cost of owning a car. P_{AC} is the marginal cost for driving a car per trip. λ_f and λ_q are the carbon allowance consumption quantity per unit time on cruising and queuing, respectively.

$$P_A(N_{as}) = P_C + P_{AC} + p(\lambda_f t_f + \lambda_q Q_s/N_{as} - G_s/N) + \frac{\beta\gamma(N_{as} + \Delta s * T_s)}{(\beta+\gamma)s} + \alpha t_f \tag{18}$$

$$P_T(N_{ts}) = P_C + P_{TP} + \alpha T_l + a(N - N_{as}) + b - pG_s/N \tag{19}$$

In user equilibrium, the travel cost for car commuters and transit commuters is the same, thus, we get *Equation 20*. Substituting *Equations 18 and 19* into *Equation 20*, we can get *Equation 21*, which can be simplified to *Equation 22*.

$$P_A(N_{as}) = P_T(N_{ts}) \tag{20}$$

$$P_C + P_{AC} + \frac{\beta\gamma}{(\beta+\gamma)s}N_{as} + \alpha t_f + p\lambda_f t_f + \frac{p\lambda_q Q_s}{N_{as}} + \frac{\beta\gamma\Delta s * T_s}{(\beta+\gamma)s} = P_C + P_{TP} + \alpha T_l + a(N - N_{as}) + b \tag{21}$$

$$(a + \frac{\beta\gamma}{(\beta+\gamma)s})N_{as} + p\lambda_q Q_s/N_{as} = P_{TP} - P_{AC} + \alpha(T_l - t_f) - p\lambda_f t_f + aN + b - \frac{\beta\gamma\Delta s * T_s}{(\beta+\gamma)s} \tag{22}$$

The queue start time t_m and end time t_n, total queue time for all the travellers Q_s and total carbon emissions G_s are calculated as follows:

$$t_m = t^* + \frac{\gamma(st_s + s_aT_s - st_a - N_{as})}{(\beta + \gamma)s} \tag{23}$$

$$t_n = t^* + \frac{\beta(N_{as} + st_a - st_s - s_aT_s)}{(\beta + \gamma)s} \tag{24}$$

$$\begin{aligned} Q_s = \int_{t_v}^{t_N} q(t)s(t)dt = \frac{\beta}{2(\alpha + p\lambda_q)}\left[s(t_s - t_m)^2 + s_a(t^* - t_s)(t^* + t_s - 2t_m)\right] \\ + \frac{\gamma}{2(\alpha + p\lambda_q)}\left[s_a(t_a - t^*)(2t_n - t^* - t_a) + s(t_n - t_a)^2\right] \end{aligned} \tag{25}$$

$$\begin{aligned} G_s = \lambda_f T_f + \lambda_q Q_s = \lambda_f t_f N_{as} + \frac{\beta\lambda_q}{2(\alpha + p\lambda_q)}\left[s(t_s - t_m)^2 + s_a(t^* - t_s)(t^* + t_s - 2t_m)\right] \\ + \frac{\gamma\lambda_q}{2(\alpha + p\lambda_q)}\left[s_a(t_a - t^*)(2t_n - t^* - t_a) + s(t_n - t_a)^2\right] \end{aligned} \tag{26}$$

For a transportation system with a given commuters number (N) and other parameters $(\alpha, \beta, \gamma, P_{TP}, P_{AC}, \lambda_q, \lambda_f, a, b, s, t_f, T_l, N, s_a, T_s, \Delta t_s)$, from *Equations 4 and 9*, the number of car travellers N_{ab} and the total carbon emissions G_b in the system before the implementation of the carbon allowances policy can be obtained. From *Equations 22 and 26*, when the total amount of carbon allowances G_s is set for distribution among travellers, the number of car travellers N_{as} after the implementation of the carbon allowances policy and the unit price of carbon allowances p can be calculated. If $G_s < G_b$, then $N_{as} < N_{ab}$, $p > 0$. That illustrates that when travelers are homogeneous and carbon allowances are distributed equally and traded freely, if the total amount of carbon allowances is set to be less than the total carbon emissions before the policy, the policy can reduce the number of car commuters and motivate travelers to use public transportation, thereby alleviating traffic congestion and reducing the system carbon emissions and environmental pollution.

2.3 Proper total amount of carbon allowances

The cost of congestion discomfort in public transit carriages increases with the number of transit commuters. However, these commuters can offset their travel costs by selling carbon allowances. Conversely, the queuing time for car travellers decreases as the number of transit commuters increases, leading to a reduction in travel time costs. Yet, car travellers must consume carbon allowances, which adds to their overall travel costs. To ensure that the benefits of both car and transit passengers are not diminished, it is essential to balance these factors by determining an appropriate total amount of carbon allowances that achieves Pareto optimisation.

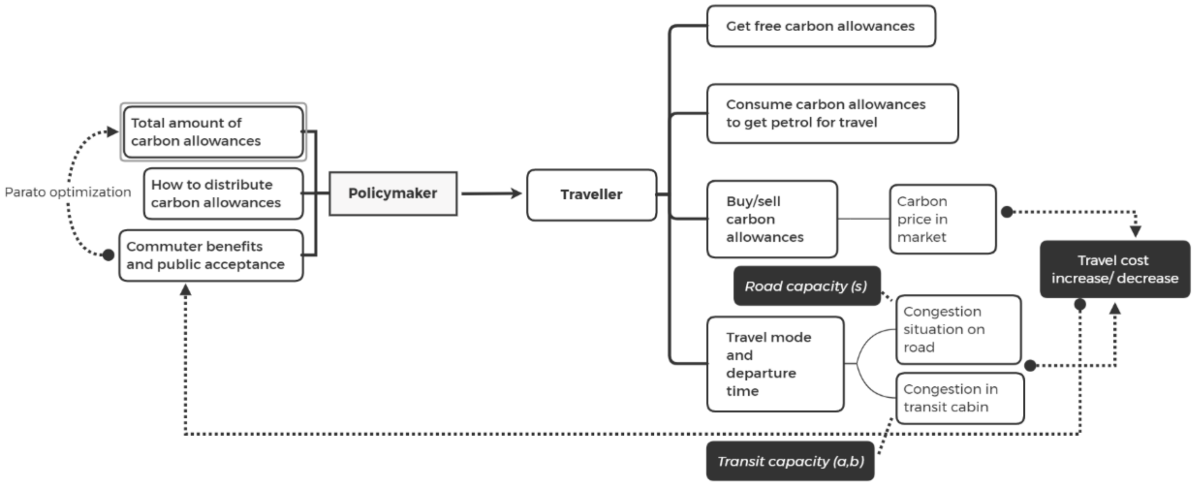


Figure 4 – Carbon allowances policy dynamics between the policymaker and travellers

To explore the optimal quantity of carbon allowances, it is crucial to consider the implications of issuing too few or too many allowances. If too few allowances are issued and the price of carbon allowances becomes too high, passengers may be forced to switch to transit, resulting in overcrowding and a poor travel experience. Conversely, if too many allowances are issued and their price is too low, the policy may fail to effectively reduce carbon emissions. Therefore, this study seeks to identify the appropriate total amount of carbon allowances that ensures a Pareto improvement. To guarantee that no party experiences a loss of benefits, we establish the following constraints.

Based on *Equations 1, 2, 18 and 19*, the difference in travel costs for travellers before and after the implementation of the carbon allowances policy can be derived, as shown in *Equation 27*.

$$P_A(N_{ab}) - P_A(N_{as}) = P_T(N_{tb}) - P_T(N_{ts}) = a(N_{as} - N_{\mathrm{ab}}) + pG_s/N \tag{27}$$

To meet the Pareto improving scheme, the travel costs for travellers after the implementation of the carbon allowances policy should not be higher than those before its implementation, therefore, *Equation 28* can be derived.

$$P_A(N_{ab}) - P_A(N_{as}) = P_T(N_{tb}) - P_T(N_{ts}) = a(N_{as} - N_{\mathrm{ab}}) + pG_s/N \geq 0 \tag{28}$$

3. NUMERICAL EXAMPLES

In this part, we consider the variable road capacity caused by traffic accidents and design the numerical examples. Two examples are presented in this section, one on a small scale and one on a larger scale.

3.1 Simple example

In the first example, the parameters are assumed and analytical solutions are provided. We use MATLAB to solve this equation.

Consider a simple example, the value of α, β, γ, P_{TP}, P_{AC}, λ_q, λ_f, a, b, s, t_f, T_l, N, s_a, T_s , Δt_s are assumed as *Table 1* shows. The value of parameters references from previous literatures (Zhang et al., 2010; Zhang et al., 2011; Wang et al., 2020) [31-33]. $\Delta s = s - s_a = 500$, the accident occurs at time $t_s = t^* - \Delta t_s$, the value of the time when the accident is solved $t_a = t_s + T_s = t^* + 0.2$.

Table 1 – The value of parameters in the scheme

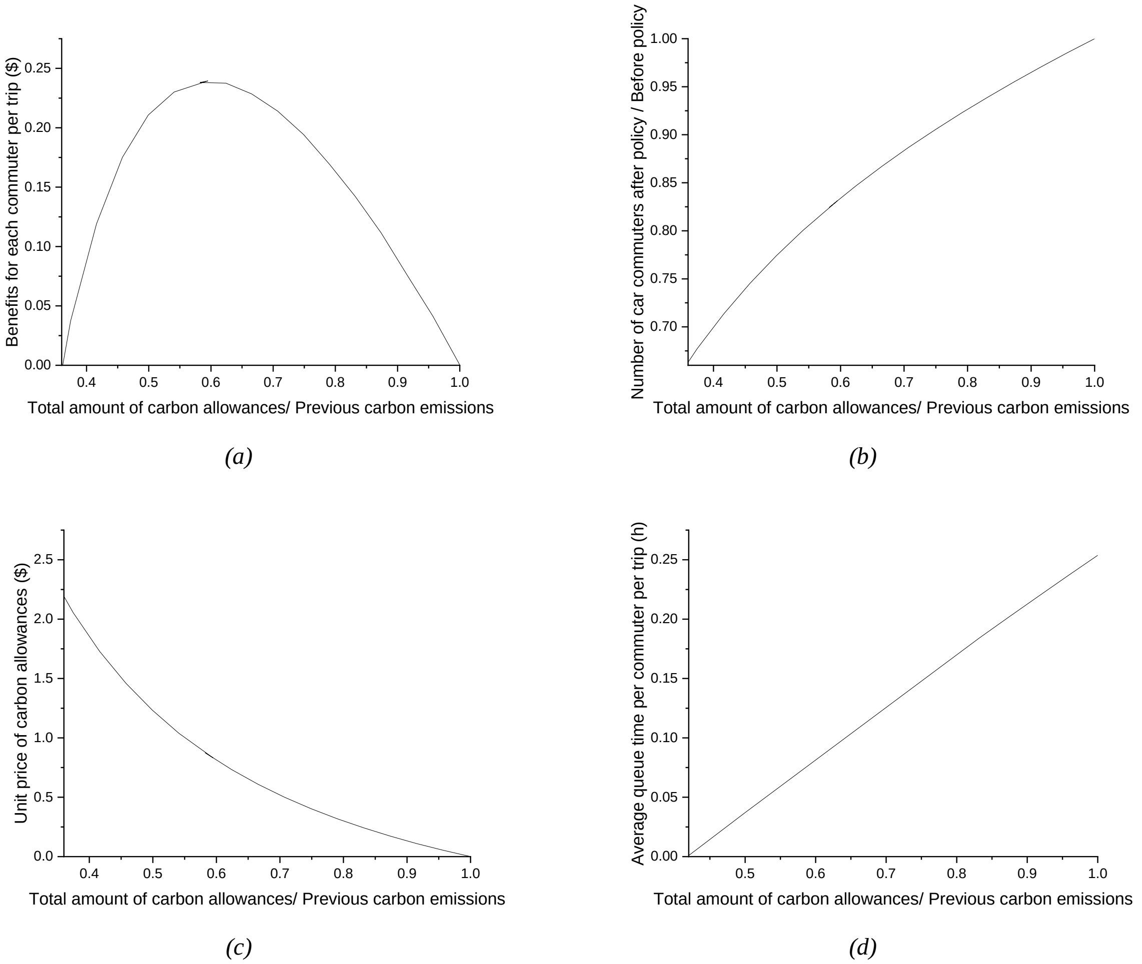
Variables	**Meanings**	**Assumption values**
N	Number of total travellers	10000
P_{TP}	Transit ticket fee	1 ($/trip)
P_{AC}	Car consuming fee	5 ($/trip)
s	Capacity of the bottleneck	2,000 (persons)
γ	Unit value of schedule delay time	21 ($/h)
α	Unit value of travel time	12 ($/h)
β	The schedule penalty for a unit time of early arrival	4 ($/h)
λ_f	The carbon allowance consumption quantity per unit time on cruising	5 (/h)
λ_q	The carbon allowance consumption quantity per unit time on queuing	7 (/h)
t_f	Travel time of free flow on the highway	0.5 (h)
T_l	Travel time by transit	1.5 (h)
a	Variable coefficient for transit crowding (slope)	0.001$/(persons*h)
b	Variable coefficient for transit crowding (intercept)	1$/(persons*h)
s_a	The value of the road capacity after accidents	1,500 (persons)
T_s	The value of the time to solve the accidents	0.4 (h)
Δt_s	The time period from the accident occurs time (t_s) to preferred arrival time (t^*)	0.2 (h)

According to *Equations 4 and 9*, we can obtain the number of car commuters N_{ab} is 5,621, and the carbon emissions G_b is 24,038 before the carbon allowances policy.

Through MATLAB calculations, it can be determined that the carbon allowances policy can effectively reduce carbon emissions in the system and not harm the interests of travellers only when the total amount of carbon allowances (G_s) is in the range [8,650, 24,038]. When G_s is within the above interval, the number of car commuters (N_{ab}) is in the range [3,731, 5,621]. We can also determine that after implementing the carbon allowances policy, under the condition of not harming the interests of travellers (Pareto improving), issuing the minimum amount of carbon allowances, in free market trading, the highest carbon allowance price (p) is 2.19. The benefits of commuters, the number of car commuters and the unit price of carbon allowance vary with the total amount of carbon allowance, as *Table 2* and *Figure 5* show.

Table 2 – Calculation results for the above example

Variables	**Values**										
Total amount of carbon allowances (G_s)	24,038	22,000	20,000	18,000	16,000	14,000	14,300	12,000	10,000	8,650	8,000
Number of car commuters (N_{as})	5,621	5,455	5,279	5,088	4,875	4,634	4,672	4,352	4,010	3,731	3,577
Unit price of carbon allowances (p)	0	0.11	0.24	0.40	0.61	0.88	0.83	1.23	1.73	2.19	2.45
Commuter benefits	0	0.076	0.142	0.194	0.229	0.238	0.239	0.211	0.119	0	-0.084



(a) *(b)*

(c) *(d)*

Figure 5 – Impact of varying total amount of carbon allowances (G_s) on key variables: a) benefits for commuters; b) number of car commuters; c) unit price of carbon allowance; d) average queue time of each driver

The model demonstrates that while a reduction in the total amount of carbon allowances leads to greater suppression of carbon emissions, it simultaneously raises the market price of carbon allowances, thereby increasing commuting costs within the simulated framework. These findings suggest that policymakers may need to balance emission reduction goals with commuter welfare considerations.

In the model, setting the carbon allowance below a threshold (8,650 units) leads to a simulated behavioural shift where more commuters opt for public transit. This not only diminishes the comfort of public transit but also degrades the overall travel experience for passengers. As this decline in comfort can be perceived as an increase in travel costs, the total travel costs for public transit commuters rise. Simultaneously, the high price of carbon allowances, driven by their limited availability, significantly increases travel costs for car commuters. Under such circumstances, the system fails to achieve Pareto efficiency, as it cannot reach an optimal state without disadvantaging one group or another. As *Table 2* shows, under the model's utility parameters, PCA levels below 8,650 units result in negative benefits, and the system fails to achieve Pareto efficiency.

In determining the total carbon allowance, policymakers must weigh two key objectives: prioritising the welfare of commuters or minimising total carbon emissions from the transportation sector. As illustrated in *Table 2*, within the simulated framework, the model identifies two policy trade-offs: if the aim is to minimise carbon emissions while satisfying Pareto efficiency conditions, a total carbon allowance of 8,650 units could be selected, which reduces total emissions by 64% compared to levels before the implementation of the carbon allowance policy. On the other hand, if the objective is to maximise commuter benefits while still achieving emission reductions, an allowance of 14,300 units could be chosen, which results in a 41% reduction in emissions relative to pre-policy levels.

This balancing act underscores the complexity of policy formulation. If the total carbon allowance is set too low, emissions may indeed decrease, but the strain on the public transit system could generate new challenges, ultimately compromising the policy's overall effectiveness. Therefore, model-based balancing between emission reduction and welfare preservation may enhance policy feasibility, though real-world calibration would require contextual adjustments.

Moreover, the flexibility of the policy warrants attention. In different transportation environments and economic contexts, the optimal total carbon allowance may vary. Policymakers should dynamically adjust the distribution of carbon allowances based on actual conditions and long-term objectives to ensure the policy continues to reduce emissions without compromising public welfare. Through such dynamic adjustments, the carbon allowance policy can better respond to real-world needs, facilitating its broader implementation and providing stronger support for global efforts to combat climate change.

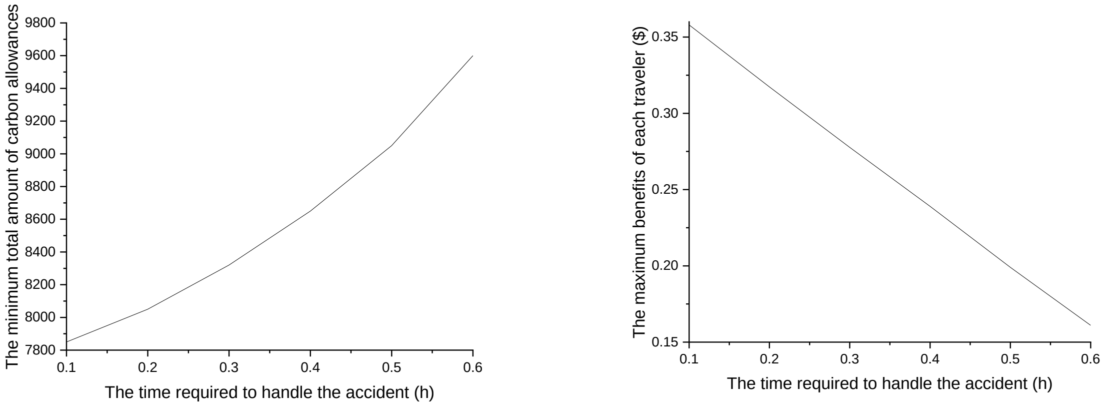
Next, we consider the impact of varying accident emergency response capabilities on different roads. Assuming the values of other variables remain as shown in *Table 1*, the values of s_a, T_s , Δt_s change with different road emergency response capabilities. Within the tested parameter ranges ($s_a \in [100,\ 1900], \Delta t_s \in [0.1, 0.6]$), MATLAB simulations suggest that variations in the road capacity after accident (s_a) and the time period from accident occurs time to preferred arrival time (Δt_s) have limited influence on optimal carbon allowance amount under the current model setup. Assuming the time required to handle the accident (T_s) varies within the range [0.1, 0.6], the time for the accident to be completely resolved is $t_a = t_s + T_s$. After calculations by MATLAB, the following results are obtained. *Table 3* shows the minimum total amount of carbon allowances in the Pareto optimisation condition with different T_s values. In *Table 4*, as T_s varies, we compare the maximum benefits of travellers.

Table 3 – Calculation results of Pareto optimisation case

Variables	**Values**					
Time required to handle the accident (T_s)	0.1	0.2	0.3	0.4	0.5	0.6
Total amount of carbon allowances (G_s)	7,850	8,050	8,320	8,650	9,050	9,600
Number of car commuters (N_{as})	3,570	3,618	3,673	3,731	3,793	3,873
Unit price of carbon allowances (p)	2.614	2.488	2.342	2.185	2.018	1.822
The benefits of travelling	0	0	0	0	0	0

Table 4 – Calculation results of peak traveller welfare case

Variables	**Values**					
Time required to handle the accident (T_s)	0.1	0.2	0.3	0.4	0.5	0.6
Total amount of carbon allowances (G_s)	14,300	14,300	14,300	14,300	14,300	14,300
Number of car commuters (N_{as})	4,716	4,707	4,692	4,672	4,646	4,613
Unit price of carbon allowances (p)	0.883	0.861	0.844	0.831	0.822	0.817
The benefits of travelling	0.358	0.317	0.278	0.239	0.199	0.161



(a) *(b)*

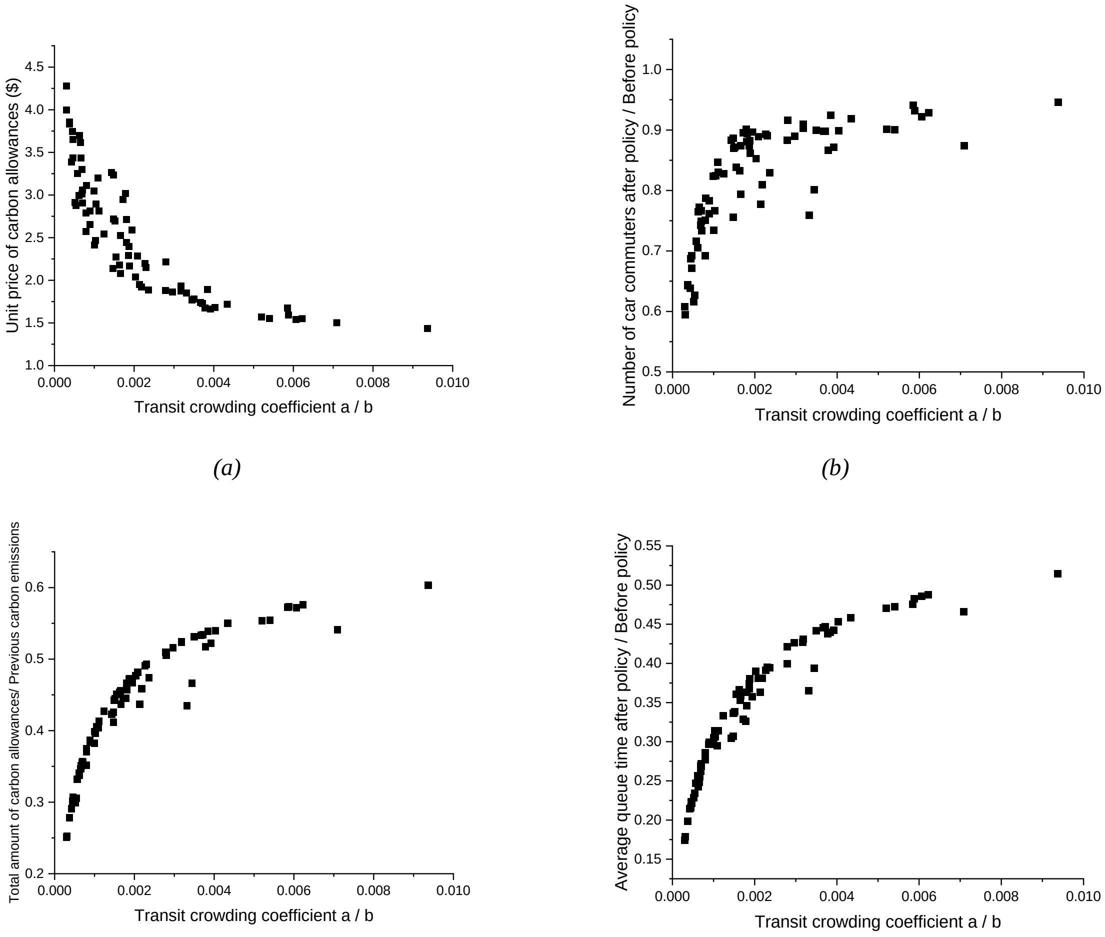
Figure 6 – Impact of time required to handle accidents (T_s) on Pareto optimisation case and peak traveller welfare case: a) the minimum amount of carbon allowances that satisfies Pareto optimisation; b) the maximum benefits of each traveller

From *Figure 6*, it can be seen that when accidents are handled promptly and the time required is short, the minimum amount of total carbon allowances to be distributed is lower without affecting traveller benefits, meaning that the carbon emissions of the transportation system can be reduced further. However, when accidents are not handled promptly and the time required is long, the maximum traveller benefits decrease under the carbon allowances policy.

3.2 Example with 80 random parameters

The above analytical results are made under special settings of parameters, and the effect of road capacity variation is analysed. To illustrate the policy's efficiency under more general settings, and also to observe the effects of other different parameters, we ignore the accidents and assume $s_a = s$, the values of variable parameter a (slope, which illustrates passenger's sensitivity for uncomfortable cost in a crowd cabin), fixed parameter b (intercept, which illustrates a fixed uncomfortable cost for crowd in a cabin) are randomly generated to constitute total 80 sets of parameters within intervals [0.001, 0.01], [0.1, 5] respectively, keeping other parameter values same as the values in *Table 1*. The unit price of carbon allowances and the lowest total amount of carbon allowances to be allocated in the conditions of Pareto optimisation are calculated, and the following observations are made.

Similarly, keeping other parameters the same, the values of parameters s, N random vary within intervals [2,000, 3,000], [5,000, 20,000] respectively, t_f, T_l within [0.1, 1] and [1, 3] respectively. The scope of parameters references from previous works of literature (Zhang et al., 2010; Zhang et al., 2011) [31-32]. We assume public transportation commuters need more travel time (compared with the free flow time for car commuters) and arrive at t^*, which also represents the real transportation situation in most cases. The results are made into figures to make it easier to understand.



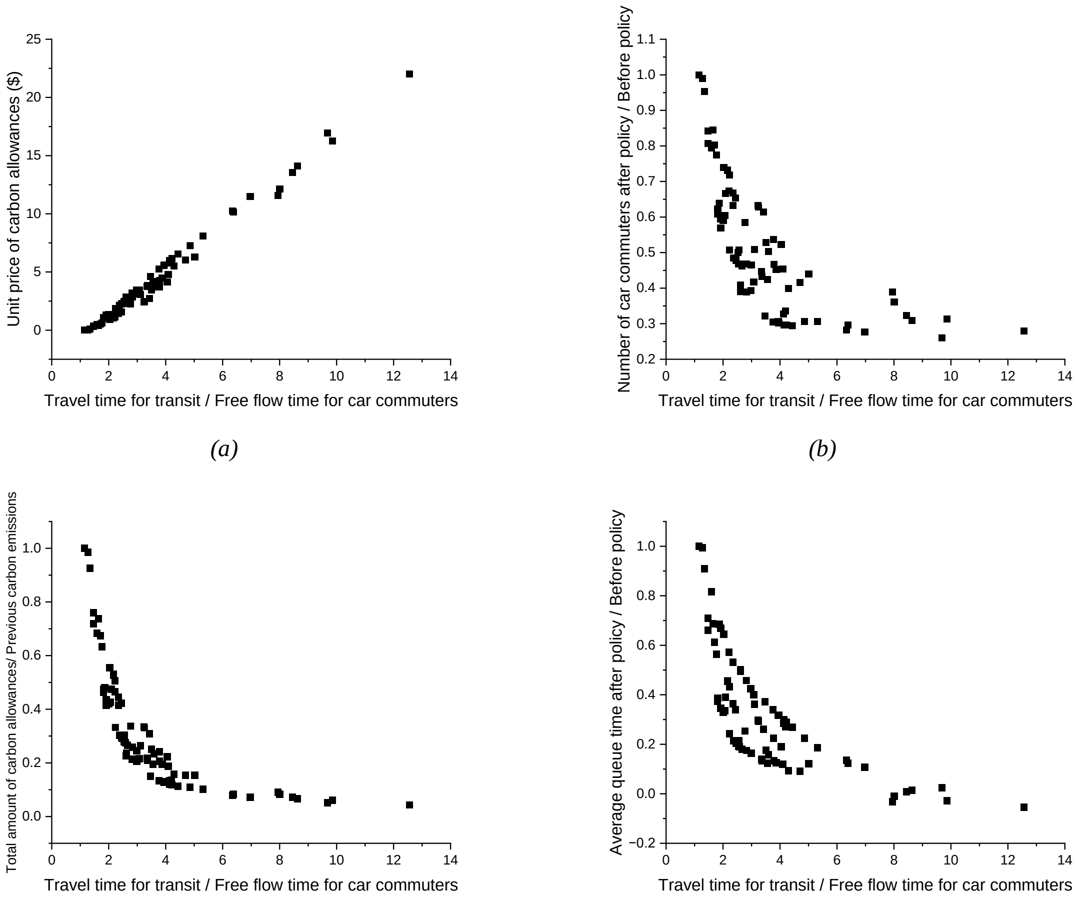
(a) *(b)*

(c) *(d)*

Figure 7 – Impact of passenger sensitivity for crowd cost on key variables: a) carbon allowances price; b) comparison of car commuter numbers; c) reduction rate of carbon emissions; d) queue time comparison

In the cases analysed under the study's framework, the implementation of the carbon allowances policy suggests a potential reduction in total carbon emissions within the transportation system while maintaining commuter welfare within the modelled parameters. This indicates that, under the assumptions of a well-designed policy framework, for homogeneous commuter preferences, environmental benefits may align with individual benefits. However, real-world applicability would require further validation of these assumptions. *Figure 7* further reveals an important phenomenon: when travellers are more sensitive to crowding in public transit, the reduction in carbon emissions under the Pareto optimal conditions is relatively smaller, and the price of carbon allowances per unit is lower. This phenomenon may be attributed to the model's representation of crowding-sensitive travellers, who are assumed to avoid using transit as their primary commuting mode even before the carbon allowances policy is introduced. Therefore, after the policy is implemented, although there may be an increase in transit ridership, the overall change is not significant since the number of passengers already using public transit was initially low.

This finding highlights the potential critical role of crowding sensitivity in travellers' mode choice decisions within the modelled scenarios. When crowding becomes a significant consideration for commuters, the impact of a carbon allowance policy aimed at encouraging public transit usage through economic incentives may be limited. To enhance the effectiveness of such a policy, policymakers should also focus on improving the quality of public transportation services, particularly in reducing crowding on transit carriages. If crowding can be alleviated, the model suggests that a higher share of travellers could potentially shift toward public transit. This shift, under the study's assumptions, may contribute to carbon reduction goals and improve transportation efficiency.

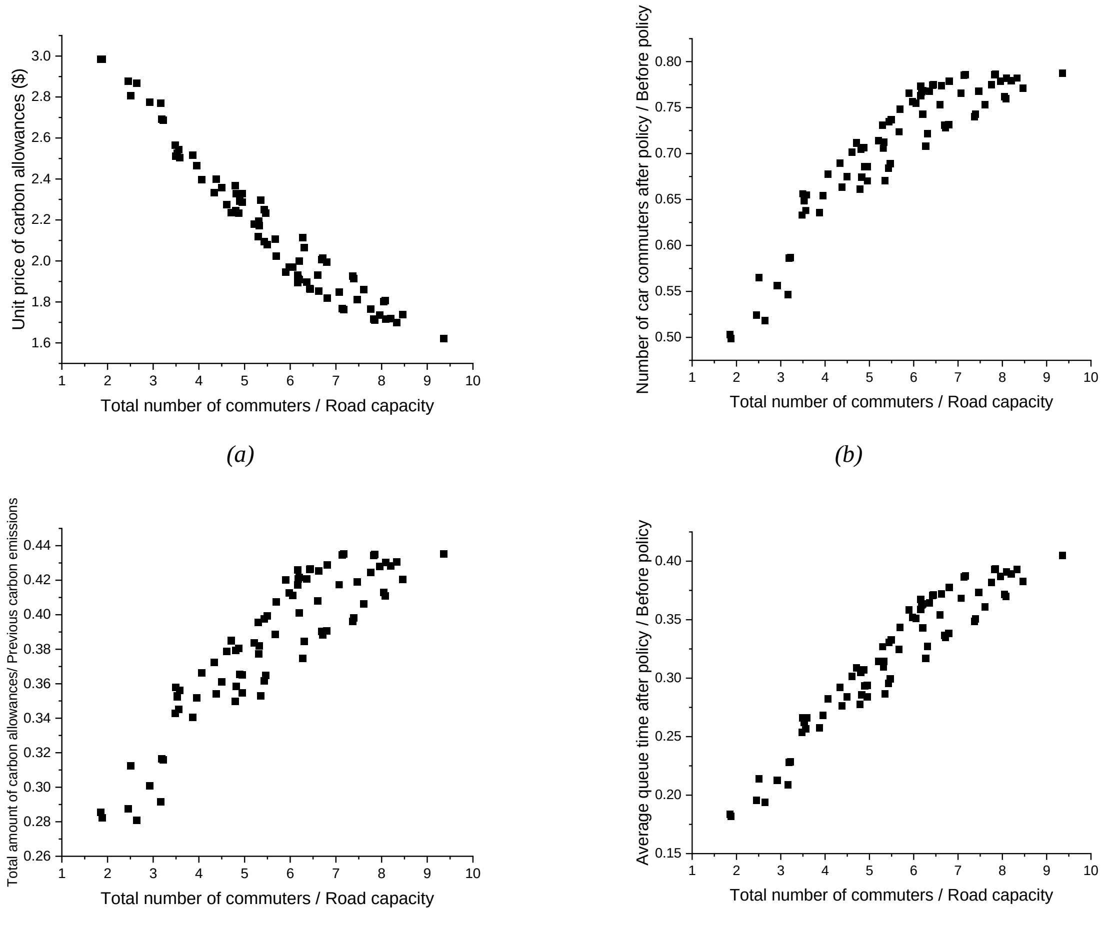


(a) *(b)*

(c) *(d)*

Figure 8 – Impact of transit travel time to free flow time ratio (T_l/t_f) on key variables: a) carbon allowances price; b) comparison of car commuter numbers; c) reduction rate of carbon emissions; d) queue time comparison

From *Figure 8*, the model reveals that when the travel time of transit is relatively longer, the unit price of carbon allowances is higher, and the reduction of carbon emissions is relatively more effective. Within the model's framework, when the difference between free flow time t_f and transit time T_l is relatively small, simulated travellers are more likely to choose the transit even before the carbon allowances policy is implemented. As a result, the policy induces only limited behavioural changes in such scenarios. These findings underscore the importance of transit travel time in influencing the choice of transportation mode within the modelled system. A shorter transit travel time not only makes the transit a more attractive commuting option but also encourages more drivers to shift to public transit, even before the policy is applied, which helps alleviate road congestion. In contrast, when the transit travel time is longer, driving remains competitive before the policy. Therefore, after the policy, the reduction in carbon emissions becomes more pronounced. This suggests that, under the model's assumptions, in such scenarios, the policy may have a relatively stronger effect on incentivising mode shifts. For urban planners and policymakers, findings imply that investments in transit service quality, particularly reducing transit time, could complement carbon allowances policy by enhancing public transit attractiveness. However, real-world effectiveness would depend on contextual factors such as commuter preferences or existing infrastructure. This not only leads to reduced road congestion but also contributes to the long-term sustainability of the transportation system. About the discontinuity observed in *Figure 8* at $T_l/t_f = 3$ emerges from the model's parameterisation: when T_l/t_f is constant, varying absolute values of t_f and T_l lead to nonlinear interactions in the simulation, affecting carbon allowances price, number of drivers and total carbon emissions.



(a) *(b)*

(c) *(d)*

Figure 9 – Impact of commuter number to road capacity ratio (N/s) on key variables: a) carbon allowances price; b) comparison of car commuter numbers; c) reduction rate of carbon emissions; d) queue time comparison

Figure 9 illustrates that within the modelled scenarios, lower road capacity correlates with smaller carbon emission reductions and lower unit carbon allowance prices. This pattern may arise from pre-policy behavioural dynamics: when the road capacity is not enough, a higher share of commuters in the simulation already opt for public transit due to limited driving alternatives. Consequently, the effectiveness of the policy is smaller. This outcome also highlights that when the road system's capacity is constrained, the attractiveness of public transit reaches a saturation point even before the policy is applied. In such cases, the additional incentives provided by the carbon allowances policy lose their effectiveness, as there is limited room for further shifts from driving to public transit. Therefore, the policy's impact on emission reduction is not as significant as expected.

Moreover, these findings suggest that to achieve more substantial carbon reductions under conditions of low road capacity, the carbon allowances policy may need to be combined with other transportation improvements. For instance, enhancing public transit service quality or expanding road capacity could provide more diverse travel options, making it easier for commuters to shift from private vehicles to public transit. These complementary measures may amplify the effectiveness of the carbon allowances policy, leading to greater reductions in emissions and overall improvements in the transportation system's efficiency.

4. CONCLUSIONS

This study has explored the potential impacts of a personal carbon trading system on commute behaviour, mode shifts and traffic congestion within the transportation system. As such, it makes a novel contribution to the field, as many previous studies have not engaged with the dynamics of specific behaviour change impacts.

For the implementation of this policy, one of the key design concerns for the government is the setting of the total amount and distribution methods of carbon allowances. In this study, we analysed the impact of the total amount of carbon allowances on the effectiveness of the policy on both choices and, for public acceptance, used the Pareto optimisation approach to find the optimum total amount of carbon allowances. We identified the total allowance quantity that both maximises carbon emission reduction and is publicly acceptable. Additionally, we found the allowance amount that achieves a moderate reduction in emissions while ensuring the highest level of public approval.

Numerical analysis demonstrates that the carbon allowances policy does indeed reduce total carbon emissions as expected. However, several factors influence its effectiveness. First, if the road network is already congested due to inadequate infrastructure, the policy may not significantly shift travel behaviour toward public transport. In heavily congested areas, many commuters may have already opted for public transit prior to policy implementation. Second, the issue of overcrowding in public transport must be addressed. Using a simple model of crowding aversion in our study, we found that if public transit capacity cannot be expanded and overcrowding is not mitigated, it becomes challenging to significantly reduce the number of car commuters without harming passengers' overall experience. Furthermore, when road capacity is frequently affected by traffic accidents, the carbon emission reductions achieved by the carbon allowances policy are lower compared to situations where road capacity remains stable. However, when accidents are promptly addressed, the policy's effectiveness in reducing emissions is less impacted, thereby maintaining its intended benefits.

There are further developments which are necessary to continue to develop this approach for more impactful policy insights. For this study, we only used homogeneous travellers, which is simplified compared with the real transportation system. Further study can investigate the PCAs policy in the transportation system with heterogeneous travellers. Further to this, one could expand from a simple mode choice to a network scale representation. We also held the number of travellers fixed, whereas the acceleration of homeworking, which occurred during the Covid-19 pandemic, suggests that this would be an important third option.

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