



Exploring Cooperative Lane Change Decisions in Vehicle-to-Infrastructure – A Potential Conflict Analysis Approach

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ABSTRACT

Vehicle-to-infrastructure (V2I) technology enables information interaction between vehicles and among vehicles and infrastructure, significantly enhancing the efficiency of lane-changing processes and stabilising traffic flow. Current research primarily focuses on single lane-changing events in fixed micro-level scenarios or studies involving small-scale vehicle fleets, neglecting the randomness of lane-changing vehicle arrivals and potential conflicts during lane-changing. This paper proposes a lane-changing decision model based on potential conflict analysis, specifically tailored to mandatory lane-changing requirements in high-density traffic conditions. The model comprises sub-models for lane-changing decision triggering, influence range calculation and lane-changing priority determination, capable of dynamically adjusting the lane-changing sequence, mitigating lane-changing conflicts, and improving driving safety and traffic efficiency. Simulation experiments indicate that, when compared to lane-changing patterns in real-world traffic scenarios, this model reduces travel time by 23.30%, delays by 21.95% and the number of stops by 23.84%, thereby providing a novel approach for lane-changing decision-making and control in V2I environments.

KEYWORDS

lane-changing model; vehicle-to-infrastructure technology; lane-changing conflict; lane-changing impact range; lane-changing priority.

1. INTRODUCTION

With the continuous development of intelligent transportation systems, vehicle-to-infrastructure technology has demonstrated immense potential, standing out in alleviating traffic congestion and enhancing the efficiency of urban road networks. It has been widely applied in various traffic scenarios. In increasingly crowded urban road networks, the mandatory lane-changing behaviour of vehicles has become a core link that affects the overall urban traffic order due to the interaction of multiple factors and the dynamic changes in traffic flow environments.

Lane-changing models are an important component of micro-traffic flow models. However, in the context of vehicle-road cooperation, vehicles can obtain real-time road condition information and changes in traffic conditions, leading to changes in lane-changing behaviour. Traditional lane-changing models such as the Gipps model [1], MITSIM model [2], minimum safety spacing (MSS) model [3], and hidden Markov model (HMM) [4] are no longer suitable for analysing traffic characteristics in the environment of vehicle-road cooperation. This is because these models are primarily based on basic information, such as the relative position and speed

between vehicles for modelling, and they do not fully consider the impact of real-time road condition information and changes in traffic conditions brought about by vehicle-to-infrastructure technology on lane-changing behaviour. Therefore, they are no longer suitable for analysing traffic characteristics in the context of vehicle-to-infrastructure. Therefore, it is particularly important to study vehicle lane-changing models under vehicle-road cooperation. These models usually involve technologies such as vehicle perception, path planning, decision-making and control, among which the formulation of lane-changing decisions is crucial [5].

Currently, the research on vehicle lane-changing decision-making models mainly includes two types: traditional rule-making models and predictive control models based on sensor data. Among them, the vehicle-road coordinated lane-changing model based on rules usually includes three stages: perception, judgement and execution [6]. The perception stage acquires the information of the forward traffic flow through the vehicle sensors and processes it according to preset rules [7]. The judgement stage evaluates various possibilities using preset decision-making rules based on perception to determine the best lane-changing strategy. The execution stage is the actual operation of the selected lane-changing strategy [8]. David et al. [9] used a quantitative response equilibrium framework to simulate lane change operations and proposed a lane-changing decision rule based on game theory. Tian et al. [10] used the continuous cellular automata (CCA) model to model and analyse the entrance and exit ramps of urban highways, and designed the main road merging rules, ramp inflow and outflow rules, lane change process analysis and cooperative following (LC-CCF) model and improved the Nagatani lane-change rules to control the overall operation of road vehicles. Zhao et al. [11] conducted an analysis of the strengths and weaknesses of existing car-following models, introducing the safety distance parameter for the most hazardous braking scenario.

Decision-making based on sensor data mainly focuses on the perception and analysis of surrounding environmental information. By obtaining information about surrounding vehicles through sensors and utilising machine learning algorithms to analyse these data, more accurate lane-changing decisions can be made [12]. Xie et al. [13] employed deep learning techniques to process the surrounding environmental information of vehicles, enabling accurate recognition of lane-changing intentions. Ma et al. [14] designed a low-cost and low-complexity sensor system for platoon vehicles, which integrates various sensor data to determine subsequent lane-changing operations. Ali et al. [15] examined the impact of connected vehicle environment (CVE) on safety during mandatory lane-changing operations and concluded that CVE-enabled driving, through the utilisation of driving assistance systems that leverage surrounding traffic information, can assist drivers in enhancing the safety and efficiency of lane-changing.

Based on the research papers on vehicle lane-changing models and behaviours, numerous scholars have made significant progress in the field of lane-changing research. However, there are still some limitations. In terms of research objects, existing studies on vehicle-road collaborative lane-changing models have primarily focused on lane-changing processes in fixed scenarios. By designing typical scenarios, such as lane-changing behaviours between dual lanes or between urban and expressways [16], and assuming the presence of one or several lane-changing vehicles, specific motion states and lane-changing trajectories for the lane-changing vehicles and surrounding vehicles are planned. Due to the idealised nature of these lane-changing scenarios, the proposed models are only applicable to specific lane-changing scenarios and cannot be compatible with other scenarios and processes. In terms of model application, current lane-changing research includes both single-vehicle and multi-vehicle lane-changing, but most studies on multi-vehicle collaborative lane-changing models focus on a single lane-changing process in a fixed scenario, without considering the randomness of the arrival of lane-changing vehicles and their temporal-spatial evolution characteristics [17]. In real-world traffic environments, lane-changing scenarios and processes are constantly changing. There is still relatively little research on defining the beginning and end of the lane-changing process based on temporal-spatial evolution, as well as the interaction between vehicles during lane-changing, especially the dynamic interactive influence between lane-changing vehicles and other surrounding lane-changing vehicles.

Given this, this paper focuses on the study area of signalised intersection entrances to conduct research on lane-changing decisions. It delves into the temporal and spatial characteristics of vehicle lane changes, comprehensively considers the randomness of arrival and temporal-spatial evolution characteristics of lane-changing vehicles and proposes a vehicle-road collaborative lane-changing decision model based on potential conflict analysis [18]. The research focuses on the mandatory lane-changing demand under high-density conditions, aiming to improve traffic efficiency while ensuring lane-changing safety. Finally, simulations are conducted to verify the control effectiveness of the proposed model [19].

The contributions of this paper are as follows:

- 1) Propose a vehicle-to-infrastructure lane-changing decision-making approach based on potential conflict analysis. This integrates a trigger model considering vehicle arrival randomness and spatiotemporal characteristics, calculates influence ranges, adjusts lane-changing sequences to resolve conflicts, and determines the least disruptive lane-changing plan.
- 2) Propose concepts of vehicle lane-changing influence range and priority. Analyse vehicle interactions in a vehicle-road collaborative environment, defining influence ranges based on traffic density and the target lane's preceding vehicle's position and speed. Provide calculation methods for local vehicle interactions. Determine potential conflicts based on influence ranges, quantify conflict factors, and propose collaborative lane-changing rules using lane-changing priority and real-time vehicle states.

The organisation of the rest of this paper is as follows. In the second section, the proposed model will be explained and analysed. The third section provides a description of the simulation experiment settings and the experimental analysis results of the proposed model. In the final section, the conclusions of the work done in this paper are given.

2. THE PROPOSED MODEL

In this study, aiming at the mandatory lane-changing requirements under high-density conditions, this paper proposes a vehicle-to-road cooperative lane-changing decision-making approach based on potential conflict analysis. Supported by the detection and calculation of the vehicle-to-road cooperative system, the approach can obtain the arrival information of lane-changing vehicles in real time. Then, according to the characteristics of spatiotemporal evolution, a lane-changing decision trigger model is designed to calculate and control the lane-changing process. This section presents more details on how to apply the model in the vehicle-to-road cooperative environment. The model notations are listed in *Table 1*.

Table 1 – Model notations

Notion	Full description
i	Lane number
j	The vehicle j , and vehicle number decreases sequentially with the direction of the traffic flow
$X_{i,j}(t)$	The longitudinal driving distance of the j vehicle in the i lane at time t
$Y_{i,j}(t)$	The lateral driving distance of the j vehicle in the i lane at time t
$V_{i,j}(t)$	The speed of the j vehicle in the i lane at time t
$V_{xi,j}(t)$	The longitudinal driving speed of the j vehicle in the i lane at time t
$V_{yi,j}(t)$	The lateral driving speed of the j vehicle in the i lane at time t
$\Delta d_{(i,j)-(i+1,j+1)}(t)$	The longitudinal distance between the j vehicle in the i lane and the $j+1$ vehicle in the $i+1$ lane at time t
L_0	The length of the solid line portion of the lane dividing line
L	The length of the entrance lane
$\Delta L_{i,j}(t)$	The distance between the vehicle and the intersection stop line at time t
L_d	The desired distance between the vehicle and the vehicle in front of the target
L_{sc}	The impact distance of the vehicle lane-changing
D_{safe}	The safety clearance that needs to be met by the following vehicle
T	The safe time interval
$\Delta V_{(i+1,j)-(i,j)}$	The speed difference between the lane-changing vehicle and the preceding vehicle in the target lane
a_{max}	The maximum acceleration of the lane-changing vehicle
b	The comfortable deceleration of the lane-changing vehicle
W	The lateral required safe distance

Notion	Full description
L_{exp}	The lane-changing influence distance of the vehicle under the desired speed
$\Delta V_{exp-(i,j)}$	The difference between the lane-changing vehicle's speed and the desired lane-changing speed
P_{ln}	The lane number impact factor
Δn	The number of lanes that the vehicle needs to cross
n	The total number of lanes in the direction of the current traffic flow
P_{kl}	The impact factors of traffic flow density and the distance to the stop line
K	Traffic density
K_j	Jam density
α	The impact coefficient of traffic flow density
β	The impact coefficient of the distance to the stop line
γ	Lane-changing process control parameters
$L(\theta)$	The likelihood function of the sample
V	Average speed within the interval
V_m	The speed at the maximum traffic volume, namely the critical speed
P_r	Lane-changing priority
λ, μ	Lane-changing priority weighting factor
D_f	The minimum oblique safety distance that needs to be maintained with the preceding vehicle in the target lane
θ	The angle between the centreline of the lane-changing vehicle and the centreline of the target lane
D_{fg}	The spacing required for non-cooperative lane changing
V_{max}	The speed limit of the road
T_c	Vehicle lane-changing time
$\varepsilon(t)$	The angle between the centreline of the lane changing vehicle and the longitudinal direction of the road at time t
L_A	Vehicle length
D_B	The required distance between $C_{i+1,j+1}$ and $C_{i,j}$
t_c	The time taken by $C_{i,j}$ to travel from the starting position of lane-changing to the critical collision point M
τ	Acceleration integration variable
d_{min}	The safe following distance after emergency braking
a_m	The maximum deceleration rate of a following vehicle
t_d	The response latency of intelligent and connected automobiles

2.1 Lane-changing decision-making ideas

In this study, all vehicles are considered as intelligent connected vehicles, with smooth information interaction between vehicles and between vehicles and roadside facilities, without considering communication delays. When planning the lane-changing trajectory of the vehicle, the longitudinal and lateral motions of the vehicle are decoupled, meaning that the longitudinal and lateral motions of the vehicle are relatively independent. Additionally, once the vehicle receives the decision-making results, it can fully follow the control instructions. To facilitate the establishment of a lane-changing safety distance model for vehicles, the impact of vehicle shape on lane changes is ignored. It is assumed that the vehicles involved in the model are all standard compact cars, and other vehicle types are not considered. A schematic diagram of this scenario is shown in *Figure 1*.

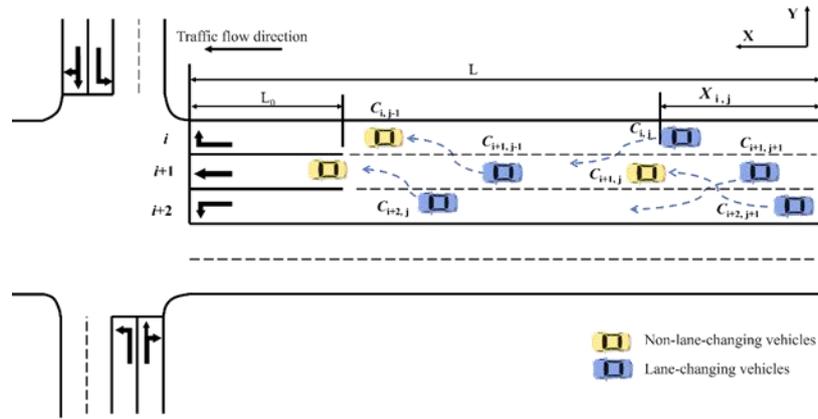


Figure 1 – Relationship among multi-lane vehicles

In actual road conditions, multiple vehicles in multiple lanes are all dynamically driving, and the interaction between vehicles is significant. Taking the position of vehicle $C_{i,j}$ at time t as the reference point, with the traffic flow direction as the X-axis and the direction perpendicular to the traffic flow as the Y-axis, the positional relationship between vehicles is described as shown in Table 1.

The relationship among the vehicles in Figure 1 can be summarised as follows: (1) Vehicle $C_{i+1,j-1}$ is changing lanes from lane $i+1$ to lane i . At the same time, there is another lane-changing vehicle, $C_{i+2,j}$, in lane $i+2$, but the lane-changing trajectories of these two vehicles do not intersect. (2) Vehicle $C_{i+2,j+1}$ is changing lanes from lane $i+2$ to lane $i+1$. During this process, there are two other lane-changing vehicles in lanes i and $i+2$. Among them, the lane-changing trajectory of vehicle $C_{i,j}$ does not intersect with that of vehicle $C_{i+2,j+1}$, while the lane-changing trajectory of vehicle $C_{i+1,j+1}$ intersects with that of vehicle $C_{i+2,j+1}$.

The process of cooperative lane-changing among multiple vehicles involves various scenarios and complex lane-changing situations, and its distribution patterns cannot be directly described through mathematical models. This paper analyses and categorises many potential collaborative scenarios based on actual traffic conditions. By analysing the characteristics of vehicle lane-changing, the initial and target positions of lane-changing, two potential conflict lane-changing scenarios that have a significant impact on traffic flow and can reflect information interaction are finally summarised. See Figure 2 for details.

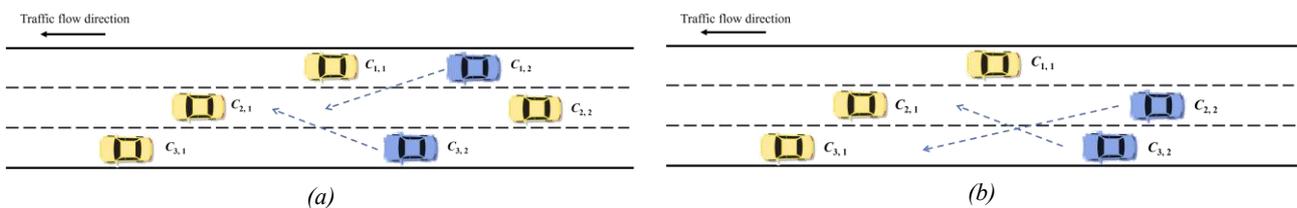


Figure 2 – Typical scenarios of potential conflict at the lane change process: a) the first lane-changing scenario; b) the second lane-changing scenario

In the first lane-changing scenario, $C_{1,2}$ and $C_{3,2}$, the initial positions of the two vehicles are separated by one lane, and their target lanes after generating lane-changing intention are the same lane. In the second lane-changing scenario, $C_{2,2}$ and $C_{3,2}$, the initial positions of the two vehicles are on adjacent lanes, and the target lane-changing positions are the initial lanes of each other. If the two vehicles change lanes in a short period of time, their lane-changing trajectories will inevitably intersect, posing a risk of conflict. In real-world situations, if one vehicle changes lanes first, the other vehicle may give up lane-changing due to insufficient safety distance required, which cannot meet the lane-changing demand.

When the traffic flow density is high, lane-changing behaviours of vehicles will, to a certain extent, affect the normal operation of the traffic flow, which is then in an unstable state. Vehicles in the original lane and the target lane may be disturbed by lane-changing behaviours and must reduce their speeds. This slowdown effect will propagate upstream in the traffic flow during the traffic operation. This paper mainly focuses on the mandatory lane-changing requirements under high-density conditions at signal-controlled intersections and studies a vehicle-road cooperative lane-changing decision-making model based on potential conflict analysis. The detailed flowchart of lane-changing decision-making is shown in Figure 3.

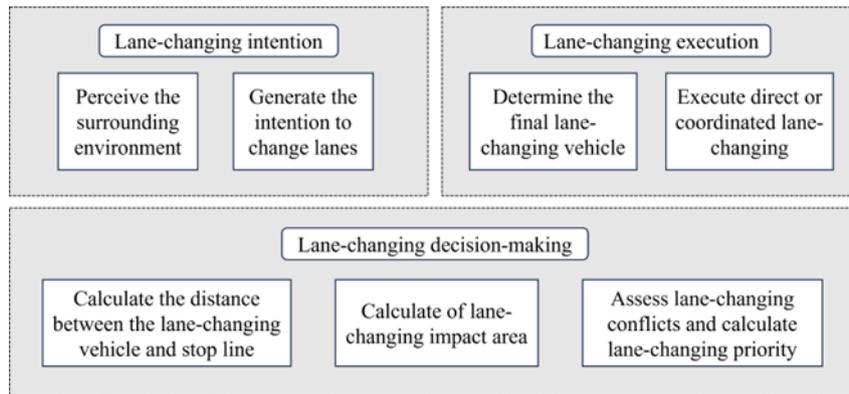


Figure 3 – Detailed flow chart for lane-changing decision

2.2 Calculation of lane-changing impact area

The interaction area between vehicles is dynamically changing, and this dynamic interaction area is referred to as the lane-changing influence range. The lane-changing influence range of a vehicle is mainly determined by the relationship between driving speed, vehicle spacing and safety distance. Since the flow differences at signal-controlled intersections vary significantly at different times, the IDM model, which can be applied well from free flow to complete congestion flow, is adopted to describe the car-following behaviour of lane-changing vehicles. The IDM model considers the speed difference between the lane-changing vehicle and the preceding vehicle, as well as the distance between the two vehicles [20]. Vehicle $C_{i,j}$, as the observed vehicle, determines its lane-changing influence distance, as shown in Figure 4.

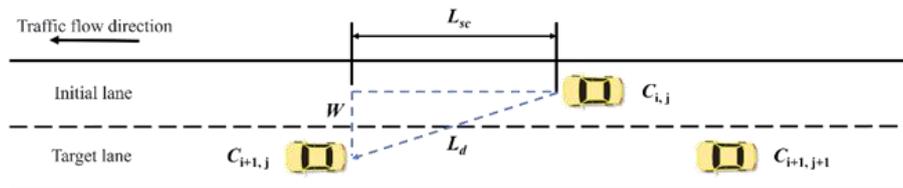


Figure 4 – The influence range of the lane-changing vehicle

The desired distance between the vehicle and the target front car during lane changing is obtained from the following Equation 1.

$$L_d = D_{safe} + V_{i,j}T + \frac{V_{i,j} \Delta V_{(i+1,j)-(i,j)}}{2\sqrt{a_{max} b}} \tag{1}$$

The impact distance of the vehicle lane-changing is shown in Equation 2.

$$L_{sc} = \sqrt{L_d^2 - W^2} = \sqrt{\left(D_{safe} + V_{i,j}T + \frac{V_{i,j} \Delta V_{(i+1,j)-(i,j)}}{2\sqrt{a_{max} b}} \right)^2 - W^2} \tag{2}$$

The expected distance between the vehicle and the preceding car in the target lane during lane changing primarily depends on the relative running state between the two vehicles, which is mainly determined by the running state of the preceding car. Therefore, it is necessary to analyse the corresponding expected distance based on different running states of the preceding car. According to Equation 2, the expected distance equation includes a balancing term $D_{safe} + V_{i,j}T$, indicating that there should be at least a minimum gap between the two vehicles. It is worth noting that if the speed difference between the preceding car and the lane-changing car exceeds the speed difference between the desired speed and the lane-changing car, and if calculated according to the above formula, the planned area of the lane-changing impact distance would be much larger than the required distance, thus increasing unnecessary strategic operating costs. Therefore, when the speed of the preceding car in the target lane is greater than the desired lane-changing speed, it can be shown as Equation 3.

$$L_{\text{exp}} = D_{\text{safe}} + V_{i,j} T + \frac{V_{i,j} \Delta V_{\text{exp}-(i,j)}}{2\sqrt{a_{\text{max}} b}} \quad (3)$$

The lane-changing impact area is a rectangular region defined by the lane-changing impact distance as its length and the total number of lanes as its width. Under the condition of vehicle-road coordination, the spatial motion states of vehicles can be updated in real time. The decision-making model calculates the lane-changing impact area of each lane-changing vehicle based on the received motion states and inter-vehicle distances of surrounding vehicles, sorted in descending order of distance to the stop line. If vehicle $C_{i+1, j}$ is within the lane-changing impact area of vehicle $C_{i, j}$, then vehicle $C_{i, j}$ is also within the lane-changing impact area of vehicle $C_{i+1, j}$, indicating that the impact between vehicles is mutual. If the running state of any vehicle changes, it will trigger changes in the running states of other vehicles within its lane-changing impact area. If the distance between two lane-changing vehicles is outside the lane-changing impact area, they are not considered within the scope of conflicting lane changes.

2.3 Lane-changing priority calculation

To ensure the safety and efficiency of the vehicle lane-changing process, it is necessary to analyse the relationship between vehicles within the range of influence and screen out conflicting vehicles during lane changing. Conflicting vehicles mainly include three types:

- 1) Lane-changing vehicles with intersecting lane-changing trajectories;
- 2) Lane-changing vehicles with conflict counts greater than or equal to 2;
- 3) Lane-changing vehicles with conflict counts equal to 1 and lower lane-changing priority.

The lane-changing conflict scenarios within the range of influence are simplified into three lanes with two lane-changing vehicles, as shown in *Figure 5*. Assuming that the lane-changing vehicle is currently located in lane 1 and intends to change lanes to the left, it may conflict with vehicles changing from lane 2 to lane 1 and from lane 3 to lane 2. Assuming that the lane-changing vehicle is currently located in lane 2, if it intends to change lanes to the right, it may conflict with vehicles changing from lane 1 to lane 2. If it intends to change lanes to the left, it may conflict with vehicles changing from lane 3 to lane 2. Assuming that the lane-changing vehicle is currently located in lane 3 and intends to change lanes to the right, it may conflict with vehicles changing from lane 2 to lane 3 and from lane 1 to lane 2.

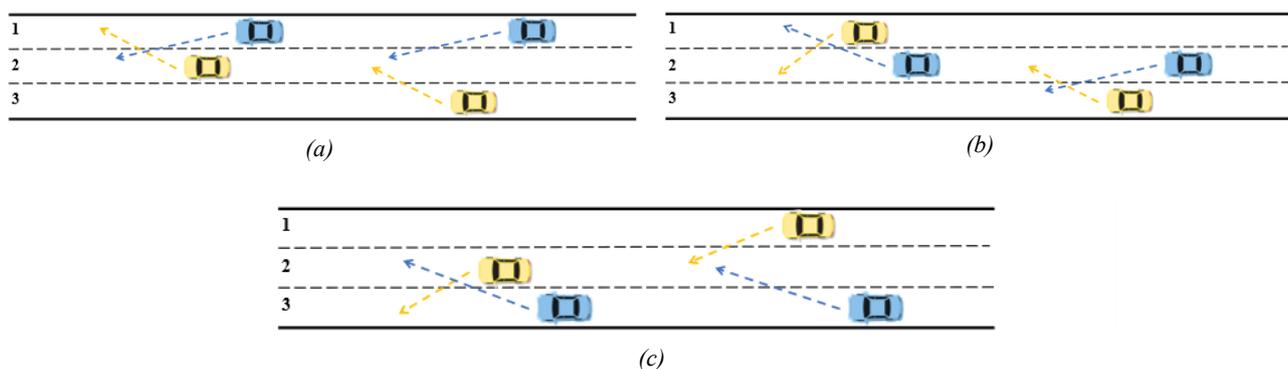


Figure 5 – Lane-changing conflict scenarios: a) the lane-changing vehicle is currently in lane 1; b) the lane-changing vehicle is currently in lane 2; c) the lane-changing vehicle is currently in lane 3

Based on the range of influence during lane-changing, it is necessary to determine whether there are conflicting cross-lane-changing vehicles. If there are no other lane-changing vehicles within the range of influence, the lane-changing manoeuvre can be executed directly after determining that it meets safety conditions. If there are other lane-changing vehicles but no conflicts occur between them, each vehicle can execute lane changing after satisfying safety conditions. If there are lane-changing conflict vehicles with intersecting trajectories, it is necessary to count the respective numbers of lane-changing conflicts:

- 1) When the conflict counts are both greater than or equal to 2, lane changing will have a significant impact on the entry lane, and the vehicles should temporarily refrain from executing lane changing;
- 2) When the conflict counts between vehicles are both equal to 1, it is necessary to calculate the lane-changing priority.

When conflict counts between lane-changing vehicles are both 1, planning the lane-changing sequence is crucial to avoid interference. Therefore, this paper introduces the concept of lane-changing priority, which considers factors such as the number of lanes to be crossed, traffic density and distance from the stop line. Vehicles intending to change lanes send requests, and those crossing multiple lanes should do so gradually to minimise traffic disturbance. Longer lane-changing processes may significantly affect traffic flow, so such vehicles are given higher priority, as indicated in Equation 4.

$$P_{ln} = \frac{\Delta n + 1}{n} \tag{4}$$

In actual road conditions, vehicle density varies continuously. In low-density scenarios, the distance between vehicles in the target lane is large, minimising the impact of lane changes on traffic flow, regardless of the vehicle’s distance from the stop line. As traffic density increases, if the original lane’s conditions fail to meet the driver’s speed demands and adjacent lane spacing permits, vehicles may accelerate after changing lanes, increasing the likelihood of lane changes. However, this gradually increases traffic interference. As vehicles approach the stop line, their desire to change lanes intensifies, accelerating with shortened distance. When near the guiding lane’s solid line, lane changes adversely affect at least two lanes’ capacity. With increasing traffic flow, vehicle interaction intensifies, speeds drop in both original and adjacent lanes, and distances shrink. Lane changes neither improve the driving environment nor meet safety standards, significantly interfering with traffic flow regardless of the vehicle’s proximity to the stop line.

Therefore, when the density is high, to meet the mandatory lane-changing demand of vehicles making turns, it is advisable to guide them to change lanes as early as possible to avoid the situation where the closer the vehicle is to the guiding lane line, the greater the lane-changing disturbance. Additionally, when the distance between vehicles in the target lane is small, the negative impact of lane changing increases, necessitating a higher priority to avoid the situation where the vehicle cannot change lanes due to insufficient spacing, further reducing the impact on other vehicles. It follows that the influence of the lane change process is limited by the density of the traffic flow and the distance from the stop line, as shown in Equation 5.

$$P_{kl} = e^{\frac{K}{K_j}} \left[\alpha \frac{K}{K_j} + \beta \left(1 - \frac{\Delta L}{L} \right) \left(K - \frac{K^2}{K_j} \right) + \gamma \right] \tag{5}$$

After determining the influence coefficients, it is necessary to calibrate their weights to reflect the degree of influence of different traffic parameters on lane-changing priority. Existing studies mostly employ qualitative analysis methods to assign specific numerical values to the weights of influence coefficients [16]. However, qualitative analysis can only roughly represent the relationship between influence coefficients and has its scope of application and limitations. Therefore, the maximum likelihood estimation method is adopted to determine the specific weights of different influence coefficients for a more comprehensive and systematic weight judgement. This approach allows for a more accurate assessment of the relationship between different influencing factors and lane-changing priority. The main steps are as follows:

- 1) Define a likelihood function that aligns with the distribution of the data;

$$L(\theta_1, \theta_2, \theta_3, \dots, \theta_k) = \prod_{i=1}^n p(x_i; \theta_1, \theta_2, \theta_3, \dots, \theta_k) \tag{6}$$

- 2) Transform the likelihood function $L(\theta)$ into a log-likelihood function and simplify it;

$$\begin{aligned} \ln L(\theta_1, \theta_2, \theta_3, \dots, \theta_k) &= \ln \left[\prod_{i=1}^n p(x_i; \theta_1, \theta_2, \theta_3, \dots, \theta_k) \right] \\ &= \sum_{i=1}^n \ln [p(x_i; \theta_1, \theta_2, \theta_3, \dots, \theta_k)] \end{aligned} \tag{7}$$

- 3) When the conflict counts between vehicles are both equal to 1, it is necessary to calculate the lane-changing priority.

$$\frac{\partial \ln L(\theta)}{\partial \theta_j} = 0 (i=1, 2, 3, \dots, k) \tag{8}$$

This paper mainly focuses on lane-changing guidance under high-density conditions. Traffic flow density varies with the change in traffic volume, which in turn leads to changes in vehicle speed. By analysing the distribution of speed changes before changing lanes during peak hours in the surveyed area, $V \sim N(9.16, 17.17)$ was obtained. When the traffic density is high, the speed-density relationship is calculated using the logarithmic model proposed by Greenberg:

$$V = V_m \ln\left(\frac{K_j}{K}\right) \tag{9}$$

According to relevant theorems of normal distribution theory, if X is a random variable with a normal distribution, then $\exp(X)$ follows a lognormal distribution [21]. The mean and variance of the lognormal distribution are shown in Equations 10 and 11.

$$E(X) = e^{\mu + \frac{\sigma^2}{2}} \tag{10}$$

$$D(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1) \tag{11}$$

So, the traffic flow density $K \sim N(e^{17.745}, e^{52.66} - e^{35.49})$. Based on relevant theorems of normal distribution theory, the linear combination of multiple independent normal distributions still satisfies the normal distribution. That is, when X_1 and X_2 are independent of each other and $X_1 \sim N(E_1, \sigma_1^2)$, $X_2 \sim N(E_2, \sigma_2^2)$, $\alpha X_1 + \beta X_2$ will follow the normal distribution $N(\alpha E_1 + \beta E_2, \alpha^2 \sigma_1^2 + \beta^2 \sigma_2^2)$. Therefore, the lane-changing priority P_{kl} follows the following distribution:

$$P_{kl} \sim N\left[e^{17.745}\alpha + 0.207\beta, (e^{52.66} - e^{35.49})\alpha^2 + 0.003\beta^2\right] \tag{12}$$

The probability density function of P_{kl} is shown in Equation 13.

$$f(\mu, \sigma) = \frac{1}{\sqrt{2\pi[(e^{52.66} - e^{35.49})\alpha^2 + 0.003\beta^2]}} \exp\left(-\frac{(x - e^{17.745}\alpha + 0.207\beta)^2}{2(e^{52.66} - e^{35.49})\alpha^2 + 0.003\beta^2}\right) \tag{13}$$

The above indicators can provide analytical judgements for lane-changing decisions. The differences in influencing factors determine the sequence of vehicles' lane changes, assist in determining the optimal lane-changing order, and enable quantitative assessments of lane-changing disturbances, operational efficiency and safety. Through the above analysis of lane-changing priority, it can be concluded that:

$$\begin{cases} P_r = \lambda P_m + \mu P_{kl} \\ \lambda + \mu = 1 \\ \lambda, \mu \in [0, 1] \end{cases} \tag{14}$$

The state of the lane-changing vehicle determines the values of λ and μ . Facing different traffic environments, dynamic adjustments are made based on lane-changing needs while ensuring the safety of lane changes. For lane-changing vehicles that need to cross fewer lanes in low-density traffic flow, λ is larger and μ is smaller. Conversely, when the traffic flow density is high and the lane-changing vehicle is closer to the stop line, λ is smaller and μ is larger. Lane-changing priority not only reflects the ability of vehicle-to-vehicle communication but also helps avoid conflicts and traffic accidents. When multiple vehicles need to change lanes simultaneously, to minimise the overall disturbance to the entry lane, it is necessary to assess lane-changing priority, coordinate the lane-changing sequence of each vehicle, avoid conflicts and congestion, and maximise the utilisation of road capacity.

2.4 Lane-changing execution

Since the Gipps model can consider the situation involving the forced lane change of vehicles when there is an obstacle on the road ahead, the safety distance model of the vehicle changing lanes in the Gipps model is mainly discussed in this paper [1]. Given that in the actual lane-changing process, the leading vehicles in the original lane and the target lane are generally in a passive state, mainly exerting constraints on the lane-

changing vehicle. Therefore, the spacing between the lane-changing vehicle and the leading vehicle in its own lane must always satisfy $\Delta d_{(i, j)-(i+1, j+1)} \geq D_{safe}$. The minimum diagonal safety distance that needs to be maintained with the leading vehicle in the target lane can be converted into Equation 16.

$$D_f = \sqrt{D_{safe}^2 + W^2} \tag{15}$$

$$\tan \theta = \frac{W}{D_{safe}} \tag{16}$$

The diagonal distance between the lane-changing vehicle and the leading vehicle in the target lane only needs to be greater than or equal to D_f . On the other hand, the following vehicle in the original lane exhibits a car-following behaviour, and its next-second change can be calculated and updated based on the displacement calculation formula in the Gipps model [1]. When determining whether a vehicle can directly execute a lane change, it is not necessary to consider the impact of the lane-changing process on the normal driving of surrounding vehicles. Not all lane-changing scenarios require lane-changing coordination. To reduce the operational cost of the strategy, it is necessary to design a lane-changing spacing trigger logic to minimise unnecessary action assessments. It is set that when the spacing between the following vehicle in the target lane and the lane-changing vehicle exceeds D_{ig} , no collaboration strategy needs to be executed during the lane change.

$$D_{ig} = V_{max} T_c + D_{safe} \tag{17}$$

If the current spacing between the vehicle’s position and the following vehicle in the target lane does not meet the lane-changing requirements, the following vehicle in the target lane should actively decelerate to increase the gap in response to the lane-changing request upon receiving the lane-changing signal. When the lane-changing vehicle determines that the gap is sufficient for a safe lane change, it begins the lane-changing operation. If the lane-changing vehicle $C_{i, j}$ starts the lane-changing operation at time t_0 , and T_c is the time when the lane change is completed. Meanwhile, the following vehicle $C_{i+1, j+1}$ in the target lane remains in its lane and does not undergo longitudinal displacement. To ensure the safe lane change of $C_{i, j}$, it is necessary to prevent collisions between $C_{i+1, j+1}$ and $C_{i, j}$ during the lane-changing process, with $\varepsilon(t)$ representing the angle between the centreline of the lane-changing vehicle and the longitudinal direction of the road at time t .

$$\tan[\varepsilon(t)] = \frac{\partial Y_{i,j}(t)}{\partial X_{i,j}(t)} = \frac{\frac{\partial Y_{i,j}(t)}{\partial(t)}}{\frac{\partial X_{i,j}(t)}{\partial(t)}} = \frac{Vy_{i,j}(t)}{Vx_{i,j}(t)} \tag{18}$$

Assuming that M is the critical collision point between $C_{i+1, j+1}$ and $C_{i, j}$, through research and analysis, it is found that $C_{i+1, j+1}$ and $C_{i, j}$ are prone to oblique collisions before reaching point M. The critical state is illustrated in Figure 6.

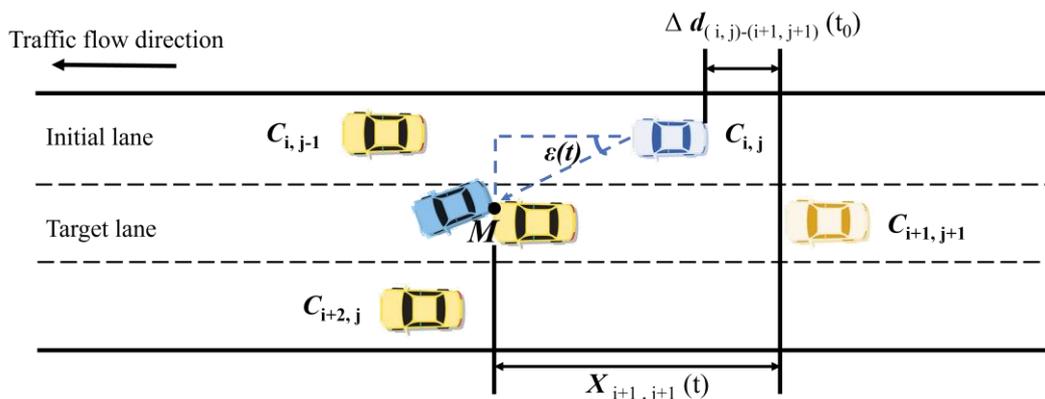


Figure 6 – Schematic diagram of critical collision between $C_{i, j}$ and $C_{i+1, j+1}$

The condition for avoiding collisions between $C_{i+1, j+1}$ and $C_{i, j}$ is shown in Equation 19.

$$X_{i+1, j+1}(t) \leq X_{i, j}(t) - L_A \cos[\varepsilon(t)] - \Delta d_{(i, j)-(i+1, j+1)}(t) \quad t \in (t_0, t_c) \tag{19}$$

When $t \geq t_c$, $\cos[\varepsilon(t)]$ reaches its maximum value, and Equation 19 can be simplified to Equation 20.

$$X_{i+1, j+1}(t) \leq X_{i, j}(t) - L_A - \Delta d_{(i, j)-(i+1, j+1)}(t) \quad t \in [t_c, T_c) \tag{20}$$

Let $D_B(t)$ represent the required distance to be maintained between $C_{i+1, j+1}$ and $C_{i, j}$. In theory, during the lane-changing process of $C_{i, j}$, when $t \geq t_c$, as long as $D_B > 0$ is ensured, collisions will not occur. However, the safety distance calculated under this scenario is slightly smaller, as it does not consider the need for the vehicles to maintain a car-following distance even after the lane change is completed. Therefore, it is necessary to consider the safe car-following distance between $C_{i+1, j+1}$ and $C_{i, j}$ during the lane-changing process, i.e. $D_B > D_{safe}$.

$$D_B(t) = X_{i, j}(t) - X_{i+1, j+1}(t) - L_A > D_{safe} \quad t \in [t_c, T_c) \tag{21}$$

$$X_{i, j}(t) - X_{i+1, j+1}(t) > D_{safe} + L_A \quad t \in [t_c, T_c) \tag{22}$$

In a vehicle-road cooperative environment, dynamic and real-time information interaction provides precise and timely traffic information for vehicle operations. If the current conditions permit, vehicle $C_{i, j}$ can actively accelerate to avoid rear-end collisions while enhancing lane-changing efficiency. At the same time, the following vehicle in the target lane actively decelerates to increase the gap. Consequently, the speeds of the lane-changing vehicle and the following vehicle in the target lane at time t are shown in Equations 23 and 24.

$$V_{i, j}(t) = V_{i, j}(t_0) + \int_0^t a_{i, j}(\tau) d\tau \tag{23}$$

$$V_{i+1, j+1}(t) = V_{i+1, j+1}(t_0) + \int_0^t a_{i+1, j+1}(\tau) d\tau \tag{24}$$

Then, the minimum value of $D_B(t)$ required to ensure no collision occurs between $C_{i, j}$ and $C_{i+1, j+1}$ represents the minimum safe distance D_b that needs to be maintained between the lane-changing vehicle and the following vehicle in the target lane.

$$D_b = \max \left\{ V_{i+1, j+1}(t) t_d + \frac{V_{i+1, j+1}(t)^2}{2a_m} + d_{\min} + L_A \right\} \tag{25}$$

3. SIMULATION EXPERIMENTS

3.1 Simulation settings

This paper selected two consecutive signal-controlled intersections within Fuzhou city as the research objects. The primary investigation direction was from east to west along Jinshan Avenue, with Intersection 1 being the intersection of Jinshan Avenue and Jinzhou Road, and Intersection 2 being the intersection of Jinshan Avenue and Jianxin Road, as shown in Figure 7. To verify the control effect of the model proposed in this article, VISUAL C++ was used to call VISSIM COM, and the proposed model was imported into simulation experiments. Simulation experiment schemes were designed for simulation scenarios with and without lane change decision models. Two factors, traffic flow and lane-changing ratio r , are selected to conduct orthogonal experiments on the scenarios with and without lane-changing decision-making models. Based on the results of the orthogonal experiments, the experimental combination with the best optimisation effect after implementing the lane-changing decision-making model is selected. Partial correlation analysis is used to change the lane-changing starting point position for simulation, and the impact of different lane-changing starting point positions k on traffic flow is analysed. The traffic flow combination settings obtained from the research are shown in Table 2, the orthogonal experimental scheme design is shown in Table 3, and the default simulation parameter settings are shown in Table 4.

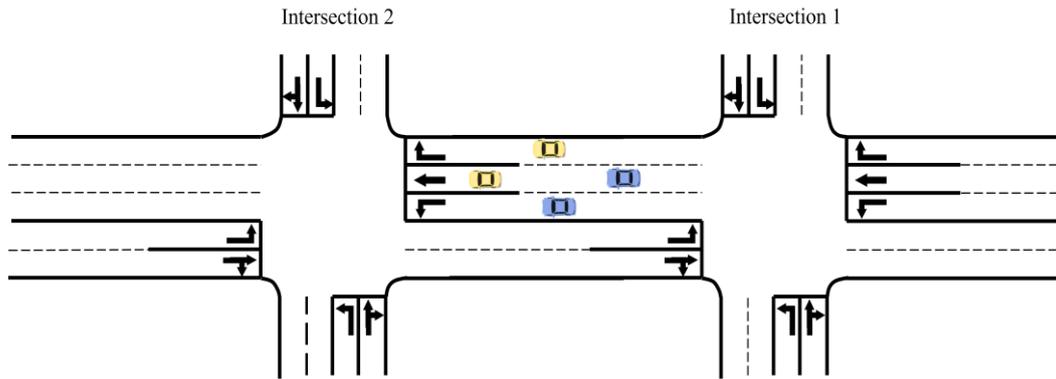


Figure 7 – Schematic diagram of the intersection

Table 2 – Different combinations of converging flows at each inlet road at peripheral intersections

Intersection number	1			2	
	east	south	north	south	north
Import road					
Traffic flow Q1(veh/h)	828	104	86	88	76
Traffic flow Q2(veh/h)	1147	161	92	127	98
Traffic flow Q3(veh/h)	1369	175	136	158	87
Traffic flow Q4(veh/h)	1643	245	247	182	226

Table 3 – Orthogonal experimental design table

Experimental plan	Traffic flow Q(veh/h)	Lane-changing ratio r
1	Q1	0.2
2	Q1	0.3
3	Q1	0.4
4	Q2	0.2
5	Q2	0.3
6	Q2	0.4
7	Q3	0.2
8	Q3	0.3
9	Q3	0.4
10	Q4	0.2
11	Q4	0.3
12	Q4	0.4

Table 3 – Simulation parameter setting

Parameter	Explanation	Valid values
V_{max} (km/h)	Maximum speed	70
V_{exp} (km/h)	Expected lane-changing speed	53
T (s)	Safe time interval	1.5
d_{min} (m)	Static safety distance	7
a_{max} (m/s ²)	Maximum acceleration	4
b (m/s ²)	Comfortable deceleration	2
K_j (pcu/km)	Jam density	124
W (m)	Lane width	3.5

3.2 Analysis of results

Adaptability analysis

The travel time index, delay index and number of stops index under various traffic flows and lane-changing ratios are presented separately. By comparing the optimisation effects under different traffic conditions, *Figures 8–10* are provided for further analysis.

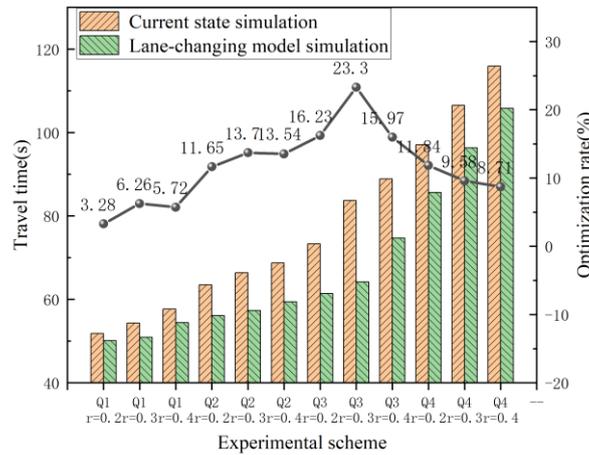


Figure 8 – Optimisation of travel time

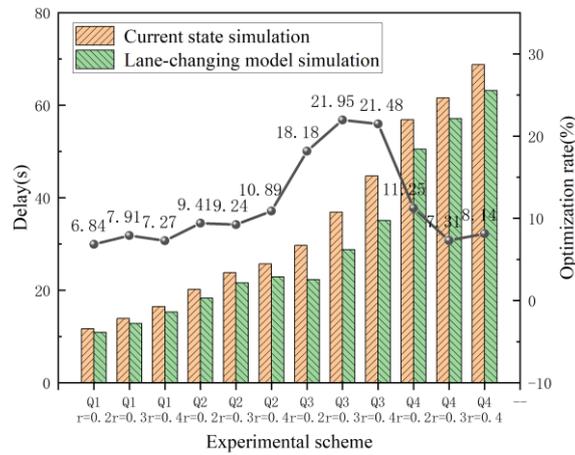


Figure 9 – Optimisation of delay

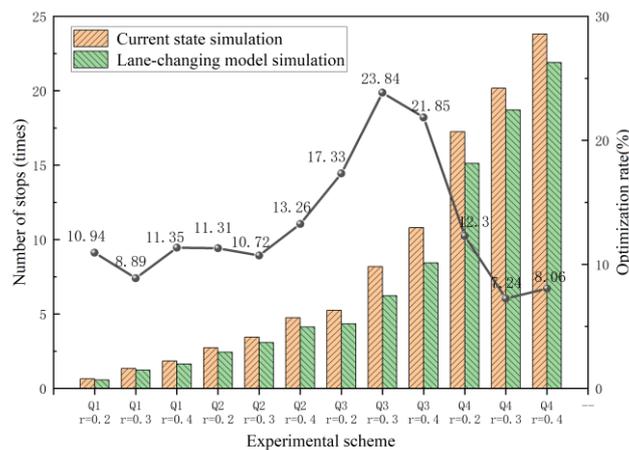


Figure 10 – Optimisation of the number of stops

After implementing the lane-changing model, by adjusting the sequence of lane-changing and avoiding conflicts among vehicles, while planning the cooperation and coordination of surrounding vehicles for lane-changing, it can effectively reduce the indicators of travel time, delay and the number of stops. When the traffic flow is Q1 and Q2, the input flow is relatively low, with high driving freedom for vehicles and large spacing between vehicles. Under such traffic conditions, lane changing and stopping are less frequent, and there is not much difference in the three indicators before and after the implementation of the lane-changing decision-making model. As the traffic flow continues to increase, due to the real-time assessment of lane changing feasibility through the different running statuses of vehicles under the condition of vehicle-to-infrastructure, vehicles can be guided to prepare for lane changing upstream of the intersection, avoiding the need to stop and wait for lane changing or even give up lane changing when vehicles approach the solid lane divider. Compared with the current situation, this increases opportunities for lane changing, and the lane-changing process becomes more efficient through the adjustment of lane-changing rules, demonstrating the advantages of the model. However, when the input flow continues to increase until it reaches a critical value, vehicles are constrained by the traffic environment, making lane changing difficult and safety conditions difficult to meet, resulting in a relatively weak optimisation effect of the model.

Based on the optimisation effects of the above simulation experiment schemes, it can be concluded that implementing the lane-changing decision-making model achieves the best optimisation effect when the input flow is Q3 and the lane-changing ratio is 0.3. Within 0~3,600 seconds, the travel time decreased by 23.30%, the average delay decreased by 21.95%, and the number of stops decreased by 23.84%. This indicates that under the same simulation duration, the proposed model can achieve safer lane-changing behaviours, which is of great significance for vehicles with mandatory lane-changing needs.

Analysis of lane-changing starting position

When the input flow is Q3 and the lane-changing ratio is 0.3, the number of conflicts among lane-changing vehicles within the influence range LSC of lane-changing vehicles at different lane-changing starting positions k within the entrance lane is counted, as shown in Figure 11.

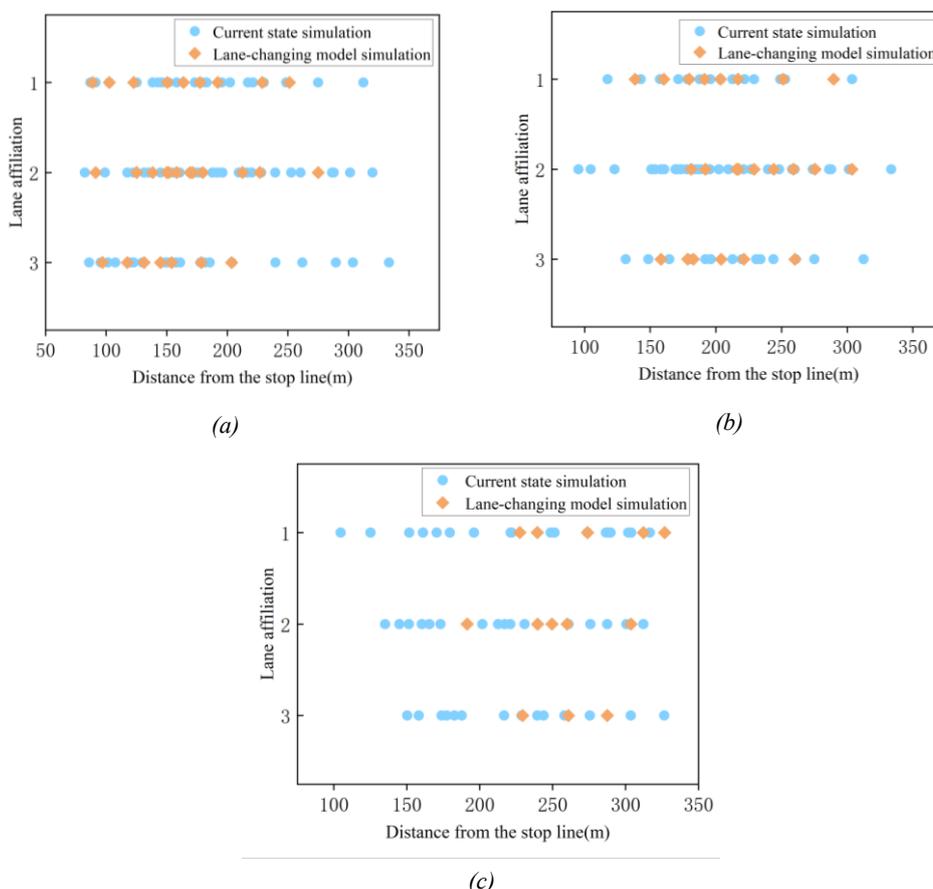


Figure 11 – Distribution of lane-changing vehicle conflicts within the impact range of lane-changing vehicles at different lane-changing starting positions: a) $k=100$ m; b) $k=200$ m; c) $k=300$ m

As the distance from the stop line decreases, the environment surrounding the lane-changing vehicle becomes more complex, and there are more intersections and conflicts with the trajectories of surrounding lane-changing vehicles. Regardless of whether the lane-changing model is implemented, the number of lane-changing conflicts within the influence range LSC of the lane-changing vehicle is relatively high. When vehicles are guided to change lanes upstream of the intersection, the number of vehicles subsequently concentrating on lane changing downstream is relatively reduced, thereby reducing the number of weaving conflicts in downstream traffic flow. Especially when the lane-changing starting position is far from the stop line, the optimisation effect of the lane-changing model under vehicle-to-infrastructure conditions is significant, which can significantly reduce the number of lane-changing conflicts and smoothly execute the lane-changing process, thus effectively improving road operation status and reducing delays.

Analysis of lane-changing decision-making process

Based on the above analysis, the vehicle-road collaborative lane-changing decision-making model can improve the efficiency of vehicle lane changing and enhance the operational status of road traffic within a relatively high range of vehicle density. When a vehicle has a lane-changing need, its lane-changing influence range and conflict count are pre-assessed. Based on the priority-based lane-changing rules, conflicting lane-changing vehicles within the influence range are provided with more optimised speed and lane-changing recommendations, further increasing the average speed of lane-changing vehicles and thus improving the traffic environment of the road segment. Taking the lane-changing process in different scenarios obtained from this experimental scenario as an example, lane-changing data samples under different decision combinations are collected to conduct a detailed analysis of the vehicle lane-changing decision-making process and examine the role played by the model. This includes three lane-changing decision-making scenarios:

- 1) There are no other lane-changing vehicles within the lane-changing influence range.
- 2) There are other lane-changing vehicles within the lane-changing influence range, but their lane-changing trajectories do not cause conflicts.
- 3) There are conflicting vehicles with intersecting trajectories within the lane-changing influence range. Specific data samples are presented in *Table 5*.

Table 5 – Lane-changing data samples under different decision combinations

Number	id	$\Delta L(m)$	V(km/h)	LSC	Pr	Conflict times-id	Lang-changing time(s)
1	1774	251.93	40.03	28.21	—	/	3.15
2	1764	250.75	36.25	30.53	—	/	3.00
3	1722	250.31	32.60	29.20	—	/	3.81
4	1769	250.10	36.14	29.43	—	/	3.10
5	1741	252.88	33.04	22.01	2.11	1-1788	4.04
6	1788	251.01	35.99	30.56	1.77	1-1741	/
7	1706	258.73	42.27	29.30	—	/	3.22
8	1710	255.66	30.38	30.78	—	/	3.62
9	1791	250.05	39.21	24.90	—	/	3.60
10	1750	250.02	43.48	28.93	—	/	3.50
11	1756	255.97	43.57	28.90	—	/	3.21
12	1761	254.53	30.24	30.77	—	/	3.51
13	1745	250.12	34.53	30.70	—	/	3.30
14	1753	255.14	36.08	30.55	—	/	3.31
15	1783	252.33	24.72	27.11	—	/	3.51
16	1687	250.57	47.54	27.41	—	/	2.54
17	1692	256.98	47.76	27.31	—	/	3.84

Number	id	$\Delta L(m)$	V(km/h)	LSC	Pr	Conflict times-id	Lang-changing time(s)
18	1839	251.56	37.16	30.41	—	/	2.54
19	1837	256.81	35.43	30.37	1.48	1-1707	3.21
20	1707	252.48	27.00	24.24	1.07	1-1837	/
21	1766	258.03	34.95	30.67	—	/	2.46
22	1716	250.10	40.77	29.70	—	/	2.63
23	1730	251.79	43.22	27.94	—	/	2.91
24	1791	257.70	43.11	29.05	—	/	3.84
25	1811	251.52	36.69	30.47	—	/	2.82
26	1846	256.69	30.48	30.50	1.36	1-1782	/
27	1782	253.49	28.82	25.78	1.43	1-1846	2.91
28	1862	261.84	36.27	30.53	—	/	2.93
29	1813	258.40	29.53	30.73	—	/	3.53
30	1896	252.33	38.90	30.11	—	/	2.73

An analysis of the lane-changing vehicle data within the lane-changing decision-making scenarios reveals the following. In scenario (1), the model determines that there are no other lane-changing vehicles within its influence range, and the vehicle can execute lane-changing directly after satisfying the safe lane-changing distance. In scenario (2), the model determines that two lane-changing vehicles are within each other's lane-changing influence range, but their lane-changing trajectories do not conflict. Therefore, there is no need to calculate the lane-changing priority, and each vehicle can execute lane changing after satisfying the safety conditions. However, in scenario (3), the model determines that two lane-changing vehicles are within each other's lane-changing influence range and their lane-changing trajectories conflict, posing a safety hazard. In this case, the priority of each lane-changing vehicle needs to be calculated. The model will process the vehicle with higher priority to execute lane changing. Compared to the current situation without implementing a lane-changing decision-making model, the lane-changing time is significantly shortened, effectively avoiding the occurrence of lane-changing conflicts, and meeting practical application requirements.

4. CONCLUSIONS

This paper proposes a vehicle-road collaborative lane-changing decision model based on potential conflict analysis, aiming to address the mandatory lane-changing needs of vehicles in high-density traffic scenarios. The model quantifies the influencing factors during the lane-changing process and calculates the lane-changing priority, ensuring that vehicles with less disturbance to the entrance lane have a higher priority for lane-changing, thereby reducing conflicts. At the same time, the model modifies the safe distance between the lane-changing vehicle and the following vehicle in the target lane to ensure lane-changing safety.

The results indicate that with the optimal optimisation effect, the travel time decreases by 23.30%, delays decrease by 21.95%, and the number of stops decreases by 23.84%. When the distance from the lane-changing starting point to the stop line is 300 m compared to 100 m, the overall travel time, delays and stops at the entrance lane decrease by 21.18%, 31.63% and 31.13%, respectively, and the average delay for lane-changing vehicles decreases by 24.88%. The model fully considers potential conflicts and lane-changing priority, making lane-changing decisions more precise and providing a new solution for vehicle-road collaborative lane-changing decisions.

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